

Choosing Stress Scenarios for Systemic Risk Through Dimension Reduction

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October 2015

Conference on Stress Testing and Macro-prudential Regulation: A
Trans-Atlantic Assessment

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Outline

- 1 Introduction
- 2 Gaps
- 3 Measurement
- 4 Variables and Factors
- 5 Main Result
- 6 Empirical Analysis
- 7 SIR pitfalls

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 3. Test BHC’s capital adequacy in the scenarios.
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- ▶ **Question:** What regulatory scenarios should we choose to achieve our goals?
 1. Which variables should we stress?
 2. In what directions?
 3. By how much should variables be stressed?
 4. How should idiosyncratic risks be accounted for?

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- ▶ Banks exposures are not formally used to pick the scenarios.

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4. Choosing scenarios for systemic risk

- ▶ Regulatory scenarios are not chosen to satisfy an explicit systemic risk objective.
- ▶ Regulatory scenarios do not use banks exposures to shared vulnerabilities in scenario design.
- ▶ Bank-tailored scenarios do not focus on banks' shared vulnerabilities.

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 - ▶ Scenario choice accounts for idiosyncratic risk.

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- ▶ Roadmap.
- ▶ Systemic Risk Measurement.
 - ▶ Methodology to identify F_1 .
 - ▶ Empirical Examples.

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$\psi = \text{Prob}(SAD_T(Cl, \Omega, X_T) > \zeta)$ is a measure of systemic risk

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► **SIR**:: Under approp regularity condns the principal components of

$$\Sigma_X^{-1} \Sigma_{E[X|SAD]}$$

1. Span the same spaces as F_1 .
2. Are ordered by their ability to explain systemic risk SAD .
3. F_1 can be identified even if SAD is nonlinear in F_1 .

Variable Selection

- ▶ SIR may in theory require enormous matrices if dimensionality of X is high because it depends on Principal Components of

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 - ▶ **Solution:** Choose $x \in X$ via Correlation Pursuit (COP) (Zhong et al 2012).
 - ▶ **Methodology:** Uses hypotheses tests to identify which variables are best for identifying factors to use in *SIR*.

Choosing a Stress Scenario

- ▶ Estimate linear statistical relation between all variables X and the factors (F_1).

$$X = \alpha + F_1\theta + \epsilon$$

- ▶ Stress-scenario formation steps.

1. Choose F_1 realization.
2. Set $X = E(X|F_1) = \alpha + F_1\theta$
3. SAD in the stress-scenario is $SAD[\Omega(E(X|F_1))]$.

- ▶ **Goal:** Choose the most plausible F_1 for a scenario such that if banks are well capitalized for the scenario, then systemic risk is low.

SAD Approximation and Main Result

- ▶ Linearize banks exposure $\omega_j(X) = X\omega_j$.
- ▶ Taylor expand SAD in $X\omega_j + Cl_j r_f$:

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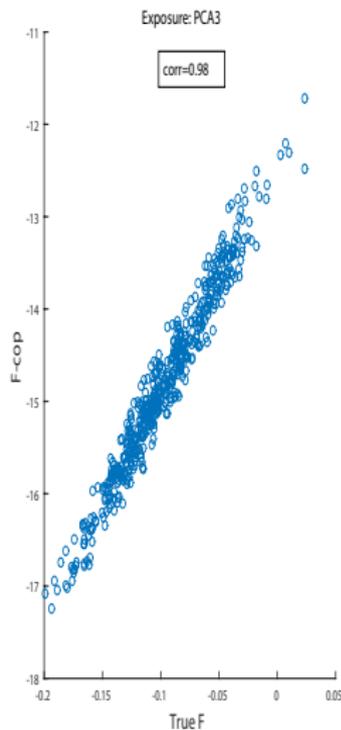
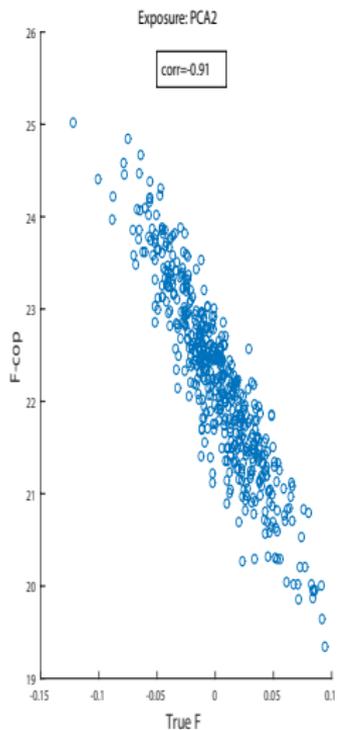
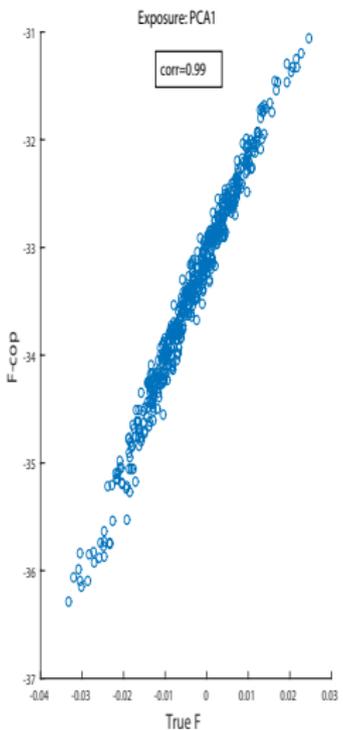
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- ▶ Estimate $H(\cdot)$, the CDF of random variable $F_1\Theta + E$.
- ▶ Find CIE^* such that $\text{Prob}(SAD \geq \zeta) \leq \psi$.
- ▶ Choose F_1^* such that $F_1^*\Theta = -CIE^* - \alpha + \sum_j D_j$.
- ▶ **Main Result:** If stress scenario is $X = \alpha + F_1^*\theta$, equivalent capital injected will be approx CIE^* , and $\text{Prob}(SAD \leq \zeta) \approx \leq \psi$.

Can SIR/COP detect the right factors-I ?

- ▶ 10 countries yield curve changes (AU,CA,CHF,GE,JP,NO,NZ,SWE,UK,US) over a 2-yr horizon simulated based on a dynamic macro term-structure model [J. Wright (2011)].
- ▶ Identified first 3 principal components (PC) of all yield curve changes.
- ▶ Created bank portfolio that loaded on PC 1,2,or 3.
- ▶ X variables were zero coupon returns over 2 years, and exponentially smoothed quarterly GDP growth and inflation in all 10 countries.
- ▶ Using a different data-sample from same DGP, tested if SIR/COP identifies the PC factors banks loaded on.
- ▶ It did.

Can SIR/COP detect the right factors-II ?



Does SIR/COP create the right stress scenarios-I

- ▶ Want SAD to be correlated with F_1 .
- ▶ Want SAD due to banks losses in stress scenarios based on F_1 , to be correlated with true *SAD*.
- ▶ Setting:
 - ▶ 6 Banks.
 - ▶ Invest in zero coupon bonds of 8 countries (AU,CA,GE,JP,SWE,CHF,GB,US), 83 variables.

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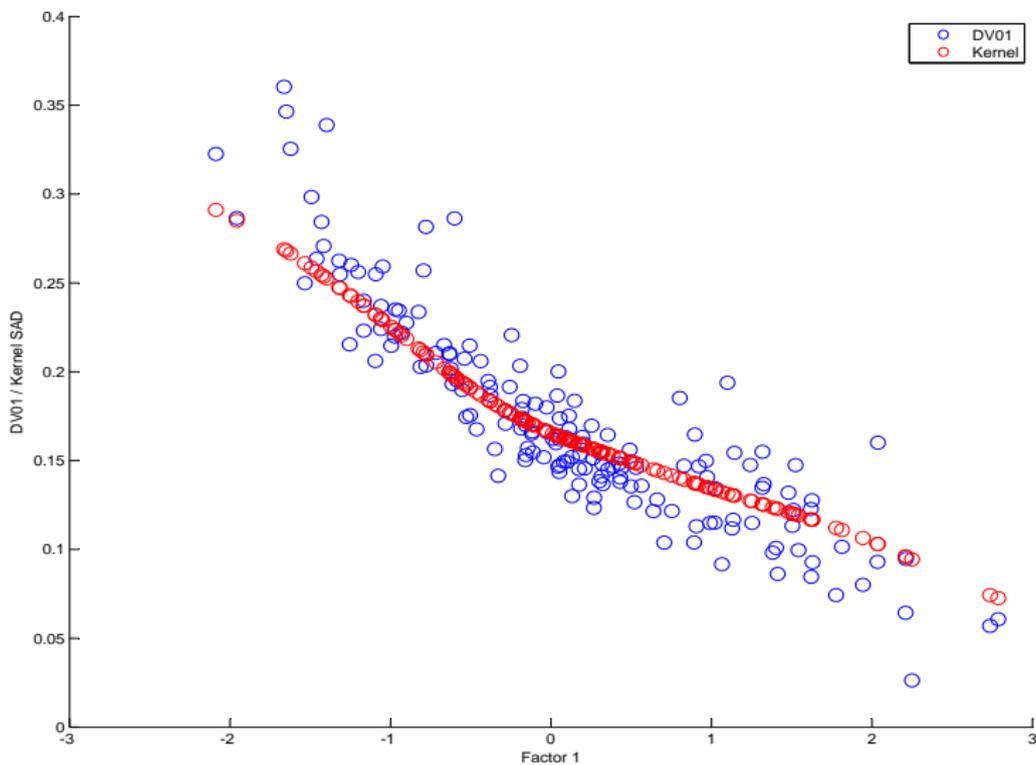
- ▶ Want SAD to be correlated with F_1 .
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- ▶ Setting:
 - ▶ 6 Banks.
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 - ▶ Maturities to 30 years.

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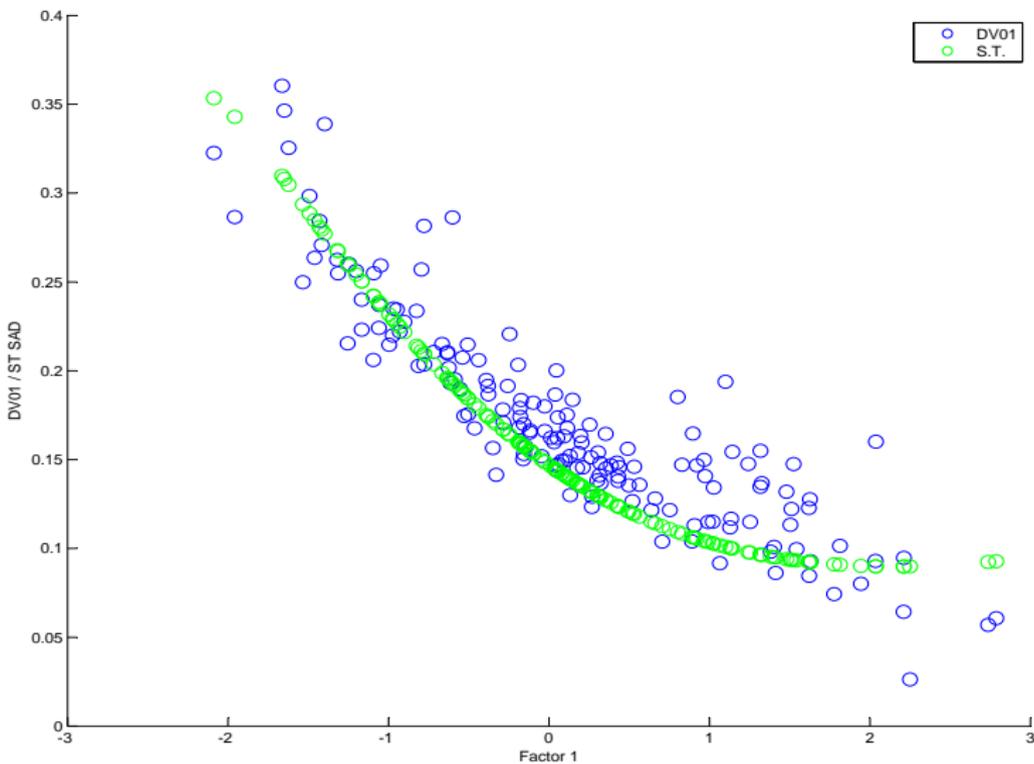
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 - ▶ Bond and FX returns are monthly.
 - ▶ Data from February 2000 to October 2013 = 165 observations.

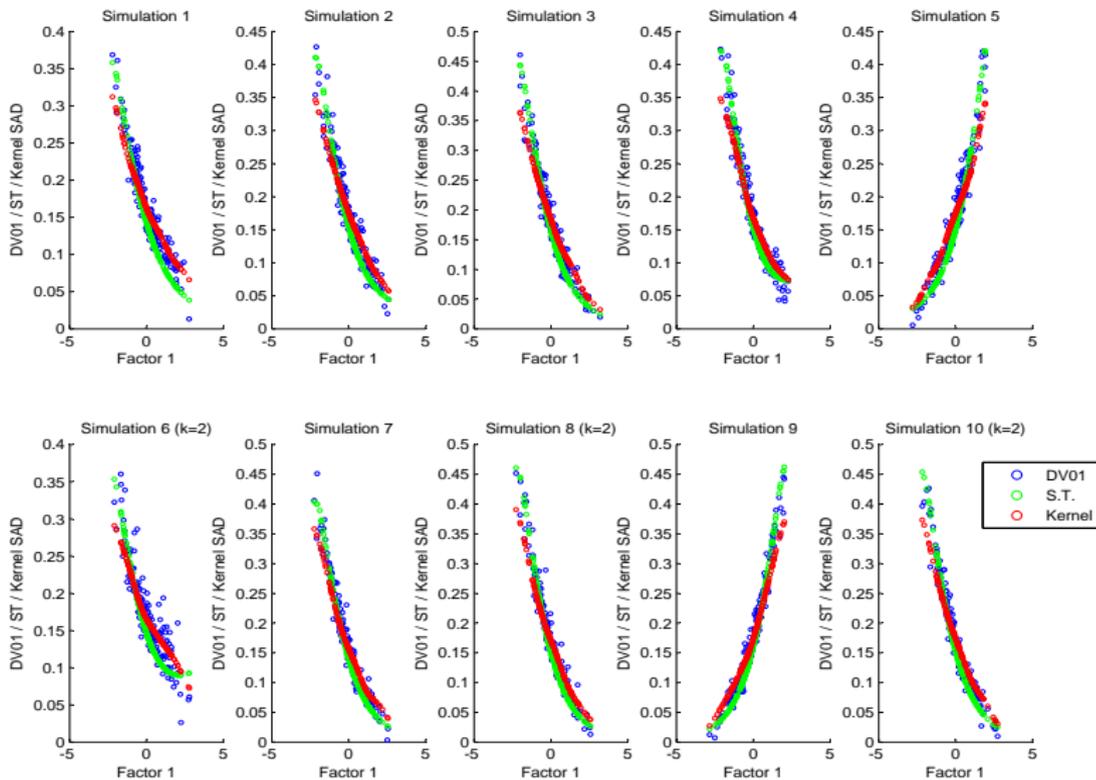
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 - ▶ Maturities to 30 years.
 - ▶ Bond return distn from historical simulation:
 - ▶ Bond and FX returns are monthly.
 - ▶ Data from February 2000 to October 2013 = 165 observations.
- ▶ Random portfolios:
 - ▶ Some with no FX risk.
 - ▶ Some with FX risk.
 - ▶ Portfolios differ in pricing approxns and generation methods too.

SAD(X) vs F_1 .

True $SAD(X)$ vs SAD based on losses in stress scenario.



$SAD(X)$ vs Kernel Reg and $SAD[\Omega(E(X|F_1))]$. Sim. 1-10. / No FX risk

Do stress-tests and capital injections based on ASAD achieve goal of low SAD with high probability

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Do stress-tests and capital injections based on ASAD achieve goal of low SAD with high probability

- ▶ No. Choosing the magnitude of F_1 based on the linear approximation of SAD (ASAD) guarantees ASAD is low with high probability.
- ▶ But, it does not guarantee SAD will be low with high probability.

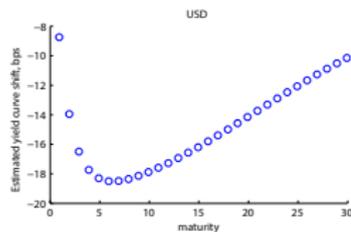
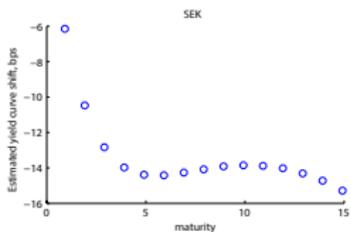
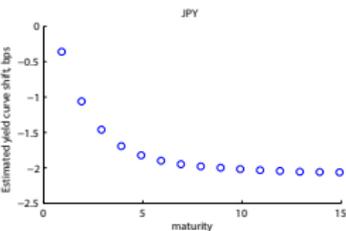
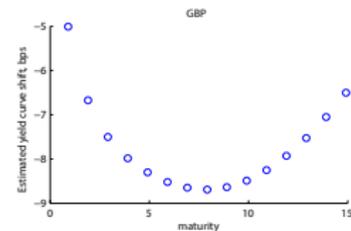
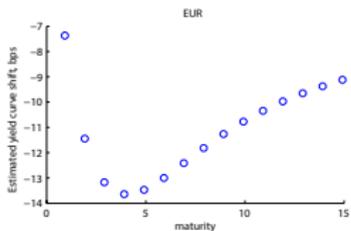
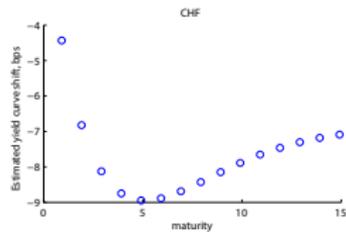
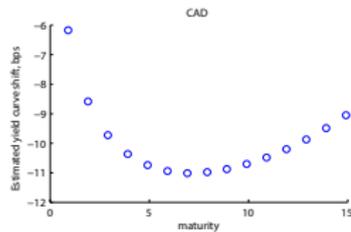
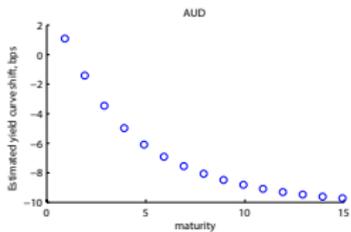
Do stress-tests and capital injections based on ASAD achieve goal of low SAD with high probability

- ▶ No. Choosing the magnitude of F_1 based on the linear approximation of SAD (ASAD) guarantees ASAD is low with high probability.
- ▶ But, it does not guarantee SAD will be low with high probability.
- ▶ Better to use ASAD to find directions to change F_1 , and then solve for magnitude of F_1 changes to satisfy systemic risk objectives.
- ▶ When multiple \tilde{F}_1 choices satisfy the objective, \tilde{F}_1 can be chosen based on additional criteria such as plausibility and minimization of capital costs.

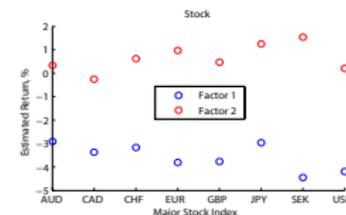
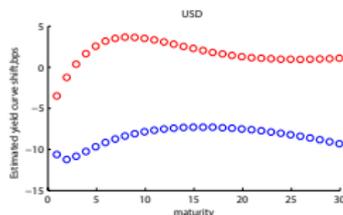
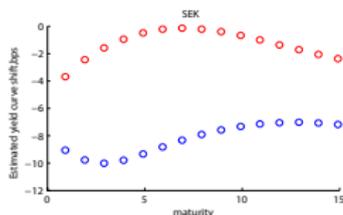
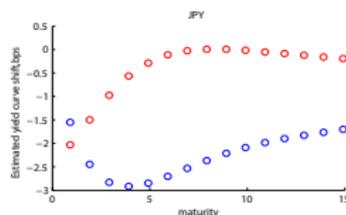
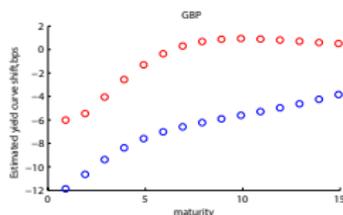
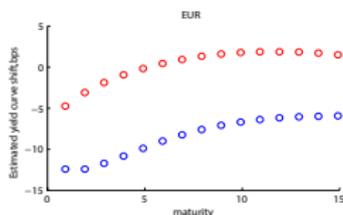
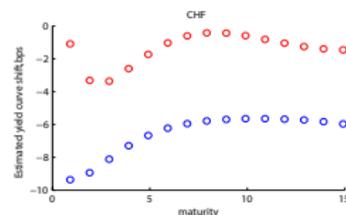
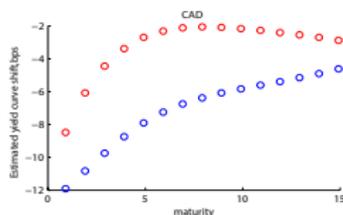
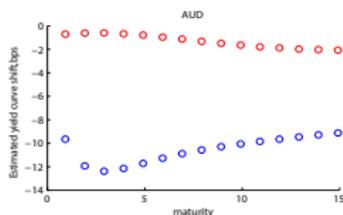
Examples of Factors Chosen by SIR

- ▶ 6 banks with only interest-rate risk positions.
- ▶ 6 banks with portfolios split 50% in interest rate exposures and 50% in stock market exposures.
- ▶ The figures will illustrate how one-standard deviation movements in the identified factors affect the X variables.
- ▶ The main point is the identified factors and consequent stresses are portfolio dependent. If banks alter their asset holdings, then the stress scenarios we apply to them should change.

Factor shocks for random bond portfolio



Factor shocks for random bond and stock portfolio



SIR Pitfall: Symmetry

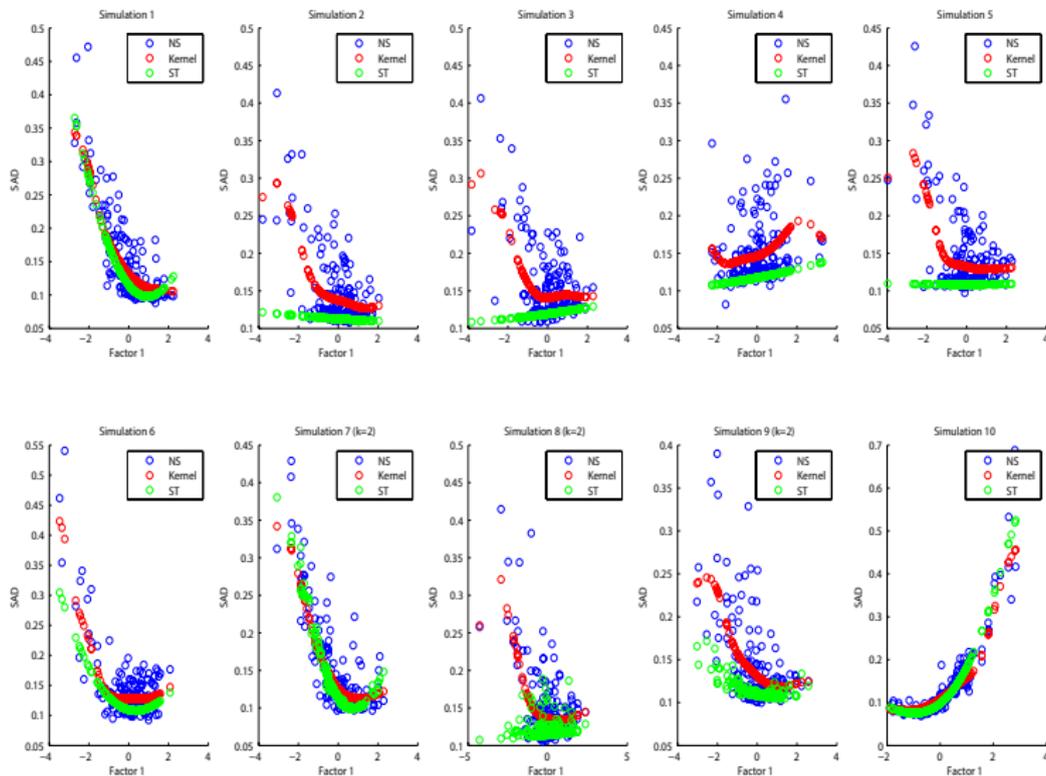
- ▶ SIR can have difficulty detecting factors when SAD is a symmetric function of X , or of the factors.

Example 1: When $SAD = X^2$, then $E(X|SAD) = 0$. In this case, SIR has trouble detecting how SAD is related to X .

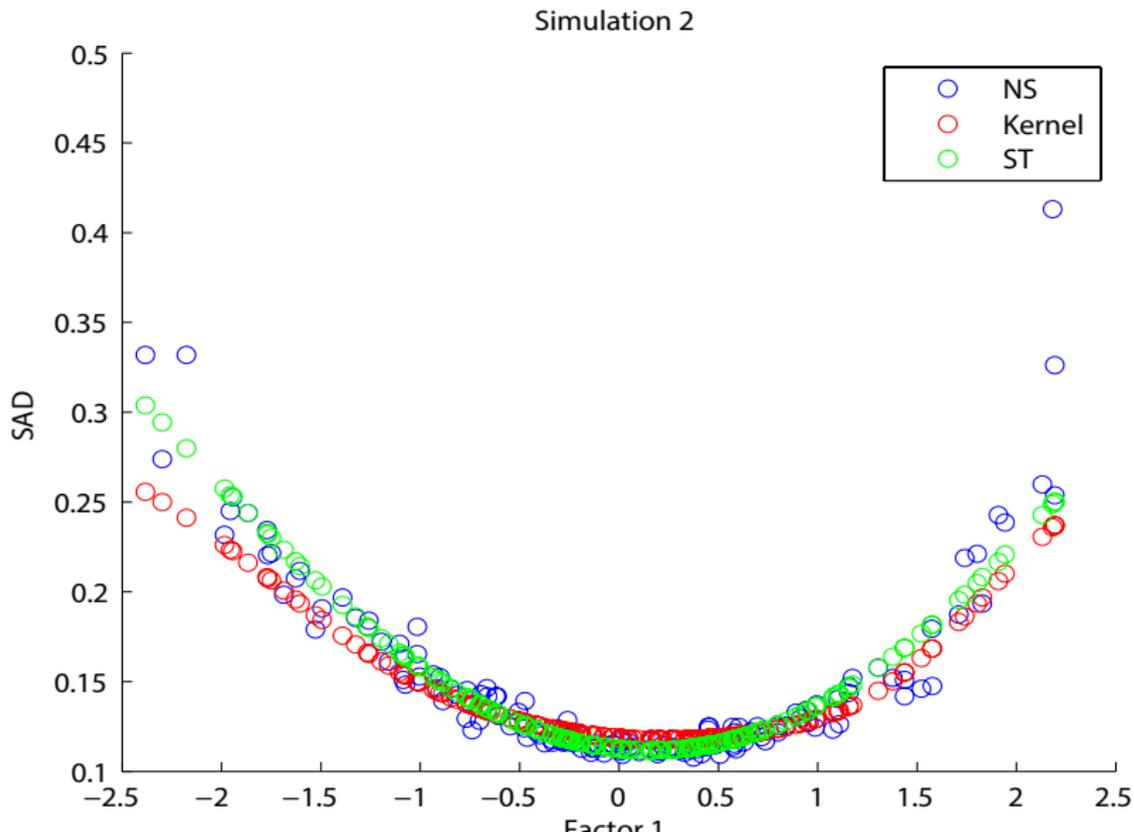
Example 2: If there are 6 large banks exposed to a single factor, and 3 are long the factor, and 3 are symmetrically short, SIR has trouble identifying the factor.

- ▶ Solution: using scatter plots of simulated P&L for the banks, compute SAD using $P\&L$ from positively or negatively correlated banks only, and identify F_1 from that.

SIR Simulations with occasional symmetry



SIR Simulations with occasional symmetry



Conclusions

- ▶ Presented A New Approach for choosing stress-scenarios.
- ▶ Contributions:
 1. Stress-scenarios are chosen so that resulting capital requirements keep systemic risk low with high probability.
 2. Variables for stress-testing are selected based on their ability to explain systemic risk.
 3. Stress factors are created based on their ability to explain systemic risk.
 4. Systemic risk scenarios are created from the factors. This is a natural way to choose stress-directions.
- ▶ Very preliminary results appear promising.
- ▶ More work is needed.