Large Orders in Small Markets: On Optimal Execution with Endogenous Liquidity Supply

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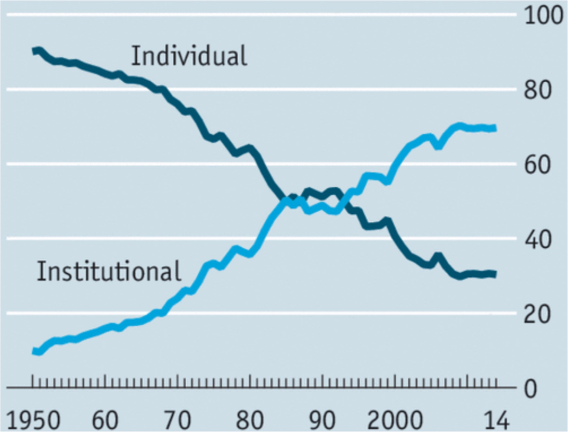
Intermediation Trends



Equity investments in the U.S. Source: The Economist.

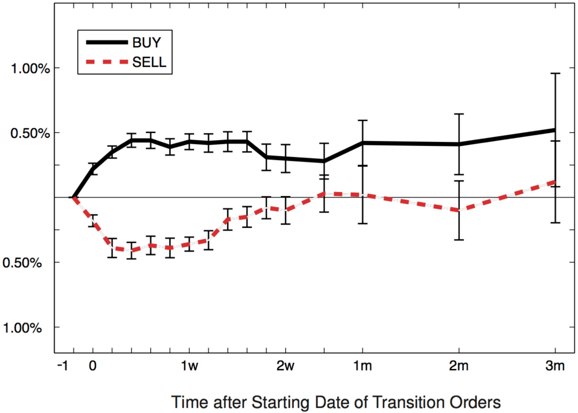
- Large investors increasingly prevalent in securities markets

Vayanos' View

- "If most large trades were motivated by information, large traders would significantly outperform the market. However, many empirical studies show that large traders do not significantly outperform, and may even underperform, the market. [...] Therefore, allocation motives must be important."

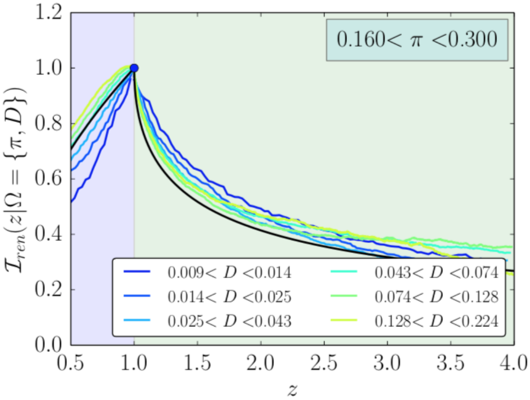
Source: Vayanos (2001, JF, p. 132)

Large Orders Impact Price



Source: Obizhaeva (2009, Fig. 1).

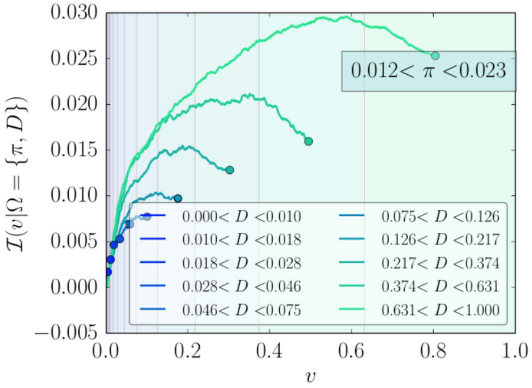
Large Orders Impact Price



- Price reverts after execution ends

Source: Zarinelli, Treccani, Farmer, and Lillo (2015). 7 million metaorders for Russell 3000 stocks in 2007-2009.

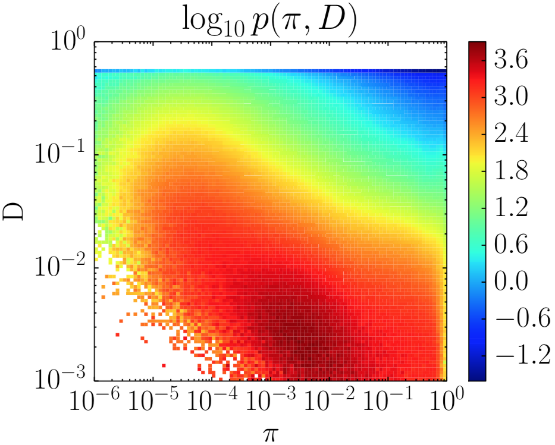
Price Impact During Execution



- Price sometime reverts *before* execution ends

Source: Zarinelli, Treccani, Farmer, and Lillo (2015), Figure 8.

Participation Rate vs Duration



Participation Rate π and Duration D are negatively correlated.

Source: Zarinelli, Treccani, Farmer, and Lillo (2015), Figure 2.

Regulatory Pressure

- Large broker-dealers might be forced to sell quickly to meet minimum liquidity ratio (Basel III)
- Cover-2 capital requirement for CCPs forces them to assess liquidity premium paid when positions of a failed account need to be sold in "close-out period."
- SEC (2016) demands open-end funds to report their liquidity risk in terms of "days-to-cash".

Unifying two Strands of Literature

- Papers on optimal execution with exogenous liquidity supply (e.g., Almgren and Chriss (2001)).
- Papers on optimal liquidity supply with exogenous demand (e.g., Amihud and Mendelson (1980)).
- No papers on both

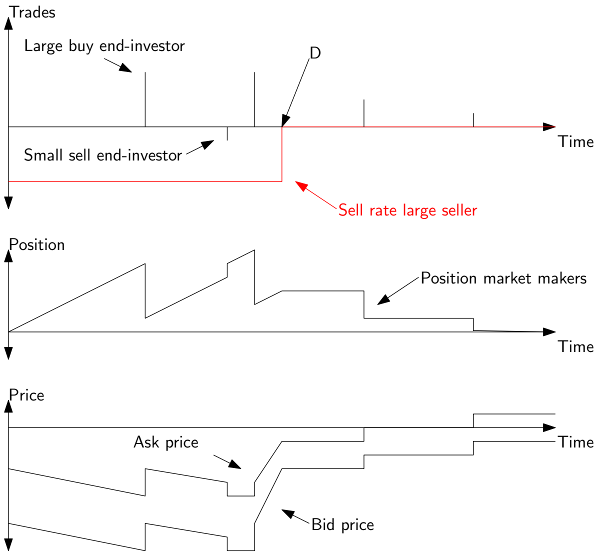
Objective

- How liquid is the market for a large seller who is (only) time constrained?
- Should he reveal this constraint?
- Do market makers benefit from large-seller's presence? And, end-user investors?
- Calibrate the model to assess economic size of these effects

Model Setting

- Strategic trading by large seller who needs to trade large position in finite time.
- Strategic trading by (Cournot) competitive market makers in response to large seller (Stackelberg)
- Information *asymmetry* on order duration
- Information *symmetry* on fundamentals
- Time is continuous and runs forever

Model Visualization



Market Environment

- Fundamental value S is common knowledge:

$$dS_t = \sigma dB_t,$$

where B_t is a standard BM

End-User Investors

- End-user investors who want to buy (sell, resp.) arrive according to a Poisson process N^B (N^S, resp.) with the same arrival intensity $\lambda > 0$
- Traded quantities by buyers and sellers depend on ask and bid prices:

$$Q^B(S, x) = c\,(S + \tilde{p} - x)\,, \quad Q^S(S, x) = c\,(x - S + \tilde{p}),$$

For $\tilde{p} > 0$:

  - $S + \tilde{p}$: maximum price at which a buyer buys from the HFT
  - $S - \tilde{p}$: minimum price at which a seller sells to the HFT

Large Seller

- Duration of liquidation D sampled from an independent exponential distribution with mean $1/\nu$.
- The large seller can engage in:
  - Stealth trading: he keeps D hidden, choosing the same liquidation rate independent of D
  - Sunshine trading: implicitly reveal D, i.e., making the liquidation rate depend on D
- Let b_t be the bid price offered by the market makers at time t, then his objective is

$$\sup_{\bar{f} \geq 0} \mathbb{E}\left[\int_0^D e^{-\beta t} \bar{f} \times (b_t - (S_t - \tilde{p}))dt\right]$$

Market Makers

- N market makers share the liquidation stream from the large seller
- Market maker n chooses how much to buy at the bid $(x_t^{b,n})$, and how much to sell at ask $(x_t^{a,n})$, aware of the price impact
- The aggregated strategies of the N market makers collectively determine the ask and bid prices via market clearing:

$$\begin{cases} \left(\sum_{n=1}^{N} x_t^{a,n}\right) dN_t^B = c(S_t + \tilde{p} - a_t) dN_t^B \\ \left(\sum_{n=1}^{N} x_t^{b,n}\right) dN_t^S = c(b_t - S_t + \tilde{p}) dN_t^S \end{cases}$$

$$\Rightarrow a_t = S_t + \tilde{p} - \frac{1}{c} \sum_{n=1}^{N} x_t^{a,n} \quad b_t = S_t - \tilde{p} + \frac{1}{c} \sum_{n=1}^{N} x_t^{b,n}$$

- $(x_t^{a,n})$ and $(x_t^{b,n})$ are Markov predictable strategies (dependent on t, $\bar{f}$, i)

The Objective of Market Makers

- Market maker n solves

$$\max_{(x_\cdot^{a,n}, x_\cdot^{b,n}) \in \mathcal{A}} \mathbb{E}\left[\int_0^\infty e^{-\beta t}\left(dW_t^{(x,n)} - \Theta\left(I_t^{(x^n,n)}\right)^2 dt\right)\right]$$

where $\mathcal{A}$ is the collection of all admissible strategies subject to:

$$dW_t^{(x,n)} = -b_t \cdot \frac{\bar{f}}{N} 1_{t \leq D} dt + a_t \cdot x_t^{a,n} dN_t^B - b_t \cdot x_t^{b,n} dN_t^S + S_t dI_t^{(x^n,n)}$$

$$dI_t^{(x^n,n)} = \underbrace{\frac{\bar{f}}{N} 1_{t \leq D} dt}_{\substack{\text{Shares liquidated} \\ \text{by institution}}} + \underbrace{x_t^{b,n} dN_t^S}_{\substack{\text{Shares bought} \\ \text{from sell investors}}} - \underbrace{x_t^{a,n}\, dN_t^B}_{\substack{\text{Shares sold} \\ \text{to buy investors}}}$$

- Focus on symmetric equilibria

Dynamic Programming

- Fix a liquidation strategy $f \equiv \bar{f} 1_{t \leq D}$.
- Given $I_t^{(x^n,n)} = i$, consider the value function

$$V_n(t, i; f)$$
$$:= \sup_{(x_\cdot^{a,n}, x_\cdot^{b,n}) \in \mathcal{A}} \mathbb{E}\left[\int_0^\infty e^{-\beta(u-t)}(dW_u^{(x,n)} - \Theta\left(I_u^{(x^n,n)}\right)^2 du) | I_t^{(x^n,n)} = i\right]$$

- Value independent of fundamental since revenue is calculated relative to the fundamental

Outline

1. Model Results
   - Stealth Trading
   - Sunshine Trading
   - Large Seller
2. Calibration Results
3. Impact of Liquidation on Others

Stealth Trading: Market Makers Policy

- Let A be the unique positive root to the following equation

$$\Theta - \beta A = \frac{8c\lambda A^2(1 + cA)}{(N + 1 + 2cA)^2}.$$

- The optimal value of market maker n is given by

$$V_n(t, i; f) = -Ai^2 + B(\bar{f})1_{t \le D} i + C(\bar{f})1_{t \le D},$$

where $B(\bar{f}) = -\bar{f}\frac{\delta - \beta}{2c\lambda}\frac{N+2cA}{N}\frac{1}{\nu + \delta}$ and $\delta = \Theta/A$.

- Best bid and ask quotes given by

$$\begin{cases} a_t(i, \bar{f}) = S_t + \dfrac{p(1 + 2cA) - 2NAi + NB(t, \bar{f})}{N + 1 + 2cA} \\ b_t(i, \bar{f}) = S_t + \dfrac{-p(1 + 2cA) - 2NAi + NB(t, \bar{f}) - \frac{\bar{f}}{c\lambda}1_{t \le D}}{N + 1 + 2cA} \end{cases}$$

Stealth Trading: Bid and Ask Price Dynamics

- Before liquidation ends, bid and ask quotes are stationary (i.e. independent of t)
- Constant bid-ask spread before termination
- Constant ask spread after termination
- Spread higher before liquidation ends
- Liquidation pressures down both bid and ask quotes
- Sudden quote corrections when liquidation ends

Sunshine Trading: Optimal Market Makers Policy

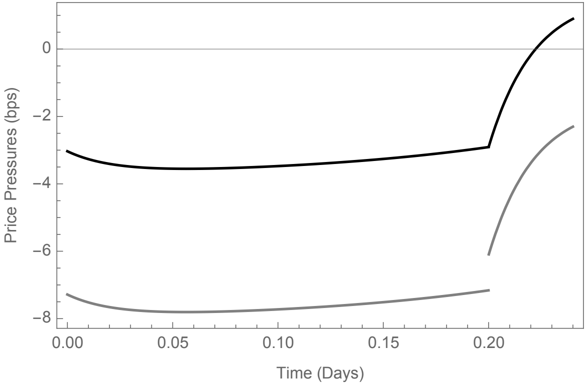
- Let A be the unique positive root to the following equation
$$\Theta - \beta A = \frac{8c\lambda A^2(1+cA)}{(N+1+2cA)^2}.$$
- Then the optimal value of market maker n is given by
$$V_n(t,i;f) = -Ai^2 + \tilde{B}(t,\bar{f})i + \tilde{C}(t,\bar{f}),$$
where $\tilde{B}(t,\bar{f}) = -\bar{f}\frac{\delta-\beta}{2c\lambda}\frac{N+2cA}{N}\frac{1-e^{-\delta(D-t)}}{\delta}1_{t\le D}$ and $\delta = \Theta/A$.
- Optimal bid and ask quotes given by
$$\begin{cases} a_t(i,\bar{f}) = S_t + \dfrac{p(1+2cA) - 2NAi + N\tilde{B}(t,\bar{f})}{N+1+2cA} \\ b_t(i,\bar{f}) = S_t + \dfrac{-p(1+2cA) - 2NAi + N\tilde{B}(t,\bar{f}) - \frac{\bar{f}}{c\lambda}1_{t\le D}}{N+1+2cA} \end{cases}$$

Sunshine Trading: Bid and Ask Price Dynamics

- Before liquidation ends, bid and ask quotes are time-dependent, continuously converging to the stationary strategies at $t = D$
- Constant bid-ask spread during liquidation
- Constant bid-ask spread after liquidation
- Liquidation widens the bid-ask spread
- Liquidation pressures down both bid and ask quotes
- No sudden price corrections to the ask price when liquidation ends

Expected Price Paths SunShine Trading

- Price pressure: the deviation from fundamental



Expected Proceeds of Large Seller

- Under stealth trading, the institutional investor's expected proceeds, if he liquidates at a rate $\bar{f} \geq 0$, is

$$\widetilde{G}(\bar{f}) := \widetilde{P}\bar{f} - \widetilde{Q}(\bar{f})^2, \tag{4.1}$$

where $\widetilde{P}, \widetilde{Q}$ are positive closed-form constants.

- Under sunshine trading, the seller's proceeds, if he liquidates at a rate $\bar{f} \geq 0$ for a given duration $D > 0$, is

$$G(D; \bar{f}) = P(D)\bar{f} - Q(D)(\bar{f})^2, \tag{4.2}$$

where $P(\cdot), Q(\cdot)$ are positive functions of D, computable in closed form.

Sunshine vs Stealth

- Fix a liquidation rate $\bar{f}$, and use it both in the sunshine and stealth trading scenarios.
- The per-share proceeds for stealth trading are lower than for sunshine trading

$$\frac{\widetilde{G}(\bar{f})}{\bar{f}} < \mathbb{E}^D\left[\frac{G(D;\bar{f})}{\bar{f}}\right]$$

- The benefits under *full sunshine* are even higher, because the institutional investor can make the liquidation rate conditional on duration D

Full Sunshine

- For sunshine trading, the large seller's expected proceeds are

$$P(D)\bar{f} - Q(D)(\bar{f})^2$$

  for some positive functions of D, $P(D)$ and $Q(D)$ that depends on $\beta, N, c, \lambda, \tilde{p}$
- The optimal liquidation rate for duration D is thus given by

$$\bar{f}^*(D) = \frac{P(D)}{2Q(D)}.$$

- The optimal expected liquidation proceeds for duration D is $\frac{(P(D))^2}{4Q(D)}$.
- Moreover, $\bar{f}^*(D)$ **is strictly decreasing in** D.

Sunshine or Stealth?

- The expected proceeds from sunshine trading, $\mathbb{E}^D[(P(D))^2/4Q(D)]$, are strictly higher than those under stealth trading, $\tilde{P}^2/4\tilde{Q}$.
- Revealing duration helps market maker reduce the execution costs
- The benefit is passed on to the large seller
- The larger seller is better off even in the case of a monopolistic market maker

Participation Rate

- Participation rate is the liquidated volume over the total volume until D.
- We define participation rate for duration D as $R(D)$:

$$R(D) = \frac{D \cdot \bar{f}^*(D)}{\mathbb{E}[\text{total volume}]}$$

- $1/R(D)$ is strictly increasing in D:

$$\frac{1}{R(D)} = \frac{N + 2cA}{N + 1 + 2cA} + \frac{2N}{N + 1 + 2cA}\frac{c\lambda\tilde{p}}{\bar{f}^*(D)},$$

- Thus, **the participation rate strictly decreases with** D.

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Calibration

- Calibration where market makers are HFTs.
- Based on Menkveld (2013)
- Results:

$$\begin{cases} N = 10, \\ \Theta = 0.140802(\mathrm{bps}/€1000), \\ c = 11.2885(€1000/\mathrm{bps}), \\ \tilde{p} = 13.61(\mathrm{bps}), \\ \lambda = 791(/\mathrm{day}), \\ \nu = 5(/\mathrm{day}). \end{cases}$$

Calibration Details

	Menkveld (2013)	Our model
Cond. price pressure (bps/€1000)	-0.026	$-\frac{2NA}{N+1+2cA}$
Std. inventory (€1000)	80.3	$\tilde{p}\sqrt{\frac{c}{2A(N+1)}}$
Half bid-ask spread (bps)	1.6	$\frac{(1+2cA)\tilde{p}}{N+1+2cA}$
Mean average No. arrivals (/day)	1582	2λ

Sunshine vs Stealth

- 4.188% improvement when large seller switches from stealth to sunshine, but maintains the same liquidation rate
- An additional 1.266% improvement is obtained if the seller picks the liquidation rate that is optimal for each D

Conclusions

- Liquidation reinforces price pressure and widens bid-ask spread
- Participation rate negatively correlates with the liquidation duration
- Price reversal occurs prior to the end of liquidation
- Sharing information on duration is beneficial for the large seller

Extensions

- Enlarge strategy space and allow for predation under sunshine trading
- What if the liquidation rate is allowed to depend on end-user arrivals?

Almgren, R., and N. Chriss (2001): "Optimal Execution of Portfolio Transactions," *Journal of Risk*, 3, 5–39.

Amihud, Y., and H. Mendelson (1980): "Dealership Market: Market-making with Inventory," *Journal of Financial Economics*, 8, 31–53.

Brogaard, J., T. Hendershott, and R. Riordan (2014): "High-Frequency Trading and Price Discovery," *Review of Financial Studies*, 27, 2267–2306.

Brunnermeier, M., and L. H. Pedersen (2005): "Predatory Trading," *Journal of Finance*, 60, 1825–1863.

Choi, J. H., K. Larsen, and D. J. Seppi (2018): "Information and Trading Targets in a Dynamic Market Equilibrium," *Journal of Financial Economics, Forthcoming*.

Glosten, L., and P. Milgrom (1985): "Bid, Ask and Transaction Prices in a Specialist Market with Heterogeneously Informed Traders," *Journal of Financial Economics*, 14, 71–100.

Hendershott, T., and A. J. Menkveld (2014): "Price Pressures," *Journal of Financial Economics*, 114, 405–423.

Ho, T., and Stoll, Hans R. (1981): "Optimal Dealer Pricing under Transactions and Return Uncertainty," *Journal of Financial Economics*, 9, 47–73.

Huberman, G., and W. Stanzl (2005): "Optimal Liquidity Trading," *Review of Finance*, 9, 165–200.

Kyle, A. (1985): "Continuous Auctions and Insider Trading," *Econometrica*, 53, 1315–1335.

Menkveld, A. (2013): "High Frequency Trading and the New-Market Makers," *Journal of Financial Markets*, 16, 712–740.

Obizhaeva, A. A., and J. Wang (2013): "Optimal Trading Strategy and Supply/Demand Dynamics," *Journal of Financial Markets*, 16, 1–32.

Vayanos, D. (1999): "Strategic Trading and Welfare in a Dynamic Market," *Review of Economic Studies*, 66, 219–254.

——— (2001): "Strategic Trading in a Dynamic Noisy Market," *Journal of Finance*, 56, 131–171.

Weill, P. (2007): "Leaning Against the Wind," *Review of Economic Studies*, 74, 1329–1354.

Zarinelli, E., M. Treccani, J. D. Farmer, and F. Lillo (2015): "Beyond the Square Root: Evidence for Logarithmic Dependence of Market Impact on Size and Participation Rate," *Market Microstructure and Liquidity*, 1, 1–32.

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Impact on Market Makers

- Suppose market makers start with zero inventory. Then, the value of one market maker is $C(0, \bar{f})$ under stealth and $\tilde{C}(0, \bar{f})$ under sunshine.
- Suppose the following liquidity condition holds:

$$N^2 + cA(2N-1) - \frac{1}{2}(1+cA)(N+2cA)^2\left(1 - \frac{\beta^2}{\delta^2}\right) \geq 0,$$

  - The above condition holds in a liquid market: low inventory costs Θ or high arrival rate $\lambda > 0$.

- Liquidation always benefits the market maker, i.e., $C(0, \bar{f}) > C(0, 0)$ and $\tilde{C}(0, \bar{f}) > \tilde{C}(0, 0)$

Impact on End Users

- Economic surplus of end users:

$$U(t,i;\bar{f}) = \mathbb{E}_{I_t=i}\left[\int_t^{\infty} e^{-\beta(s-t)}\left(\frac{1}{2}c(\tilde{p}-\tilde{a}_s)^2 dN_s^B + \frac{1}{2}c(\tilde{p}+\tilde{b}_s)^2 dN_s^S\right)\right]$$

- $U(0,0,\bar{f}) - U(0,0,0) < 0$ if $\bar{f}$ is below a threshold
- $U(0,0,\bar{f}) - U(0,0,0) > 0$ if $\bar{f}$ is above a threshold.
- Liquidation benefits end users if $\bar{f}$ is "high enough"

Impact on End Users

- High liquidation rate:
  - The additional price pressure benefits end users (Hendershott and Menkveld (2014))
- But, our model predicts that liquidation may widen the bid-ask spread, which harms end investors' surplus
- Low liquidation rate:
  - Execution costs due the widened bid-ask spread dominate the positive effects due to intensified price pressure.

Expected Proceeds of Large Seller

- Under sunshine trading, the large seller's expected proceeds, given D, are

$$\mathbb{E}^{N^S,N^B}\left[\int_0^D e^{-\beta t}\bar{f}b_t dt\right],$$

- Under stealth trading, the expected proceeds are

$$\mathbb{E}^{D,N^S,N^B}\left[\int_0^D e^{-\beta t}\bar{f}b_t dt\right]$$

Price Paths Sunshine Trading

- If $I_0^{(x^n,n)} = 0$, the expected inventory at $t \leq D$ is given by

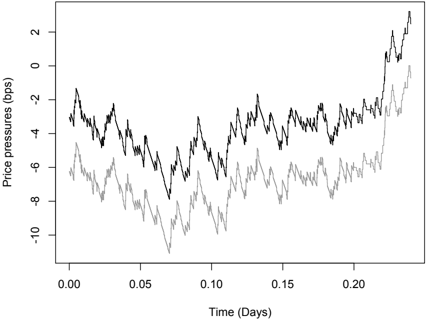
$$g(t) \equiv \mathbb{E}[I_t^{(x^n,n)}] = \frac{\bar{f}}{N} \frac{N+2cA}{N+1+2cA} \left(\frac{\beta}{\delta} \frac{1-e^{-Mt}}{M} + \frac{\delta-\beta}{\delta} \frac{e^{\delta t}-e^{-Mt}}{M+\delta} e^{-\delta S} \right),$$

where $\delta = \Theta/A$. For $t > D$, $g(t) = g(D)e^{-M(t-D)}$.

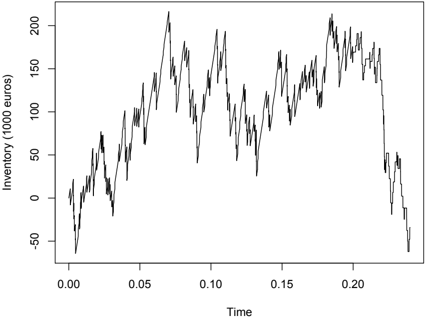
- Recall that the expected ask and bid prices are

$$\begin{cases} \mathbb{E}[a_t(i,\bar{f})] = S_0 + \dfrac{p(1+2cA) - 2NAg(t) + NB(t,\bar{f})}{N+1+2cA} \\ \mathbb{E}[b_t(i,\bar{f})] = S_0 + \dfrac{-p(1+2cA) - 2NAg(t) + NB(t,\bar{f}) - \frac{\bar{f}}{c\lambda} 1_{t \leq D}}{N+1+2cA} \end{cases}$$

Simulated Price Pressures

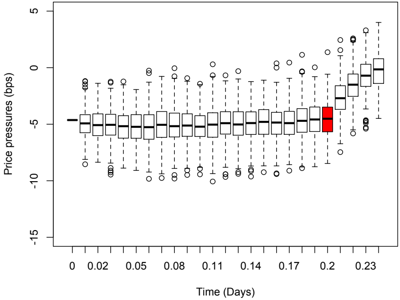


(a) Simulated Price Pressures

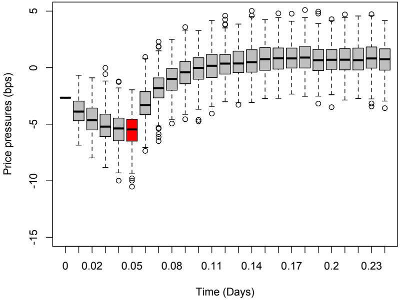


(b) Inventory

Box-plots Price Pressure

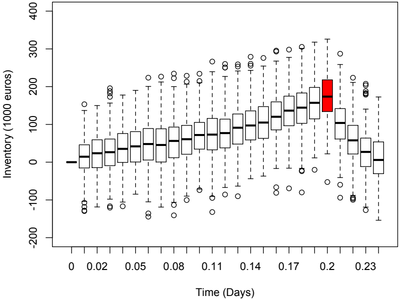


(c) Mid-quote pressure when $D = 0.2$

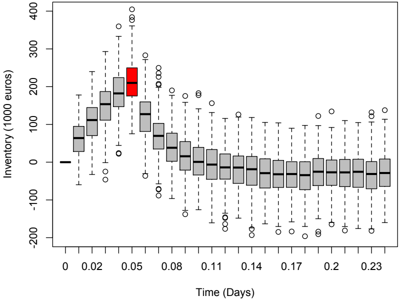


(d) Mid-quote pressure when $D = 0.05$

Box-plots Inventories



(e) Inventory when $D = 0.2$



(f) Inventory when $D = 0.05$