# Decentralized Exchange

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# Contemporaneous Markets

- **Some assets increasingly (or exclusively) traded OTC**
  - Real estate, bonds, stocks, loans, FX
- **Many (and more) types of OTC exchanges exist**
  - Liquidity pools, dealer networks, third and fourth markets
  - U.S. stock trading shifted OTC in the past ten years

    U.S. OTC bond market debt nearly tripled over the last two decades and reached approximately $3.7 trillion

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  - New online marketplaces: TradeWeb.com, BondDesk.com, MarketAxess.com

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    MiFID reform in 2007 created over 200 trading venues
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- The potential for market decentralization to improve efficiency?

# Modeling of Decentralized Markets

- **Search and matching approach** (e.g., Duffie, Garleanu and Pedersen (2005, 2008), Weill (2008), Vayanos and Weill (2008), Weill (2008), Duffie, Malamud and Manso (2009, 2011), Lagos and Rocheteau (2009), Lagos, Rocheteau and Weill (2011), Alfonso and Lagos (2012))
  - (1) Large markets
  - (2) Bilateral exchanges
- **Networks approach** (e.g., Kranton and Minehart (2001), Blume, Easley, Kleinberg and Tardos (2009), Nava (2011), Babus and Kondor (2012), Condorelli and Galeotti (2012), Elliott (2012), Fainmesser (2012))
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- **In both types of models**
  - (3) Bargaining or take-it-or-leave-it

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  - (2) Bilateral exchanges $\leftrightarrow$ Arbitrary set of exchanges
- **In both types of models**
  - (3) Bargaining or take-it-or-leave-it $\leftrightarrow$ Double auction

# Model: Market

- **Assets:** $K$ risky assets, indexed by $k$, with payoffs $\mathcal{N}(d, \Sigma)$
- **Traders:** $I$ (classes of) agents, indexed by $i$

$$U(q_i) = d \cdot \left(q_i^0 + q_i\right) - \frac{\alpha_i}{2}\left(q_i^0 + q_i\right) \cdot \Sigma\left(q_i^0 + q_i\right)$$

- **Market:** Agents trade assets in $N$ exchanges, indexed by $n$
  - Each exchange $n$ identified with a subset of agents $I(n) \subset I$ and $K(n) \subset K$ assets
  - A hypergraph $\{(I, K), \{I(n), K(n)\}_n\}$ represents the market structure
  - Hypergraphs encompass centralized markets, arbitrary networks

# Model: Market Clearing

- **Uniform-price double auction:**
  - **Strategies:** Trader $i$ submits a (net) demand schedule

$$q_i(p_{N(i)}) : \mathbb{R}^{N(i)} \to \mathbb{R}^{N(i)}$$

  - **Market clearing:** In exchange n,

$$\sum_{i \in I(n)} q_{i,n}\left(p_n, p_{N(i)\setminus n}\right) = 0$$

- **Linear Nash Equilibria** (cf. the work of Wilson (1979); Kyle (1989); Vayanos (1999); Vives (2011))
- All traders are strategic

# Individual Trader Optimization

- **A useful representation:**
  - Treat an asset traded at each exchange as a different asset

    e.g., a market with $K$ assets in each of $N$ exchanges $\leftrightarrow$ A market with $N \times K$ assets distributed $\mathcal{N}(d, \mathcal{V})$, $\mathcal{V} \in \mathbb{R}^{(N \times K) \times (N \times K)}$
  - Trader $i$ in exchanges $N(i)$ maximizes

$$U(q_i) = d \cdot \left(q_i^0 + q_i\right) - \frac{\alpha_i}{2}\left(q_i^0 + q_i\right) \cdot \mathcal{V}_{N(i)}\left(q_i^0 + q_i\right)$$

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- **Optimization by trader $i$:**
  - F.O.C. (trader $i$ in exchanges $N(i)$)

$$d - \alpha_i \mathcal{V}_{N(i)}\left(q_i^0 + q_i\right) = p_{N(i)} + \Lambda_i q_i$$

  - hence, trader $i$ submits

$$q_i(p_{N(i)}, \Lambda_i) \;=\; (\alpha_i \mathcal{V}_{N(i)} + \Lambda_i)^{-1}(d - p_{N(i)} - \alpha_i \mathcal{V}_{N(i)} q_i^0)$$

# Aggregation: Lifting

- Any symmetric matrix $X$ can be decomposed into a block form

$$X = \begin{pmatrix} X_{11} & X_{12} \\ X_{12}^T & X_{22} \end{pmatrix}$$

- Lifted matrix

$$\bar{X}_{11} = \begin{pmatrix} X_{11} & 0 \\ 0 & 0 \end{pmatrix}$$

- Similarly

$$(\bar{X}_{11})^{-1} = \begin{pmatrix} X_{11}^{-1} & 0 \\ 0 & 0 \end{pmatrix}$$

# Equilibrium Characterization

**Theorem 1** Profile $\{q_i(p_{N(i)})\}_i$ is a Linear Nash Equilibrium in a decentralized market if, and only if,

**(i)** each trader $i$ submits the schedule

$$q_i(p_{N(i)}, \Lambda_i) \;=\; (\alpha_i \mathcal{V}_{N(i)} + \Lambda_i)^{-1}(d - p_{N(i)} - \alpha_i \mathcal{V}_{N(i)} q_i^0)\,,$$

**(ii)** price impact of trader $i$ is characterized by

$$\Lambda_i = \left( \left( B - (\alpha_i \bar{\mathcal{V}}_{N(i)} + \bar{\Lambda}_i)^{-1} \right)^{-1} \right)_{N(i)}\,,$$

where

$$B \;=\; \sum_j (\alpha_j \bar{\mathcal{V}}_{N(j)} + \bar{\Lambda}_j(B))^{-1}\,.$$

- $B$ is a market-wide liquidity

# Preview

What changes equilibrium and welfare in decentralized markets?

- **Centralized markets:** $N(i) = \{(I, K)\}$ and $\mathcal{V}_{N(i)} = \Sigma$, for all $i$.

  For each $i$,

$$\Lambda_i^{\mathrm{CM}} = \beta_i \Sigma.$$

  Equilibrium implications:

  - Order reduction $\sim \Sigma$
  - Equilibrium utility $\sim$ price impact $\Lambda_i$

- **Decentralized markets:** Generically,

$$\Lambda_i \not\sim \mathcal{V}_{N(i)}.$$

# Equilibrium: Price Impacts

**Proposition 1** Price impact $\Lambda_i$ is strictly positive for each $i$, generically in $\{\{\alpha_i\}_i, \Sigma, \{N(i)\}_i\}$.

- Even in large decentralized markets, traders have non-negligible price impact

# Equilibrium: Demand Schedules

**Theorem 3** In equilibrium, the price vector for all exchanges is

$$p \;=\; A - B^{-1}\sum_{j}\left(\alpha_j \bar{\mathcal{V}}_{N(j)} + \bar{\Lambda}_j(B)\right)^{-1}\alpha_i \bar{\mathcal{V}}_{N(j)} q_j^0,$$

the trade vector of agent $i$ is

$$q_i = \left(\alpha_i \mathcal{V}_{N(i)} + \Lambda_i\right)^{-1}\left(\Pi_{N(i)}\mathbf{Q} - \alpha_i \mathcal{V}_{N(i)} q_i^0\right),$$

where

$$\mathbf{Q} \;\equiv\; B^{-1}\sum_{j}\left(\alpha_j \bar{\mathcal{V}}_{N(j)} + \bar{\Lambda}_j(B)\right)^{-1}\alpha_j \bar{\mathcal{V}}_{N(j)} q_j^0.$$

- Prices, allocations and price impacts depend on assets and preferences in all exchanges
- Contemporaneous effects of shocks and information aggregation, across exchanges

# Equilibrium Prices

- **The same asset can trade at different prices**

  **Definition** Given two exchanges $n$ and $n'$ and an asset $k$, an equivalence path connecting these two exchanges with respect to asset $k$ is a sequence of exchanges $\{n_l\}_l$ and a sequence of agent classes $\{a_l\}_{l=1}^{L-1}$ such that $n_1 = n$, $n_L = n'$, $a_l \in I(n_l) \cap I(n_{l+1})$, and $k \in K(n_l)$ for all $l$. Two equivalence paths are disjoint if the corresponding sets of agents are disjoint. Two disjoint equivalence paths form an equivalence loop.

  **Proposition** In equilibrium, asset $k$ is traded at the same price at two exchanges $n, n'$ if, and (generically) only if, there exists an equivalence loop connecting these two exchanges w.r.t. asset $k$.

  - The necessary and sufficient conditions for price discrimination in decentralized markets

- **Decentralized-market CAPM holds**

  - Prices and portfolios depend on $\{N(i)\}_i$

# Comparative Statics: Equilibrium Liquidity

- **Price Impact and Market Decentralization:**

  **Theorem** If $N'(i) \supseteq N(i)$ for all $i$ then equilibrium price impact is (weakly) lower in all exchanges.

  - Introducing new exchanges improves liquidity in existing exchanges

  **Corollary** Breaking up an exchange $n$ into groups of traders $I(n_1)$ and $I(n_2)$ such that $I(n_1) \cup I(n_2) = I(n)$ lowers liquidity.

- Remarks:
  - Price impact effects extend to all exchanges
  - Price impact is monotone w.r.t. both the set inclusion of traders (for fixed $\{K(n)\}_n$) and assets (for fixed sets of agents $\{I(n)\}_n$).

# Standardization of Assets

- **When is price impact the lowest?**

  **Definition** For two exchanges $n, n'$, an equivalence loop w.r.t. assets $K(n) \cup K(n')$ comprises an equivalence loop with respect to each asset $k \in K(n) \cup K(n')$.

  **Proposition** If a market with $K$ assets and $I$ agents, $\{q_i(.,\Lambda_i), \Lambda_i\}_i$ coincides with those in a centralized market with the same agents and assets if, and only if, for any two exchanges $n, n'$, there exists an equivalence loop w.r.t. each asset $k \in K(n) \cup K(n')$.

  - Liquidity essentially as in centralized markets $\rightarrow$ Standardization

- Empirically:
  - **Assets traded OTC** are homogenous or bespoke
  - **OTC market structure** for homogeneous assets: core-periphery (e.g., Bech and Atalay (2009); Cocco, Gomes and Martins (2009); Craig and Peter (2010); Afonso, Kovner, and Schoar (2012)); Li and Schurhoff (2012))
    for bespoke products: intermediation (e.g., Financial Stability Board (2010); TradeWeb.com)

# Standardization and Equilibrium Hypergraph

- **Equilibrium hypergraph:**

  Remove equivalence loops $\rightarrow$ Regularized market

  **Theorem** There is a one-to-one correspondence between equilibria in any market and its associated regularized market.

  **Proposition** The hypergraph of a regularized market is such that, for each $n, n'$ if $I(n) \cap I(n') \neq \varnothing$, then $K(n) \cap K(n') = \varnothing$ for all $n' \neq n$ or $|I(n) \cap I(n')| = 1$.

  - Market structures of a regularized hypergraph is a forest
  - Differences in standardization $\rightarrow$ Dichotomy of assets and market structures

# Comparative Statics: Equilibrium Utility

In decentralized markets: No general link between utility and liquidity.

Recall: In decentralized markets, $\Lambda_i \not\sim \mathcal{V}_{N(i)}$.

- **Lemma** Equilibrium utility of each agent = Compensation for Aggregate Risk + Loss from Idiosyncratic Risk+Covariance.
- **Proposition:** Indirect utility decreasing in $\Lambda_i$ if, and only if, $\Lambda_i \sim \mathcal{V}_{N(i)}$.
- In decentralized markets, agents with larger $\Lambda_i$ may have higher $U_i$
  - Utility advantage from both aggregate and idiosyncratic risk exposure
  - Different forms of intermediation (e.g., dealers, brokers, specialists)
- In decentralized markets, welfare can be higher in the Pareto sense.

# Conclusion

- Games on hypergraphs
- Endogenizing the set of exchanges (exchange creation) should not be separated from asset structure

# Thank You