

# Fire sales, price-mediated contagion and systemic risk.

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# Overview

- 1 Introduction: Price-mediated contagion and endogenous risk
- 2 Modelling fire sales
- 3 Is it relevant?
- 4 Monitoring
- 5 Comparison of fire sales and leverage targeting models
- 6 Conclusion

# Price-mediated contagion and endogenous risk

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**Goal:** Develop models for macro stress testing that can quantify such second round effects in a realistic and robust way.  
("Stresstesting 3.0")

# Systemic stress testing

System:

- $N$  banks,  $K$  *illiquid* asset classes,  $M$  *marketable* asset classes
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Data:  $N = 90$ ,  $M = 148$ ,  $K = 75$ .

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Mechanism:

- ① **Shock** to illiquid assets
- ② **Deleveraging** of marketable assets by some institutions
- ③ **Feedback effects** via price-mediated contagion  
 $\rightarrow$  potentially triggers more deleveraging (cascade).

Mathematically this is a discrete time non-linear dynamical system.

## Questions

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# Modelling fire sales

# Model balancesheet

<b>Illiquid assets</b>
<p>Residential mortgage exposures</p> <p>Commercial real estate exposure</p> <p>Retail exposures: Revolving credits, SME, Other</p> <p>Indirect sovereign exposures in the trading book</p> <p>Defaulted exposures</p> <p>Residual exposures</p>
<b>Marketable assets</b>
<p>Corporate bonds</p> <p>Sovereign debt</p> <p>Direct sovereign exposures in derivatives</p> <p>Institutional client exposures: interbank, CCPs,...</p>

Table: Stylized representation of asset classes in bank balance sheets.  
 (Data: European Banking Authority Stress Test)

- A stress scenario is defined by a vector  $\epsilon \in [0, 1]^K$  whose components  $\epsilon_\mu$  are the percentage shocks to asset class  $\mu$ .
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- Four scenarios:
  1. Spanish residential and commercial real estate losses
  2. Northern Europe residential losses
  3. Southern Europe commercial real estate losses
  4. Eastern Europe commercial real estate losses

# Fire sales model

- Total value of illiquid holdings:  $\Theta_t^i := \sum_{\mu=1}^K \Theta_t^{i\mu}$  .
- Securities:  $\Pi_t^i := \sum_{\mu=1}^M \Pi_t^{i\mu}$  .
- Common Equity Tier 1 capital:  $C_t^i$
- Initial loss:  $L_0^i := \sum_{\mu=1}^K \Theta_0^{i\mu} \epsilon_\mu$

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When a bank exceeds the leverage constraint,  $\lambda^i > \lambda_{\max}$ , it engages in fire sales of magnitude  $\Gamma^i \in [0, 1]$ :

$$\frac{(1 - \Gamma_1^i)\Pi_0^i + \Theta_0^i - L_0^i}{C_0^i - L_0^i} = \lambda_{new}^i,$$

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which yields in the **fire sales model**:

$$\Gamma_1^i = \frac{C_0^i(\lambda_0^i - \lambda_b^i)}{\Pi_0^i} \mathbb{1}_{\lambda_i > \lambda_{\max}},$$

# Price impact

The price of an asset undergoing a forced liquidation at  $t$ :

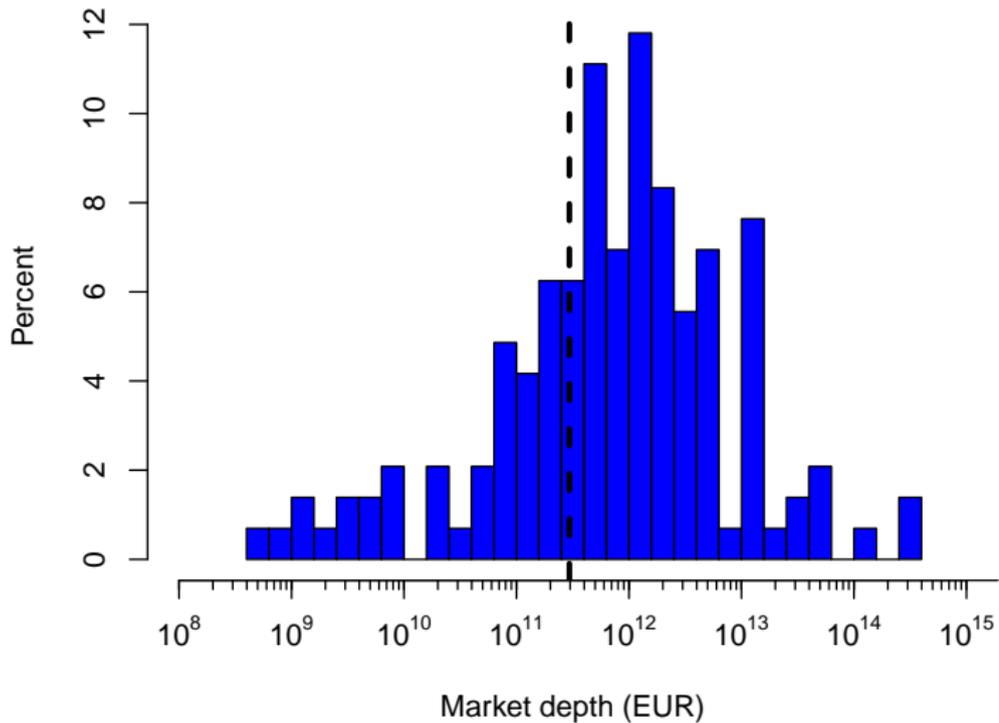
$$S_{t+1}^{\mu} = S_t^{\mu} \exp \left( -\delta_{\mu}^{-1} \sum_{j=1}^M \Pi_t^{j\mu} \Gamma_{t+1}^j \right),$$

where the market depth

$$\delta_{\mu} \sim \frac{ADV_{\mu}}{\sigma_{\mu}},$$

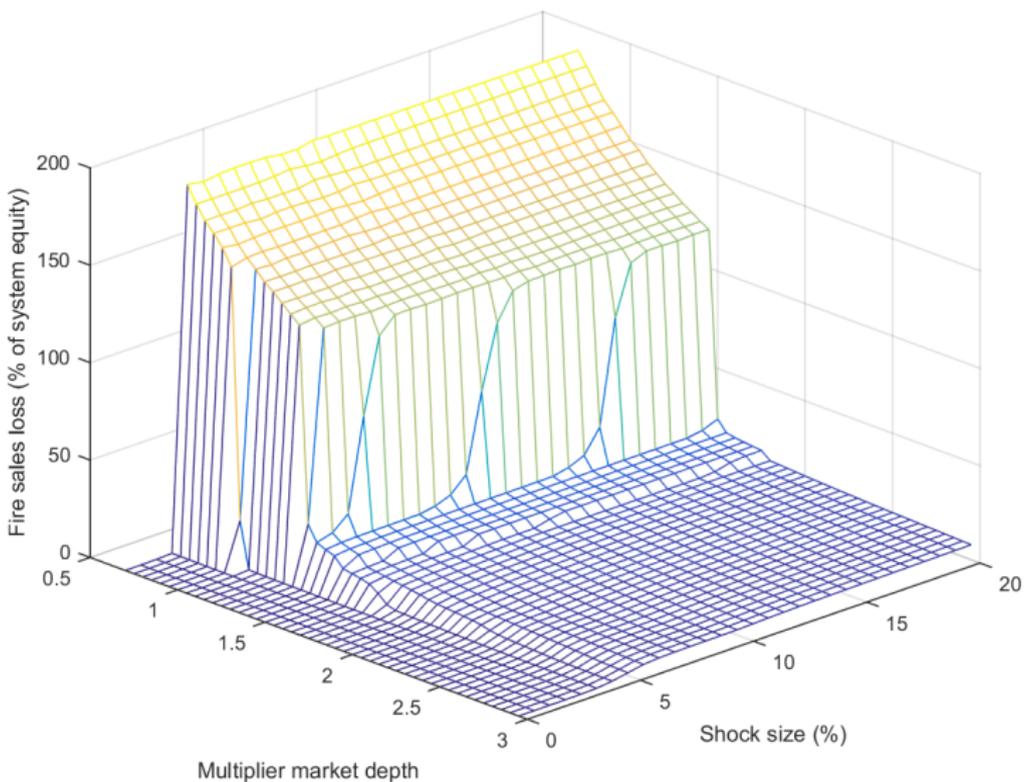
- $ADV$ : average daily volume
- $\sigma_{\mu}$  daily volatility

# Estimated market depth



Is it relevant?

# Fire sales losses and market depth



# Indirect exposures and stress test outcomes

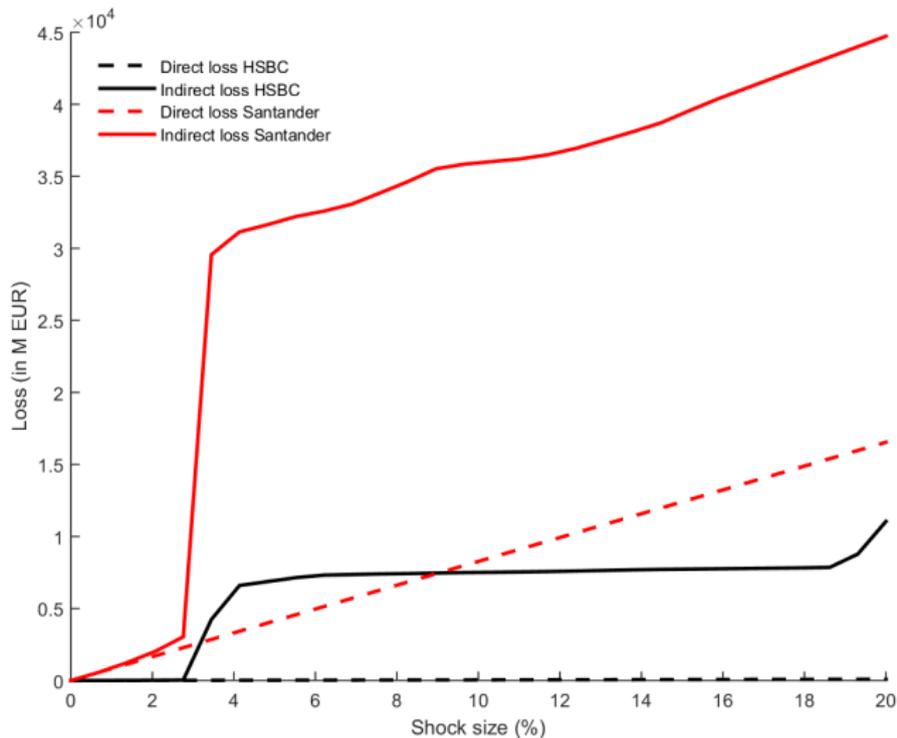
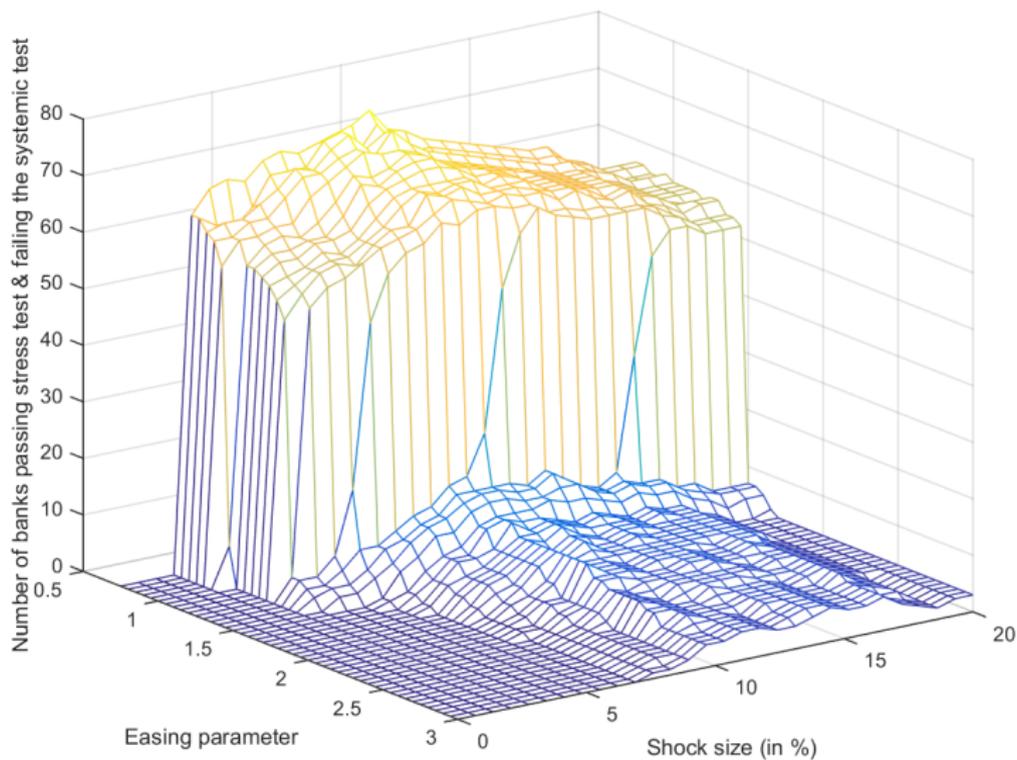


Figure: Source: EBA (public) & authors calculations.

# Indirect exposures and stress test outcomes



# Monitoring

# Fire sales losses

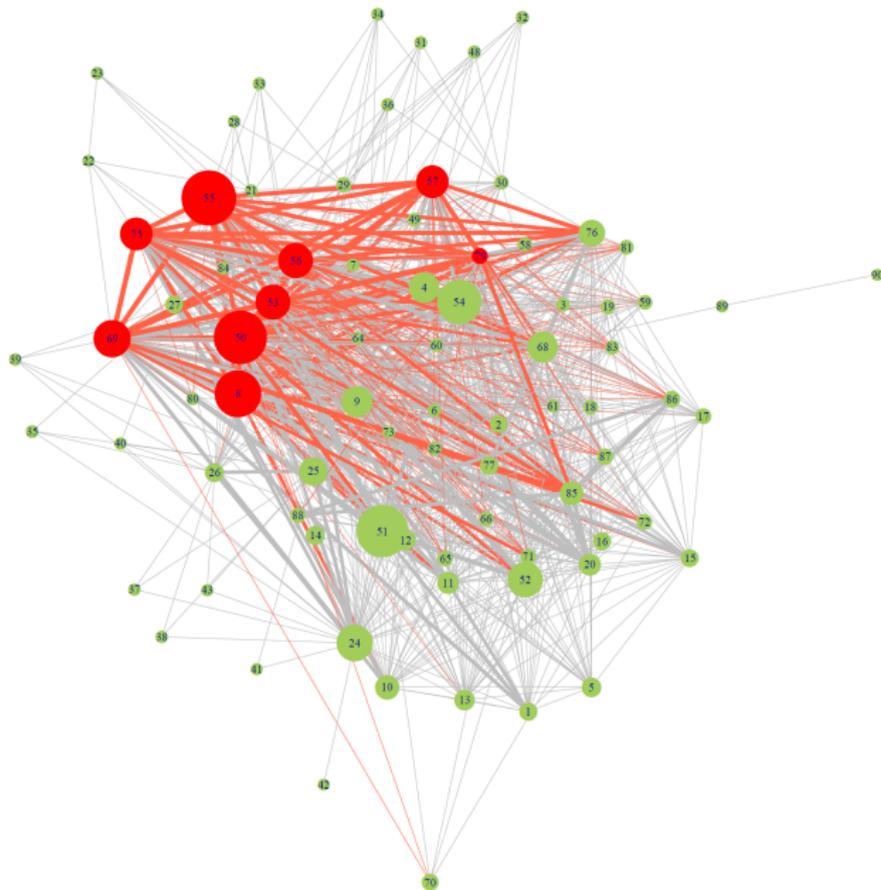
Linearising the price impact function yields

$$\begin{aligned}
 L_t^i &\approx (1 - (1 - \alpha)\Gamma_{t+1}^i) \sum_{\mu=1}^M \sum_{j=1}^N \delta_{\mu}^{-1} \Pi_t^{i\mu} \Pi_t^{j\mu} \Gamma_{t+1}^j \\
 &= (1 - (1 - \alpha)\Gamma_{t+1}^i) \sum_{j=1}^N \omega_{ij} \Gamma_{t+1}^j,
 \end{aligned}$$

where  $\omega_{ij} := \sum_{\mu=1}^M \Pi_0^{i\mu} \Pi_0^{j\mu} \delta_{\mu}^{-1}$  is the liquidity weighted overlap of portfolios  $i$  and  $j$ . This gives rise to a weighted and undirected “liquidity weighted overlap network” given by the symmetric (positive semidefinite) matrix:

$$\Omega := \Pi D^{-1} \Pi^{\top}.$$

# European banking system: liquidity weighted overlap



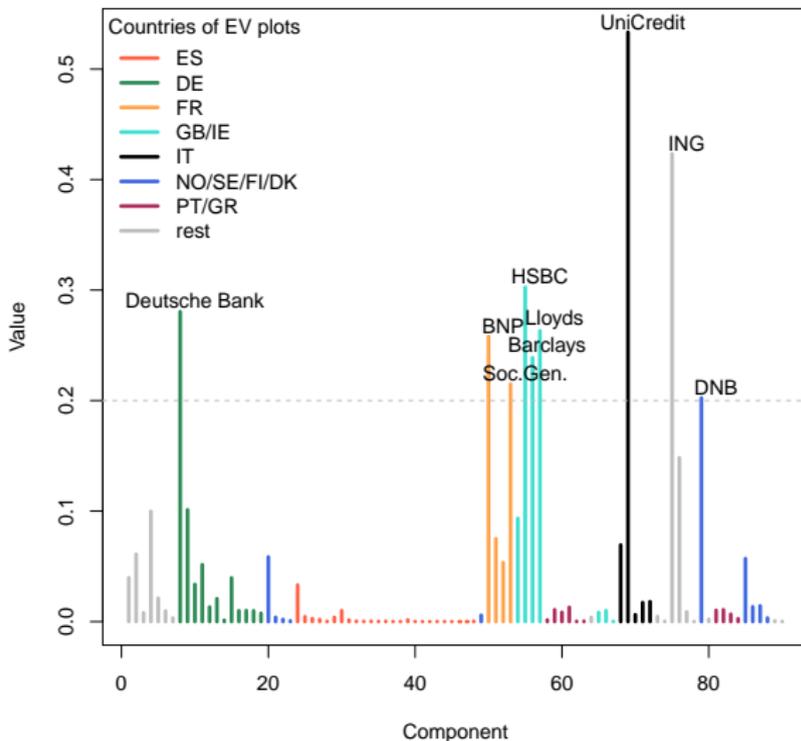


Figure: European banking system: Liquidity weighted overlaps. Source: EBA (public)

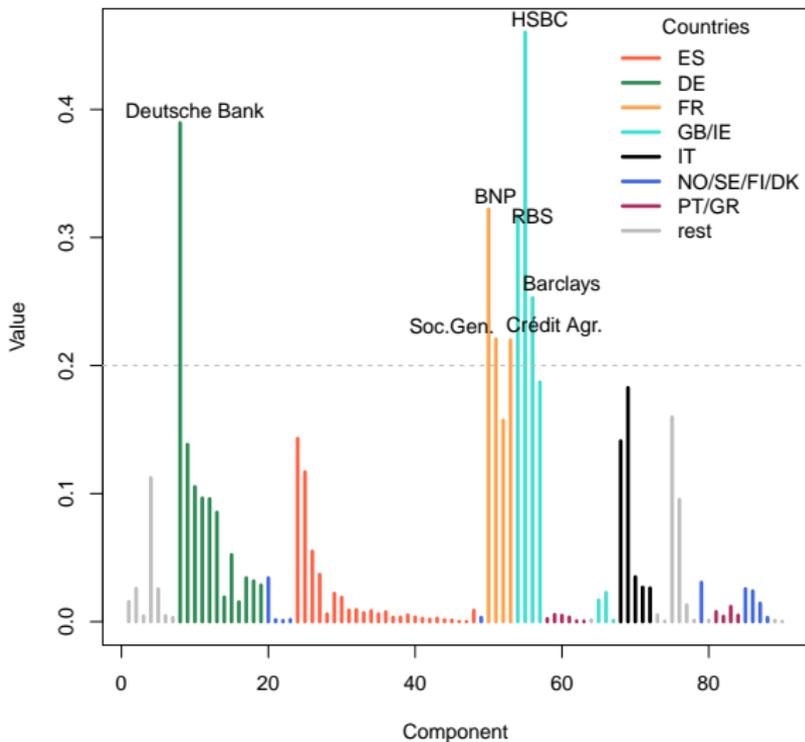


Figure: European banking system: Nominal overlaps. Source: EBA (public)

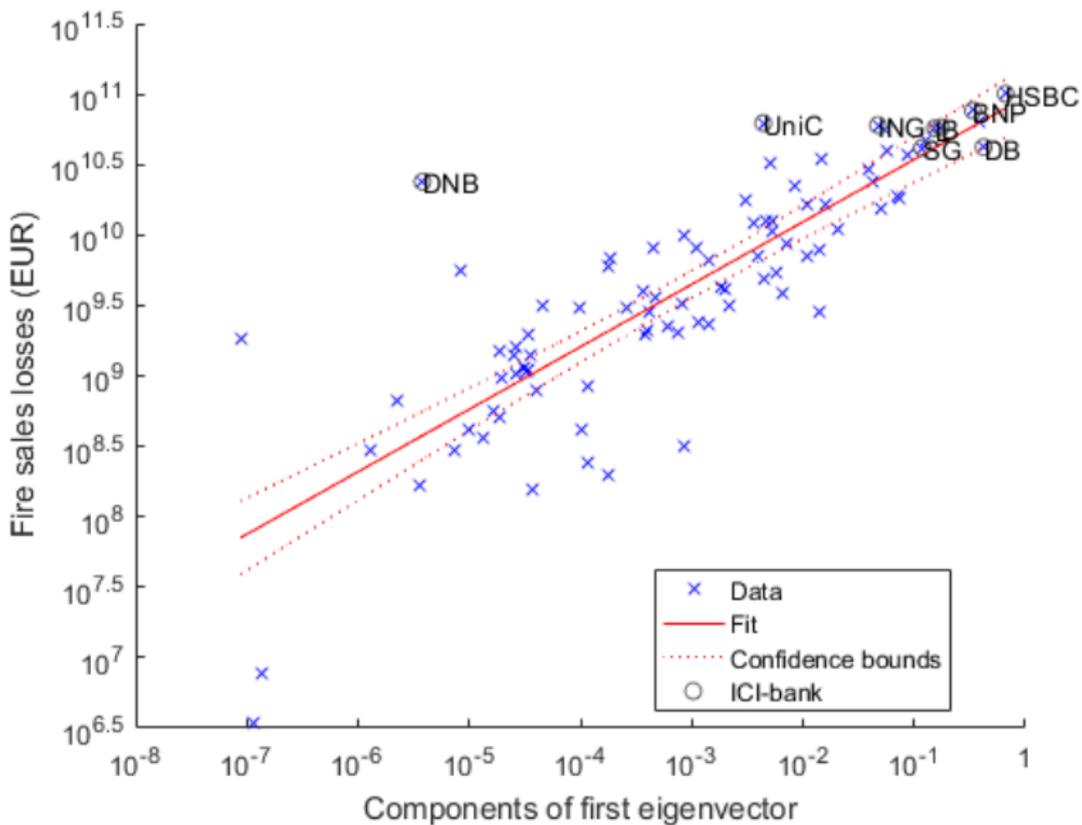


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# Constructing a Systemic Vulnerability Indicator

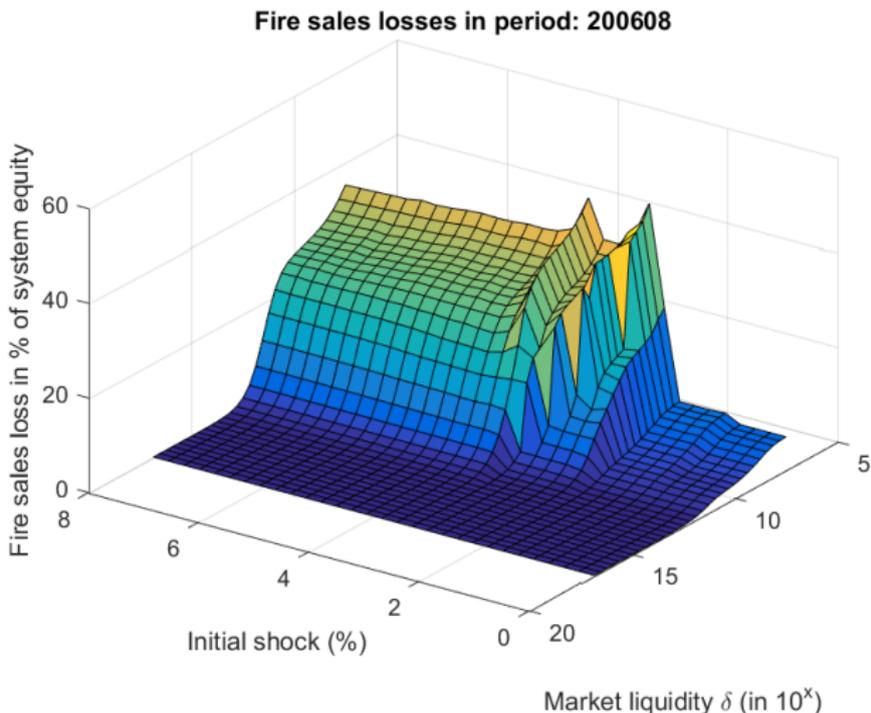


Figure: Threshold model: Fire sales losses as function of the initial shock and the market depth. Source: Statistics Norway.

# A Systemic Vulnerability Indicator

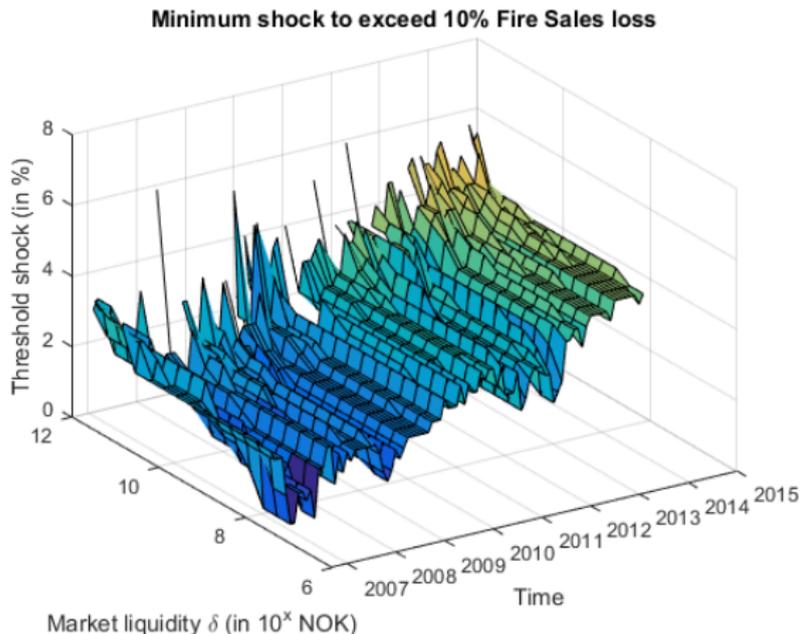


Figure: Minimum shock required to trigger large fire sales cascades, as a function of time and market depth. Source: Statistics Norway.

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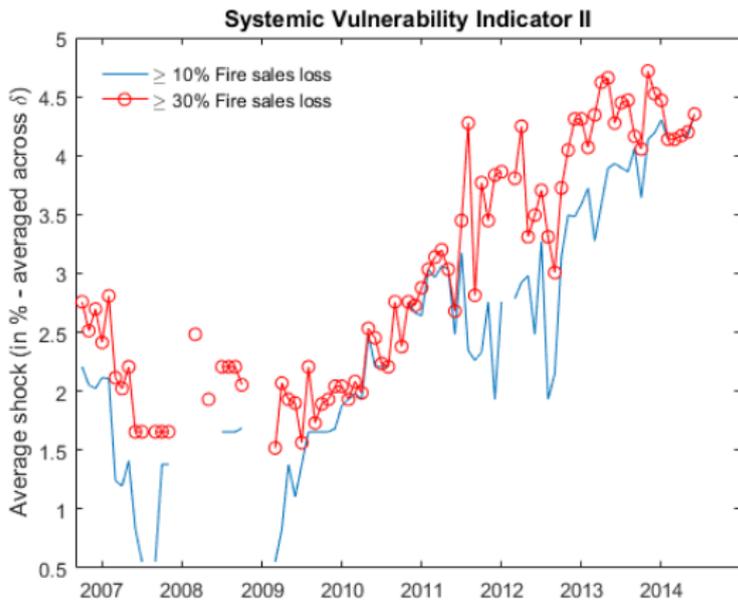


Figure: Minimum shock required to trigger large fire sales cascade, average over market depths. Source: Statistics Norway.

# Comparison to “leverage targeting” models

# Response functions

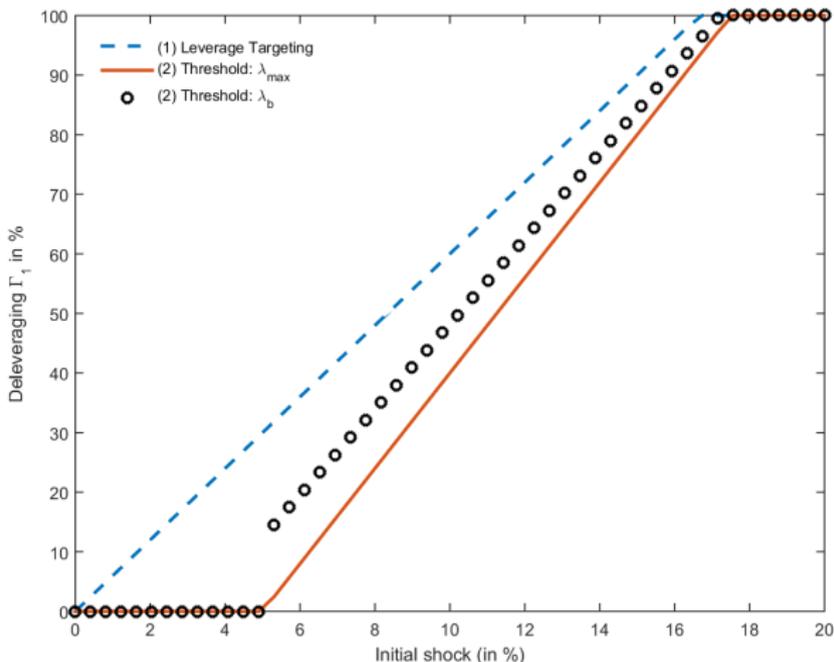
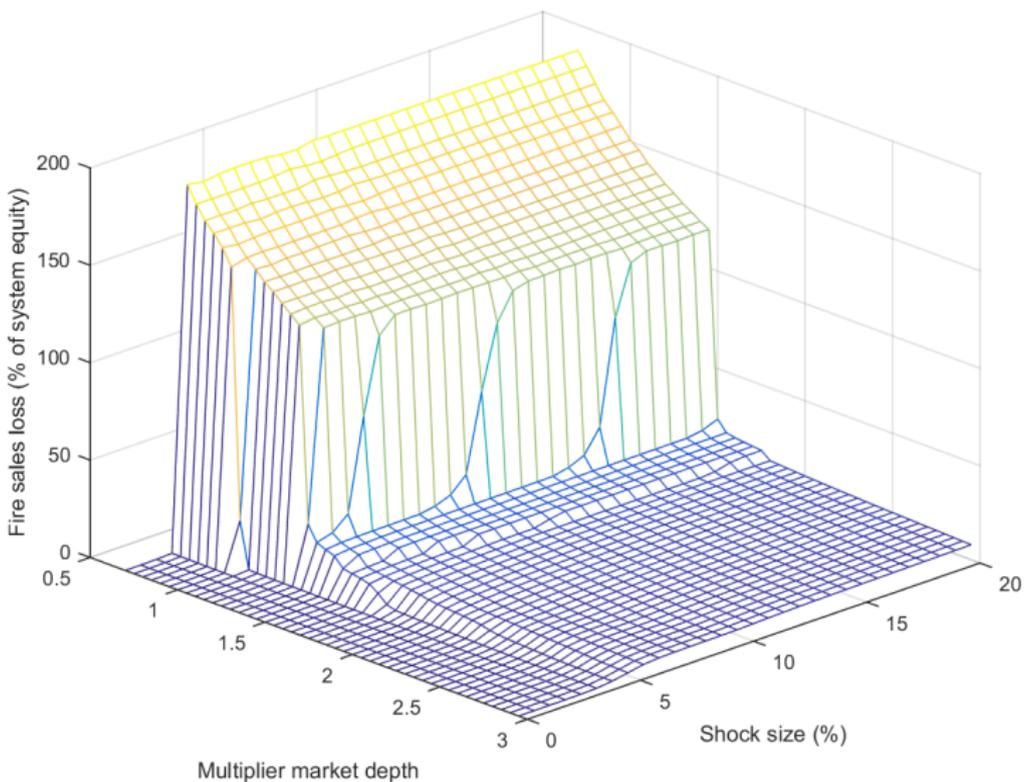
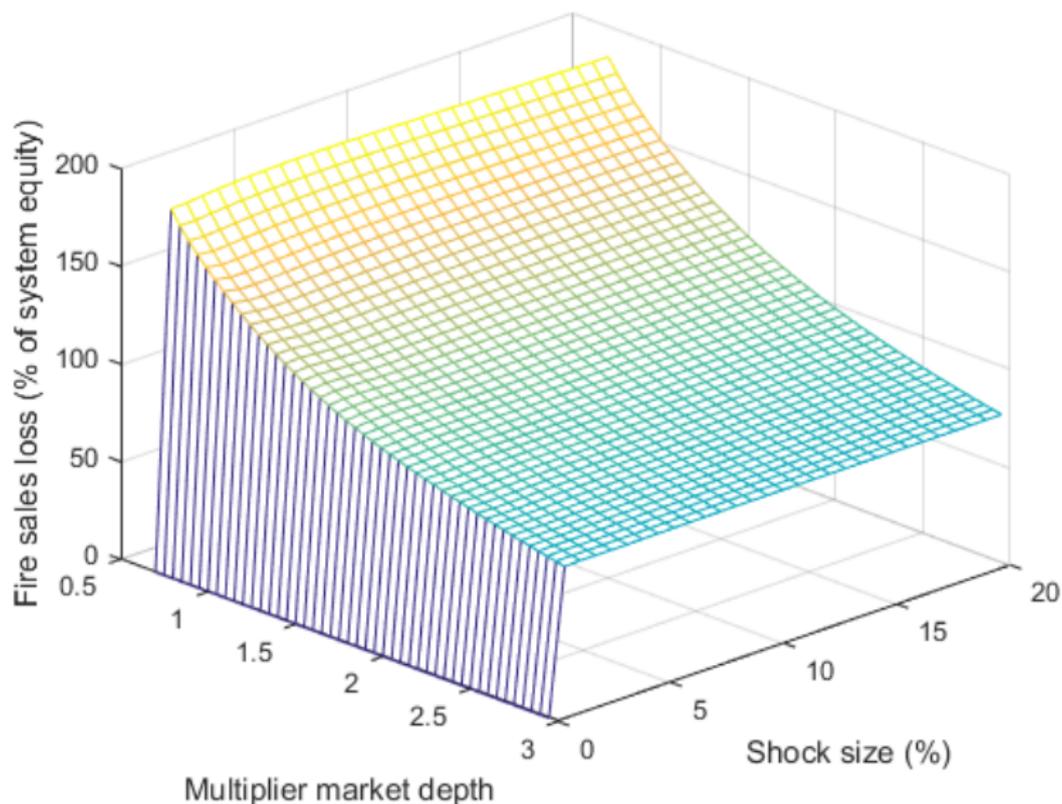


Figure: Leverage targeting response function (dashed) and two variants of the fire sales (full and circles) response functions.

# Fire sales losses and market depth



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# Distribution of fire sales losses

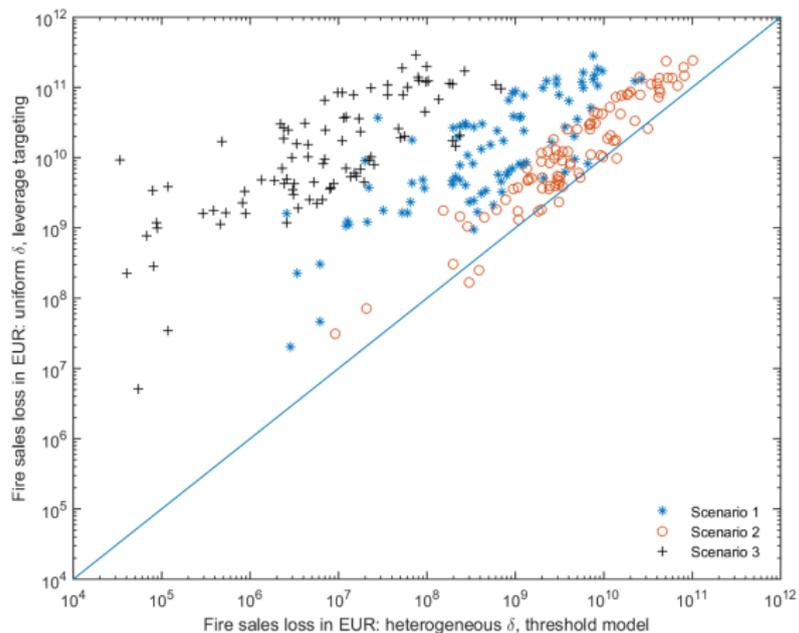


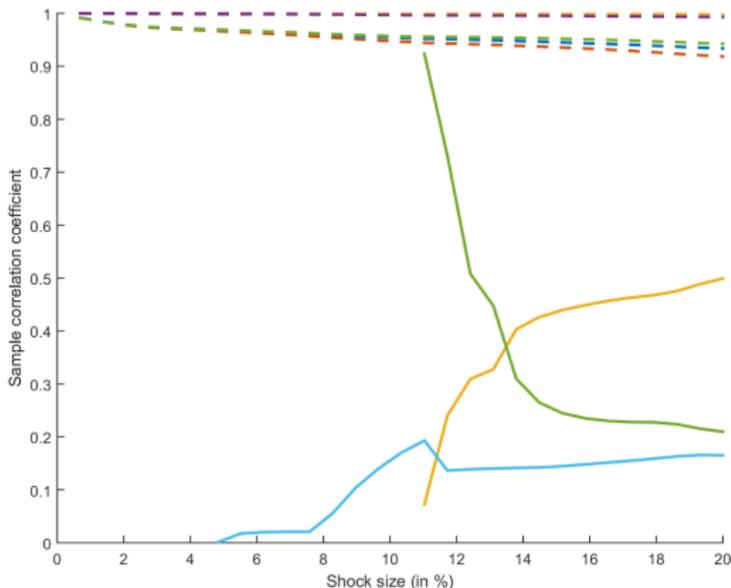
Figure: Fire sales loss for different scenarios and different model combinations.

# Sensitivity to initial stress scenario

Scenario combination	Sample correlation coefficient
1 & 2	0.0840
1 & 3	0.2130
1 & 4	-0.1449
2 & 3	-0.0509
2 & 4	0.0394
3 & 4	-0.0149

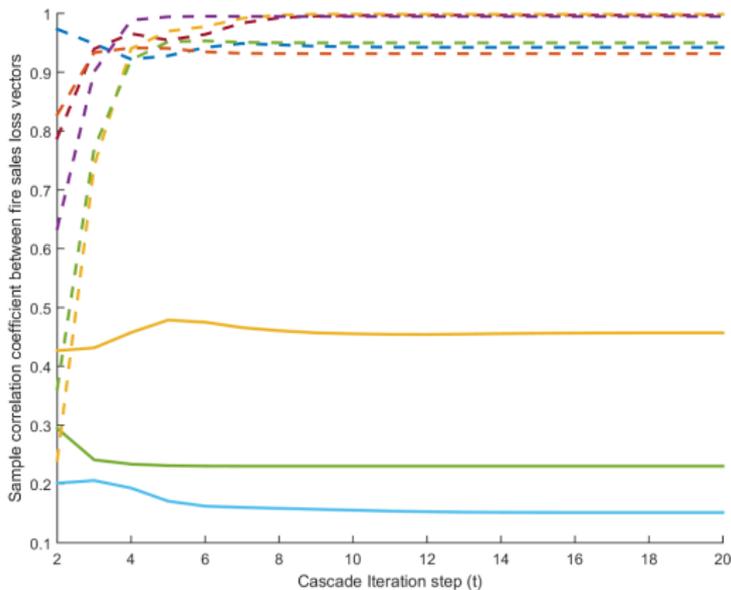
**Table:** Sample correlations between the initial loss vectors from the stress scenarios. The four stress scenarios are very different in terms of which banks are hit by the corresponding shock.

# Sensitivity to initial stress scenario



**Figure:** The pairwise sample correlation between the fire sales loss vectors of different scenarios as a function of the initial shock. Threshold model full lines - leverage targeting dashed lines.

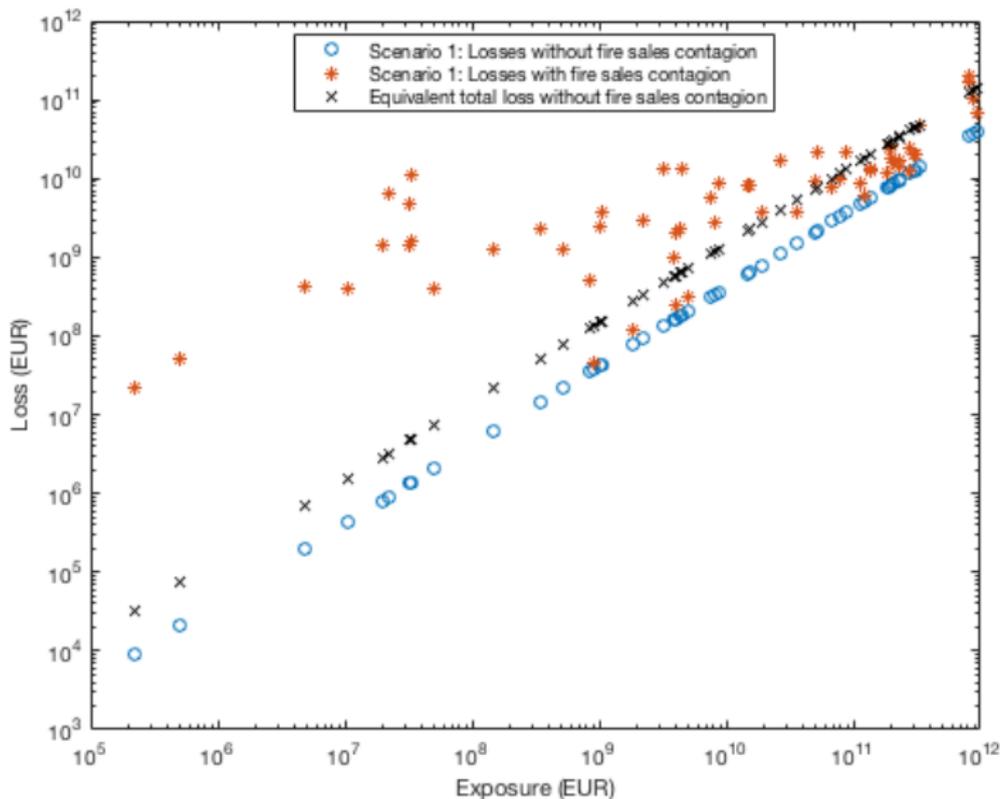
# Sensitivity to initial stress scenario



**Figure:** The evolution of the pairwise sample correlation during the fire sales cascade for a given scenario. Threshold full - leverage targeting dashed.

# Conclusion

## Account for fire sales losses “without fire sales model?”



# Conclusions

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- Liquidity-weighted overlaps lead to a bank-level indicator that may be used for monitoring and for quantifying the contribution of a financial institution to price-mediated contagion;
- The phenomenon of fire sales calls for the collection of portfolio holdings data on a broad scale (banks *and* shadow banks)

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- Contagion through fire sales cannot be accounted for simply by applying a larger macro-shock to bank portfolios;
- Results in our model differ significantly from results obtained in “leverage targeting” models.

Thank you!

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