Product Differentiation and Oligopoly: a Network Approach

Bruno Pellegrino

Columbia Business School



London School of Economics Fifth Economic Networks and Finance Conference

• Motivation: large dispersion in markups across firms

- Motivation: large dispersion in markups across firms
 - Rising level & dispersion (De Loecker, Eeckhout & Unger, 2020)

- Motivation: large dispersion in markups across firms
 - Rising level & dispersion (De Loecker, Eeckhout & Unger, 2020)
 - Rising industry concentration (Kwon et al. 2022)

- Motivation: large dispersion in markups across firms
 - Rising level & dispersion (De Loecker, Eeckhout & Unger, 2020)
 - Rising industry concentration (Kwon et al. 2022)
- **Research Question**: what's behind this heterogeneity? What's driving these trends? What are the welfare implications?

- Motivation: large dispersion in markups across firms
 - Rising level & dispersion (De Loecker, Eeckhout & Unger, 2020)
 - Rising industry concentration (Kwon et al. 2022)
- **Research Question**: what's behind this heterogeneity? What's driving these trends? What are the welfare implications?
 - Consumer surplus and deadweight loss due to oligopoly

- Motivation: large dispersion in markups across firms
 - Rising level & dispersion (De Loecker, Eeckhout & Unger, 2020)
 - Rising industry concentration (Kwon et al. 2022)
- **Research Question**: what's behind this heterogeneity? What's driving these trends? What are the welfare implications?
 - Consumer surplus and deadweight loss due to oligopoly
- **Challenge**: IO question in a macroeconomic setting:

- Motivation: large dispersion in markups across firms
 - Rising level & dispersion (De Loecker, Eeckhout & Unger, 2020)
 - Rising industry concentration (Kwon et al. 2022)
- **Research Question**: what's behind this heterogeneity? What's driving these trends? What are the welfare implications?
 - Consumer surplus and deadweight loss due to oligopoly
- **Challenge**: IO question in a macroeconomic setting:
 - Tools of empirical IO are not available (scalability, lack of data)

- Motivation: large dispersion in markups across firms
 - Rising level & dispersion (De Loecker, Eeckhout & Unger, 2020)
 - Rising industry concentration (Kwon et al. 2022)
- **Research Question**: what's behind this heterogeneity? What's driving these trends? What are the welfare implications?
 - Consumer surplus and deadweight loss due to oligopoly
- **Challenge**: IO question in a macroeconomic setting:
 - Tools of empirical IO are not available (scalability, lack of data)
 - No systematic, objective way to define product markets.

• Methodology: use network tools to bring IO into macro.

- Methodology: use network tools to bring IO into macro.
- Theory of oligopoly and markups in general equilibrium

- **Methodology**: use network tools to bring IO into macro.
- Theory of oligopoly and markups in general equilibrium
 - Forget about industries: in this model, oligopolistic firms compete in a network of product market rivalries.

- **Methodology**: use network tools to bring IO into macro.
- Theory of oligopoly and markups in general equilibrium
 - Forget about industries: in this model, oligopolistic firms compete in a network of product market rivalries.
 - ► New demand system: Generalized Hedonic-Linear (GHL).

- **Methodology**: use network tools to bring IO into macro.
- Theory of oligopoly and markups in general equilibrium
 - Forget about industries: in this model, oligopolistic firms compete in a network of product market rivalries.
 - ► New demand system: Generalized Hedonic-Linear (GHL).
- Taken to the data (and validated) for universe of US public firms, using product similarity data by Hoberg & Phillips (2016).

- **Methodology**: use network tools to bring IO into macro.
- Theory of oligopoly and markups in general equilibrium
 - Forget about industries: in this model, oligopolistic firms compete in a network of product market rivalries.
 - ► New demand system: Generalized Hedonic-Linear (GHL).
- Taken to the data (and validated) for universe of US public firms, using product similarity data by Hoberg & Phillips (2016).
- **Decompose markups** into 2 forces: productivity and centrality.

- Methodology: use network tools to bring IO into macro.
- Theory of oligopoly and markups in general equilibrium
 - Forget about industries: in this model, oligopolistic firms compete in a network of product market rivalries.
 - ► New demand system: Generalized Hedonic-Linear (GHL).
- Taken to the data (and validated) for universe of US public firms, using product similarity data by Hoberg & Phillips (2016).
- **Decompose markups** into 2 forces: productivity and centrality.
- Welfare measurement: large, increasing oligopoly deadweight loss (12.7% of total surplus in 2019), major distributional effects.

Literature

- Rising Markups and Industry Concentration: De Loecker, Eeckhout & Unger (2020), Grullon, Larkin & Michaely (2019); Kwon, Ma & Zimmermann (2021), Eeckhout & Veldkamp (2022).
- Distortions, Input/Output, Micro Origins of Aggregate TFP: Gabaix (2011); Acemoglu, Carvalho, Ozdaglar, Tahbaz-Salehi (2012); Baqaee & Farhi (2020); Bigio & La'O (2020); Edmond, Midrigan & Xu (2019); Carvalho, Elliot & Spray (2022);
- Hedonic Demand/Empirical IO: Lancaster (1968); Rosen (1974); Epple (1987) Berry, Levinsohn & Pakes (1994); Nevo (2001)...
- Network Games: Ballester, Calvo-Armengol & Zenou (2006); Galeotti, Golub, Goyal, Talamer & Tamuz (2022).
- Text Analysis/Product Similarity: Hoberg & Phillips (2016).

Theory

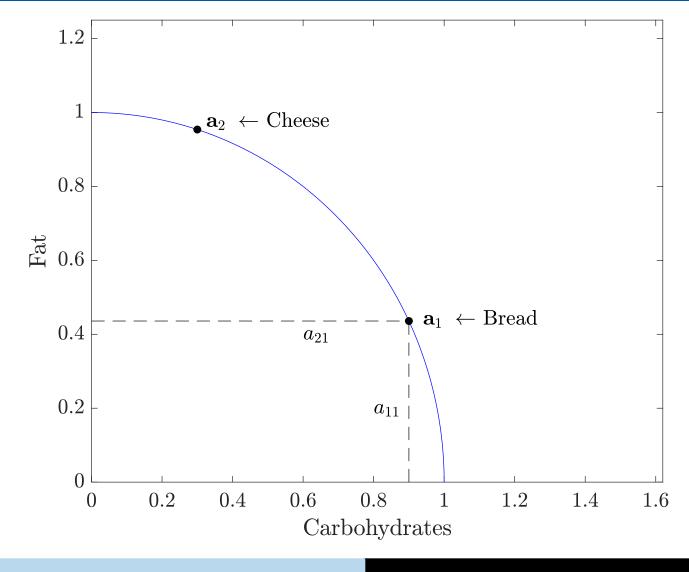
• i = 1, 2, ..., n firms that behave as oligopolists.

- i = 1, 2, ..., n firms that behave as oligopolists.
- Hedonic demand: each firm's product is a bundle of characteristics (Lancaster, 1968; Rosen, 1974; Epple, 1987; Berry, Levinsohn & Pakes 1994; etc.)

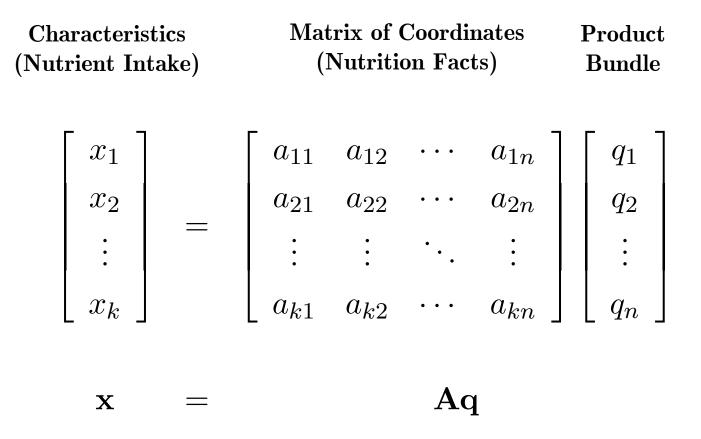
- i = 1, 2, ..., n firms that behave as oligopolists.
- Hedonic demand: each firm's product is a bundle of characteristics (Lancaster, 1968; Rosen, 1974; Epple, 1987; Berry, Levinsohn & Pakes 1994; etc.)
- 1 unit of product *i* provides:
 - 1 unit of an idiosyncratic characteristic i

- i = 1, 2, ..., n firms that behave as oligopolists.
- Hedonic demand: each firm's product is a bundle of characteristics (Lancaster, 1968; Rosen, 1974; Epple, 1987; Berry, Levinsohn & Pakes 1994; etc.)
- 1 unit of product *i* provides:
 - 1 unit of an idiosyncratic characteristic i
 - a vector of k common characteristics \mathbf{a}_i (length 1)

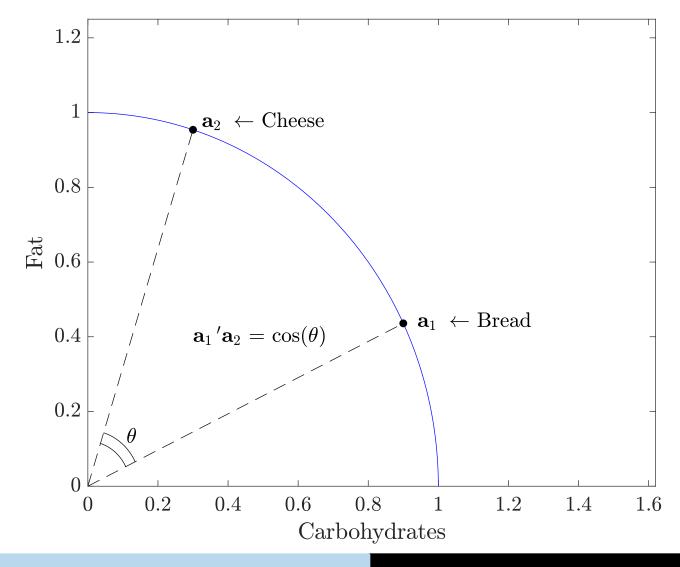
A basic example: 2 firms, 2 characteristics



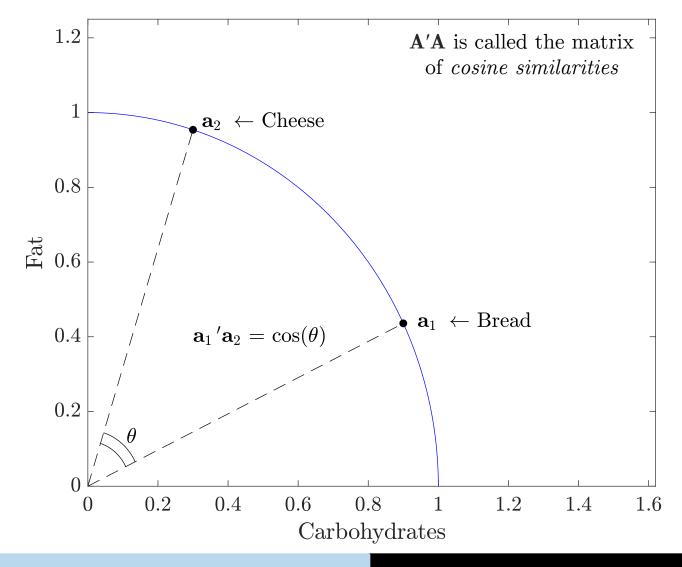
Aggregating common characteristics



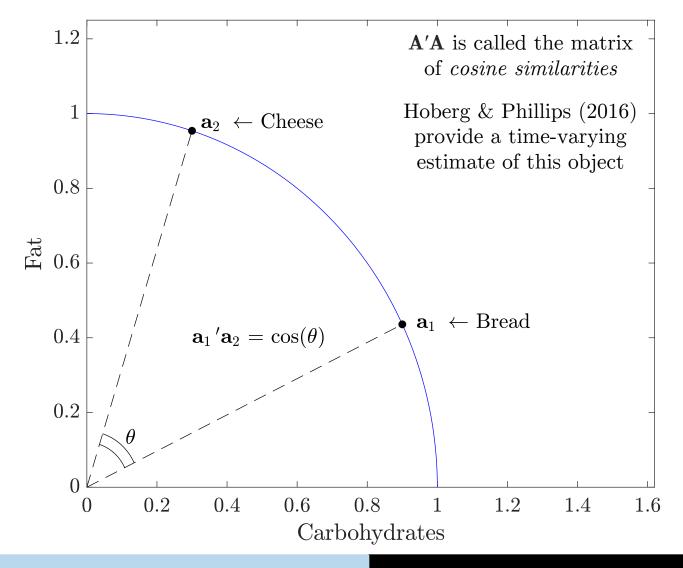
Defining Cosine Similarity



Defining Cosine Similarity



Defining Cosine Similarity



Representative Consumer-Worker-Investor

• Quadratic utility
$$U(\mathbf{x},\mathbf{y},H) =$$

$$\alpha \cdot \sum_{k=1}^{m} \left(b_k^x x_k - \frac{1}{2} x_k^2 \right) + (1 - \alpha) \sum_{i=1}^{n} \left(b_i^y y_i - \frac{1}{2} y_i^2 \right) - H$$

Representative Consumer-Worker-Investor

• Quadratic utility
$$U(\mathbf{x},\mathbf{y},H) =$$

$$\alpha \cdot \sum_{k=1}^{m} \left(b_k^x x_k - \frac{1}{2} x_k^2 \right) + (1 - \alpha) \sum_{i=1}^{n} \left(b_i^y y_i - \frac{1}{2} y_i^2 \right) - H$$

•
$$H =$$
 hours worked – numeraire

Representative Consumer-Worker-Investor

• Quadratic utility
$$U(\mathbf{x},\mathbf{y},H) =$$

$$\alpha \cdot \sum_{k=1}^{m} \left(b_k^x x_k - \frac{1}{2} x_k^2 \right) + (1 - \alpha) \sum_{i=1}^{n} \left(b_i^y y_i - \frac{1}{2} y_i^2 \right) - H$$

•
$$H =$$
 hours worked – numeraire

 Consumer faces vector of prices p and chooses demand q, subject to profits and labor income being ≥ p'q.

$$\mathbf{p} = \mathbf{b} - (\mathbf{I} + \mathbf{\Sigma}) \mathbf{q}$$

$$\mathbf{p} = \mathbf{b} - (\mathbf{I} + \mathbf{\Sigma}) \mathbf{q}$$

where $\mathbf{\Sigma} \stackrel{\mathrm{def}}{=} \alpha (\mathbf{A}' \mathbf{A} - \mathbf{I})$

$$\mathbf{p} = \mathbf{b} - (\mathbf{I} + \boldsymbol{\Sigma}) \mathbf{q}$$

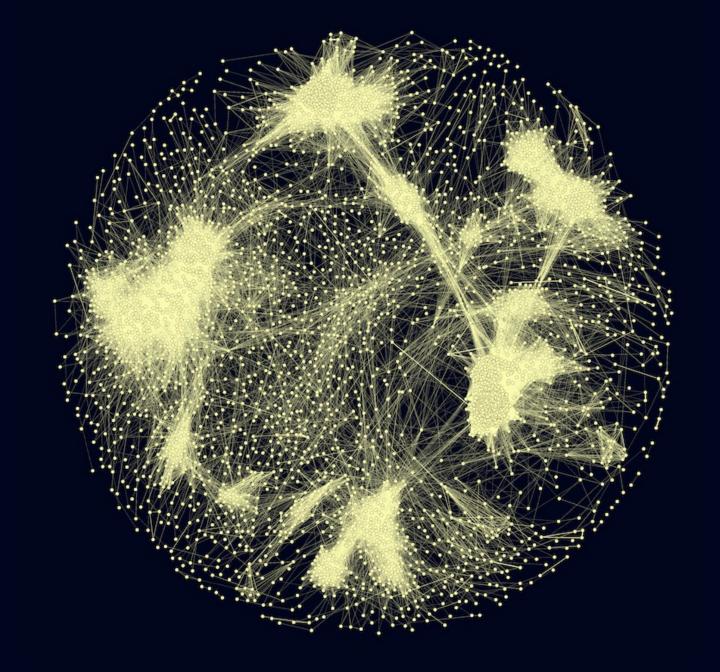
where
$$\boldsymbol{\Sigma} \stackrel{\text{def}}{=} \alpha \left(\mathbf{A}' \mathbf{A} - \mathbf{I} \right)$$

- Cournot Competition: firm *i* chooses supply q_i to maximize profits function $\pi_i \rightarrow$ (Linear-quadratic) Network game
 - Ballester, Calvó-Armengol & Zenou, 2006

$$\mathbf{p} = \mathbf{b} - (\mathbf{I} + \mathbf{\Sigma}) \mathbf{q}$$

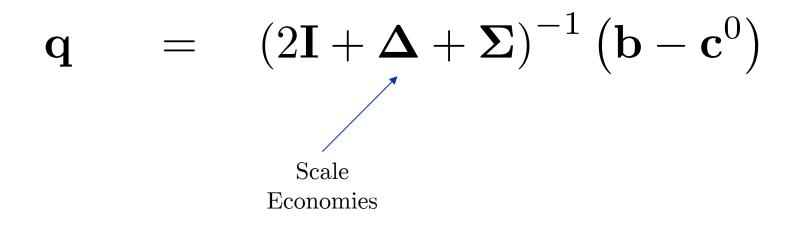
where
$$\boldsymbol{\Sigma} \stackrel{\text{def}}{=} \alpha \left(\mathbf{A}' \mathbf{A} - \mathbf{I} \right)$$

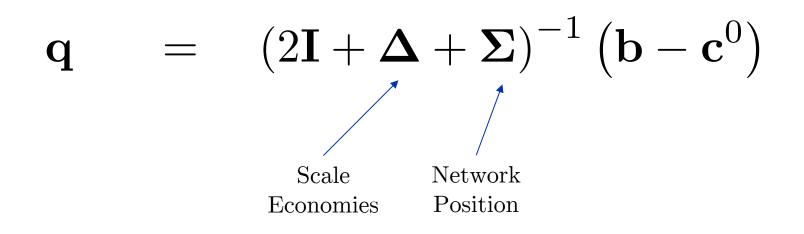
- Cournot Competition: firm *i* chooses supply q_i to maximize profits function $\pi_i \rightarrow$ (Linear-quadratic) Network game
 - Ballester, Calvó-Armengol & Zenou, 2006
- Why? the matrix of cosine similarities A'A (proportional to Σ) can be thought of as an adjacency matrix of a network

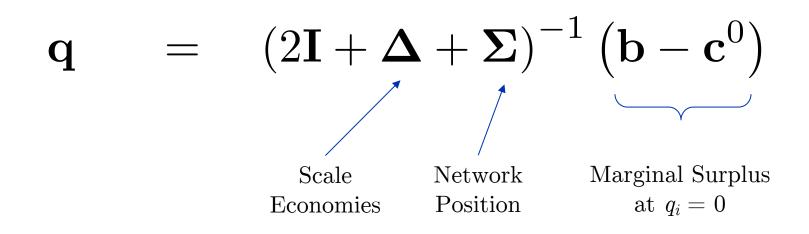


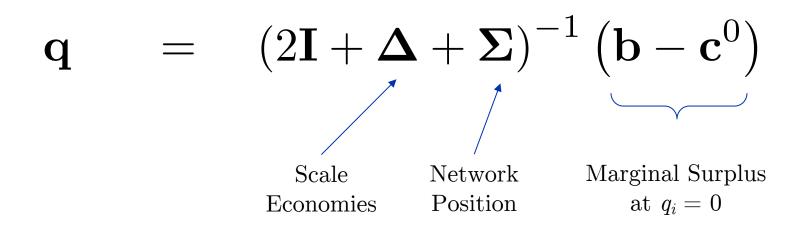
Cournot-Nash Equilibrium

$\mathbf{q} = (2\mathbf{I} + \boldsymbol{\Delta} + \boldsymbol{\Sigma})^{-1} (\mathbf{b} - \mathbf{c}^{0})$









The expression above can be shown to be a measure of network centrality (Katz-Bonacich)

Hedonic-Adjusted Productivity

$$\omega_i \stackrel{\text{def}}{=} \frac{b_i}{c_i}$$

- Accounts for product quality
- Volumetric-invariant
- Comparable across widely-different firms

Decomposing Markups

$\mu_i = \chi_i + (1 - \chi_i) \,\overline{\mu}_i$

Decomposing Markups

$\mu_i = \chi_i + (1-\chi_i)\,ar\mu_i$

Decomposing Markups

$\mu_i = \chi_i + (1-\chi_i) \, ar{\mu}_i$

Product Market Centrality

Depends on the entire matrix of cosine similarities $\mathbf{A'A}$. The profit share of surplus is a decreasing function of χ_i alone

Data and Validation

• Similarity scores constructed by text mining the "Business Description" section of 10-K filings; already standard in Finance.

- Similarity scores constructed by text mining the "Business Description" section of 10-K filings; already standard in Finance.
- Solve long-standing problems with NAICS/SIC: static, binary do not really reflect product market competition (not used in I.O.)

- Similarity scores constructed by text mining the "Business Description" section of 10-K filings; already standard in Finance.
- Solve long-standing problems with NAICS/SIC: static, binary do not really reflect product market competition (not used in I.O.)
- Construction:

$$\mathbf{v}_{i} = \begin{bmatrix} v_{i,1} \\ v_{i,2} \\ \vdots \\ v_{i,61146} \end{bmatrix} \quad \cos_{ij}^{\mathrm{HP}} \stackrel{\mathrm{def}}{=} \frac{\mathbf{v}_{i}'\mathbf{v}_{j}}{\sqrt{\|\mathbf{v}_{i}\| \|\mathbf{v}_{j}\|}}$$

- Similarity scores constructed by text mining the "Business Description" section of 10-K filings; already standard in Finance.
- Solve long-standing problems with NAICS/SIC: static, binary do not really reflect product market competition (not used in I.O.)
- Construction:

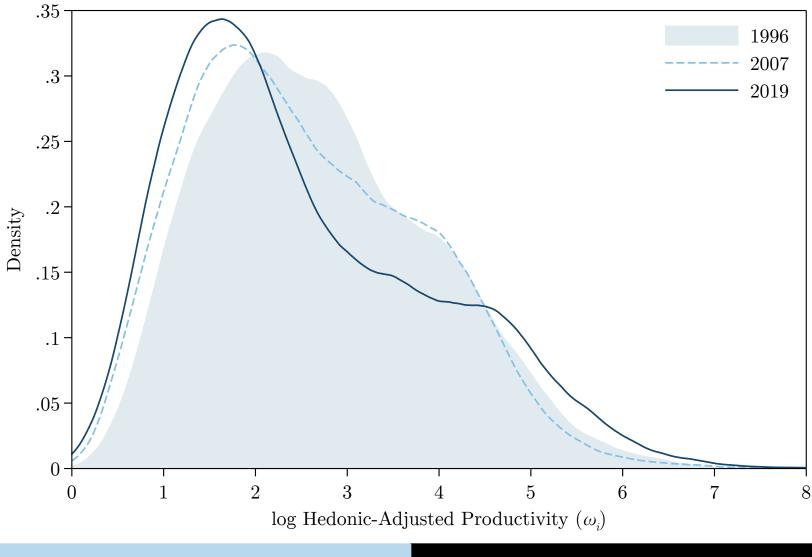
$$\mathbf{v}_{i} = \begin{bmatrix} v_{i,1} \\ v_{i,2} \\ \vdots \\ v_{i,61146} \end{bmatrix} \quad \cos_{ij}^{\mathrm{HP}} \stackrel{\mathrm{def}}{=} \frac{\mathbf{v}_{i}'\mathbf{v}_{j}}{\sqrt{\|\mathbf{v}_{i}\| \|\mathbf{v}_{j}\|}}$$

• Identification: \mathbf{a}_i and \mathbf{v}_i are collinear $\Rightarrow \mathbf{a}_i \mathbf{a}_j \equiv \cos_{ij}^{HP}$

Demand Elasticity Market Firm *i* Firm j**Micro Estimate GHL** (*text-based*) Ford Auto Ford -4.320-5.197Ford General Motors Auto 0.0340.056 Auto Ford Toyota 0.0070.017Auto **General Motors** Ford 0.0650.052 Auto General Motors General Motors -6.433-4.685Auto General Motors 0.008 0.005Toyota Auto Toyota Ford 0.018 0.025 Auto General Motors 0.008 0.008 Toyota Auto Toyota Toyota -3.085-4.851Cereals Kellogg's Kellogg's -3.231-1.770Kellogg's Quaker Oats Cereals 0.033 0.023 Cereals Quaker Oats Kellogg's 0.046 0.031Cereals Quaker Oats Quaker Oats -3.031-1.941Computers Apple Apple -11.979-8.945Computers Apple Dell 0.018 0.025Computers Dell Apple 0.027 0.047Computers Dell Dell -5.570-5.110

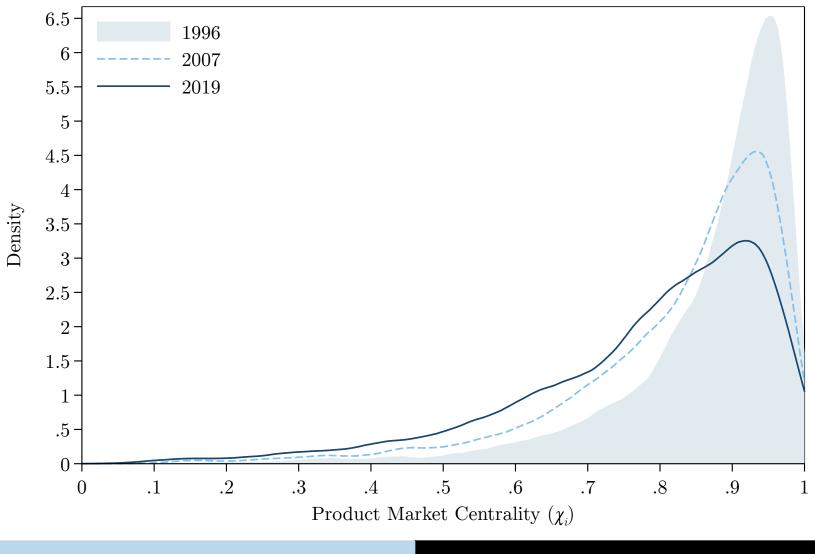
Empirics

Distribution of Hedonic-Adjusted Productivity



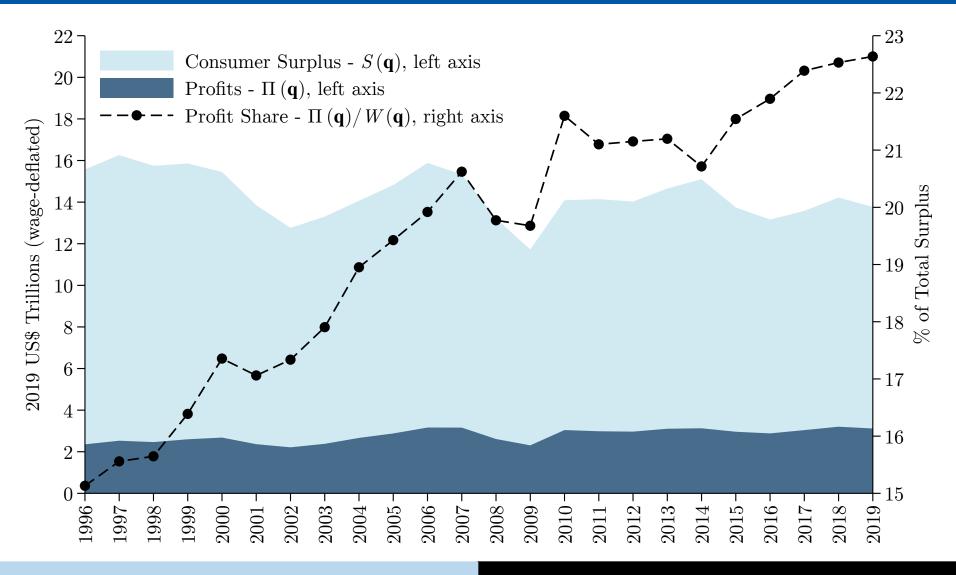
Product Differentiation and Oligopoly: a Network Approach Bruno Pellegrino (Columbia Business School)

Distribution of Product Market Centrality



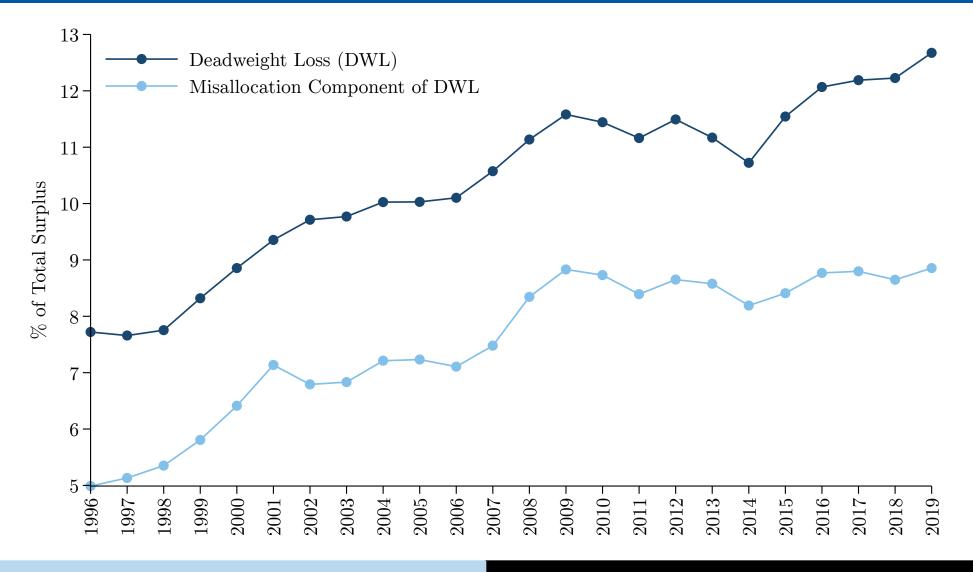
Product Differentiation and Oligopoly: a Network Approach Bruno Pellegrino (Columbia Business School)

Total Surplus and its Distribution



Product Differentiation and Oligopoly: a Network Approach Bruno Pellegrino (Columbia Business School)

Deadweight Loss from Oligopoly



• Private and foreign firms, entry and exit

- Private and foreign firms, entry and exit
 - Aggregation result: add competitive fringes of atomistic firms in the form of a representative firms.
 - Can be located using firm-sector similarity from FHP.

- Private and foreign firms, entry and exit
 - Aggregation result: add competitive fringes of atomistic firms in the form of a representative firms.
 - Can be located using firm-sector similarity from FHP.
- Non-flat marginal cost

- Private and foreign firms, entry and exit
 - Aggregation result: add competitive fringes of atomistic firms in the form of a representative firms.
 - Can be located using firm-sector similarity from FHP.
- Non-flat marginal cost
- Exclude "non-tradable" industries

- Private and foreign firms, entry and exit
 - Aggregation result: add competitive fringes of atomistic firms in the form of a representative firms.
 - Can be located using firm-sector similarity from FHP.
- Non-flat marginal cost
- Exclude "non-tradable" industries
- Bertrand

- Private and foreign firms, entry and exit
 - Aggregation result: add competitive fringes of atomistic firms in the form of a representative firms.
 - Can be located using firm-sector similarity from FHP.
- Non-flat marginal cost
- Exclude "non-tradable" industries
- Bertrand
- Multi-product firms (using Compustat Segments)

- Private and foreign firms, entry and exit
 - Aggregation result: add competitive fringes of atomistic firms in the form of a representative firms.
 - Can be located using firm-sector similarity from FHP.
- Non-flat marginal cost
- Exclude "non-tradable" industries
- Bertrand
- Multi-product firms (using Compustat Segments)
- Input-Output Linkages (using Atalay et al. 2011 IO data)

A Tale of Two Networks: Common Ownership and Product Market Rivalry

Florian Ederer BU Questrom Bruno Pellegrino Columbia GSB

4

London School of Economics Fifth Economic Networks and Finance Conference

• **Definition**: the degree to which two firms that compete in product and/or labor markets are owned by few, overlapping investors.

- **Definition**: the degree to which two firms that compete in product and/or labor markets are owned by few, overlapping investors.
- The Common Ownership hypothesis (Rotemberg, 1984):
 - Consider a standard oligopolistic market, but assume that instead of maximizing profits, firms maximize investors' value.

- **Definition**: the degree to which two firms that compete in product and/or labor markets are owned by few, overlapping investors.
- The Common Ownership hypothesis (Rotemberg, 1984):
 - Consider a standard oligopolistic market, but assume that instead of maximizing profits, firms maximize investors' value.
 - CO leads to softening of competition *without any collusion*.

- **Definition**: the degree to which two firms that compete in product and/or labor markets are owned by few, overlapping investors.
- The Common Ownership hypothesis (Rotemberg, 1984):
 - Consider a standard oligopolistic market, but assume that instead of maximizing profits, firms maximize investors' value.
 - CO leads to softening of competition *without any collusion*.
- Rising Common Ownership (Gilje, Gormley & Levit 2020; Backus, Conlon & Sinkinson, 2021) → Huge policy/research interest:
 - Consolidation in asset management industry is putting stock ownership in the hands of a few large institutional investors.

What are the welfare implications of common ownership?

What are the welfare implications of common ownership?

→ Depends on ownership as well!

• There are Z funds indexed by z = 1, 2, ..., Z. Fund z own shares s_{iz} in company *i*. Then fund z's total income is:

$$V_z \stackrel{\text{def}}{=} \sum_{i=1}^n s_{iz} \pi_i$$
 and $\sum_{z=1}^Z s_{iz} = 1$

• There are Z funds indexed by z = 1, 2, ..., Z. Fund z own shares s_{iz} in company *i*. Then fund z's total income is:

$$V_z \stackrel{\text{def}}{=} \sum_{i=1}^n s_{iz} \pi_i$$
 and $\sum_{z=1}^Z s_{iz} = 1$

• Firm *i* picks *q_i* to maximize the share-weighted profits of its investors (Rotemberg 1984 – we shall relax this later):

Common Ownersł

• There are Z funds indexed by z = 1, 2, ..., Z. Fund z own shares s_{iz} in company *i*. Then fund z's total income is:

$$V_z \stackrel{\text{def}}{=} \sum_{i=1}^n s_{iz} \pi_i$$
 and $\sum_{z=1}^Z s_{iz} = 1$

• Firm *i* picks *q_i* to maximize the share-weighted profits of its investors (Rotemberg 1984 – we shall relax this later):

$$\phi_i \stackrel{\text{def}}{=} \sum_{z=1}^Z s_{iz} V_z$$

 $\log q$

• We can write i's objective function in terms of profit weights:

$$\phi_i \propto \pi_i + \sum_{j \neq i} \kappa_{ij} \pi_j$$

• We can write i's objective function in terms of profit weights:

$$\phi_i \propto \pi_i + \sum_{j \neq i} \kappa_{ij} \pi_j \qquad \kappa_{ij} = \frac{\sum_{z=1}^Z s_{iz} s_{jz}}{\sum_{z=1}^Z s_{iz} s_{iz}}$$

• We can write i's objective function in terms of profit weights:

$$\phi_i \propto \pi_i + \sum_{j \neq i} \kappa_{ij} \pi_j \qquad \kappa_{ij} = \frac{\sum_{z=1}^Z s_{iz} s_{jz}}{\sum_{z=1}^Z s_{iz} s_{iz}}$$

• Using institutional shareholding data (forms 13-F) we can compute all of the profit weights and perform counterfactuals.

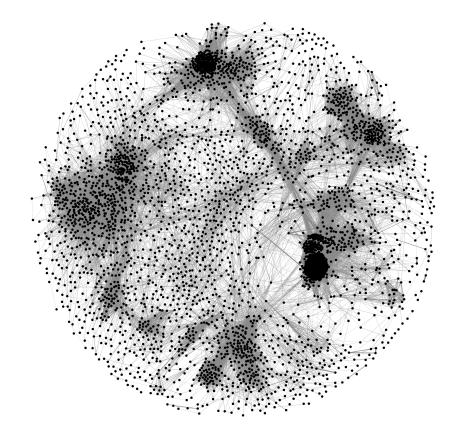
• We can write i's objective function in terms of profit weights:

$$\phi_i \propto \pi_i + \sum_{j \neq i} \kappa_{ij} \pi_j \qquad \kappa_{ij} = \frac{\sum_{z=1}^Z s_{iz} s_{jz}}{\sum_{z=1}^Z s_{iz} s_{iz}}$$

- Using institutional shareholding data (forms 13-F) we can compute all of the profit weights and perform counterfactuals.
- Equilibrium:

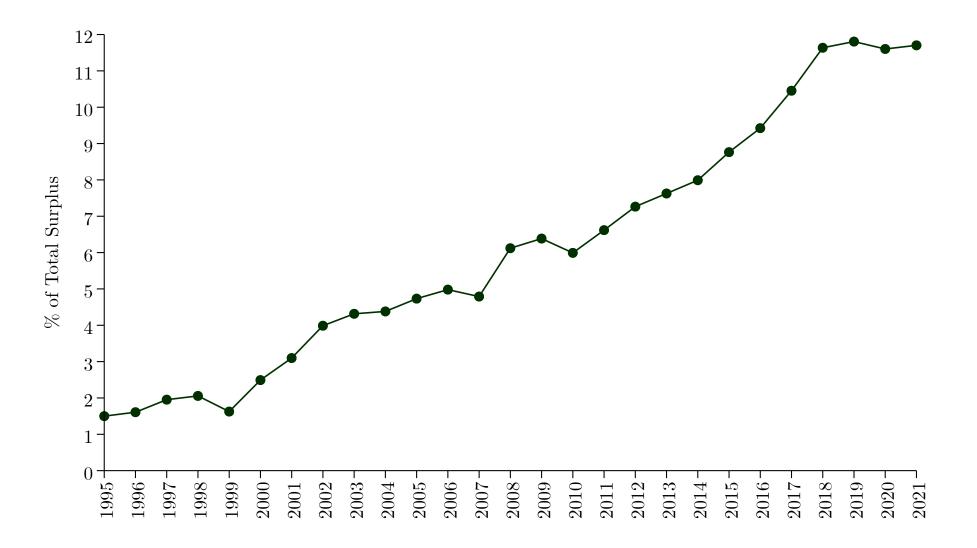
$$\mathbf{q} = (2\mathbf{I} + \boldsymbol{\Delta} + \boldsymbol{\Sigma} + \mathbf{K} \circ \boldsymbol{\Sigma})^{-1} (\mathbf{b} - \mathbf{c})$$

A Tale of Two Networks

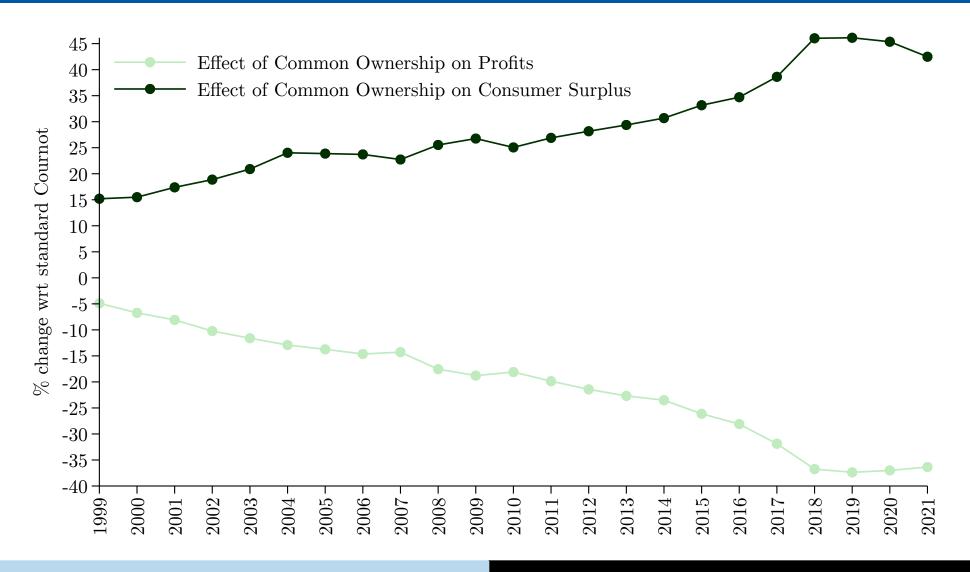


Product Market Similarity - **A'A** based on 10-K (Hoberg & Phillips, 2016) Common Ownership Weights – **K** based on 13-F data (Backus et al. 2021)

Deadweight Loss



Effect of CO on Profits and Consumer Surplus

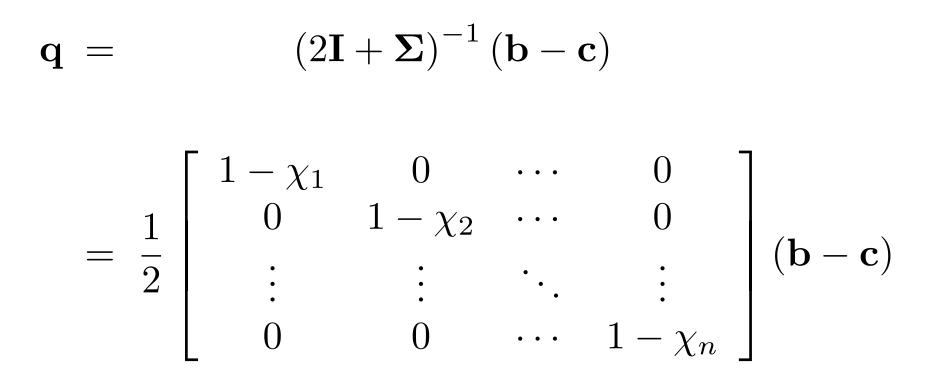


- A new GE theory of oligopoly with hedonic demand.
- Estimated for Compustat using 10-K product similarities.
- Distribution of markups is jointly determined by productivity and product market centrality.
 - Both have undergone significant changes
- Rising Oligopoly Power
 - increasing deadweight loss
 - lower consumer surplus share.

☞ I share the data! (elasticities, centrality, productivity...)

thank you

Product Market Centrality



10-K-Based Classifications of Firms in Business Services (SIC-3 = 737)

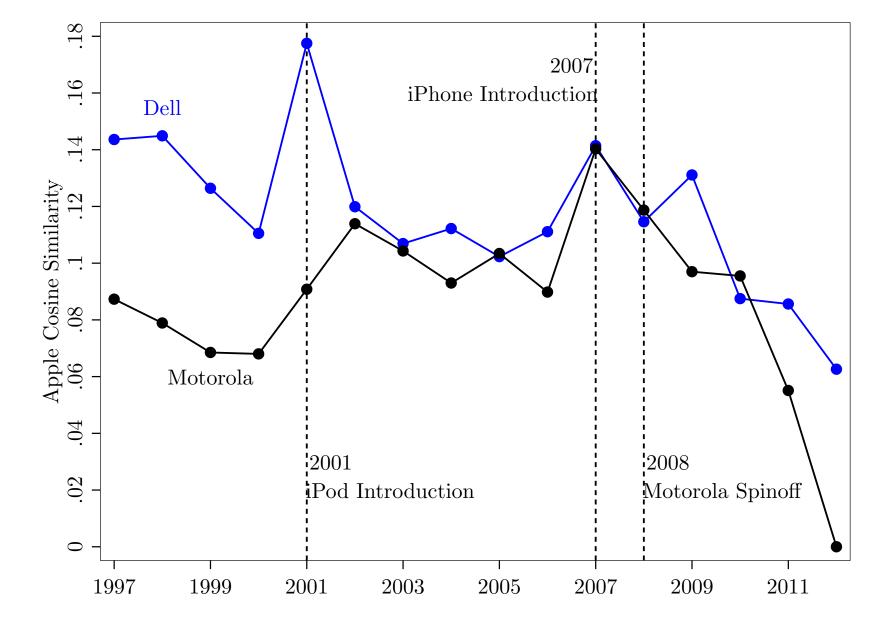
Submarket 1: Entertainment (Sample Focal Firm: Wanderlust Interactive)

43 rivals: Maxis, Piranha Interactive Publishing, Brilliant Digital Entertainment, Midway Games, Take Two Interactive Software, THQ, 3DO, New Frontier Media, . . .
SIC codes of rivals: computer programming and data processing [SIC-3 = 737] (24 rivals), motion picture production and allied services [SIC-3 = 781] (4 rivals), miscellaneous other (13 rivals)

Core words: entertainment (42), video (42), television (38), royalties (35), internet (34), content (33), creative (31), promotional (31), copyright (31), game (30), sound (29), publishing (29), . . .

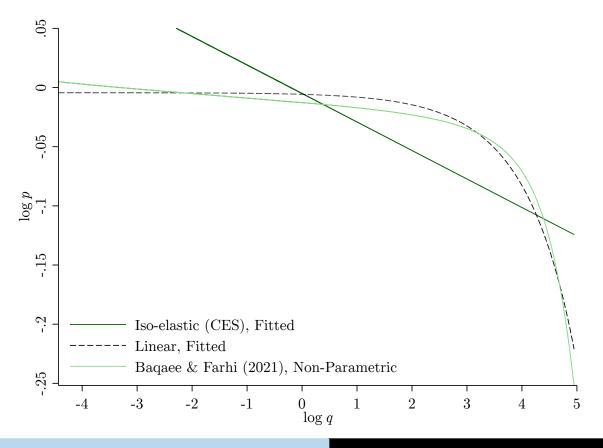
Submarket 2: Medical Services (Sample Focal Firm, Quadramed Corp.)

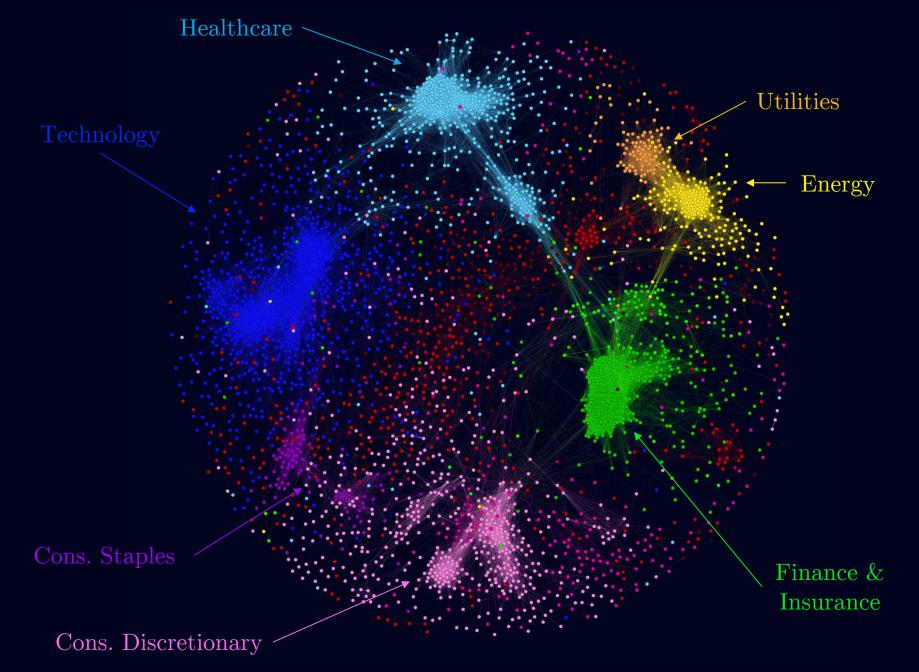
- 66 rivals: IDX Systems, Medicus Systems, Hpr, Simione Central Holdings, National Wireless Holdings, HCIA, Apache Medical Systems, . . .
- SIC codes of rivals: computer programming and data processing [SIC-3 = 737] (45 rivals), insurance agents, brokers, and service [SIC-3 = 641] (5 rivals), miscellaneous health services [SIC-3 = 809] (4 rivals), management and public relations services [SIC-3 = 874] (3 rivals), miscellaneous other (9 rivals)
- Core words: client (59), database (54), solution (49), patient (47), copyright (47), secret (47), physician (47), hospital (46), health care (46), server (45), resource (44), functionality (44), billing (44), . . .

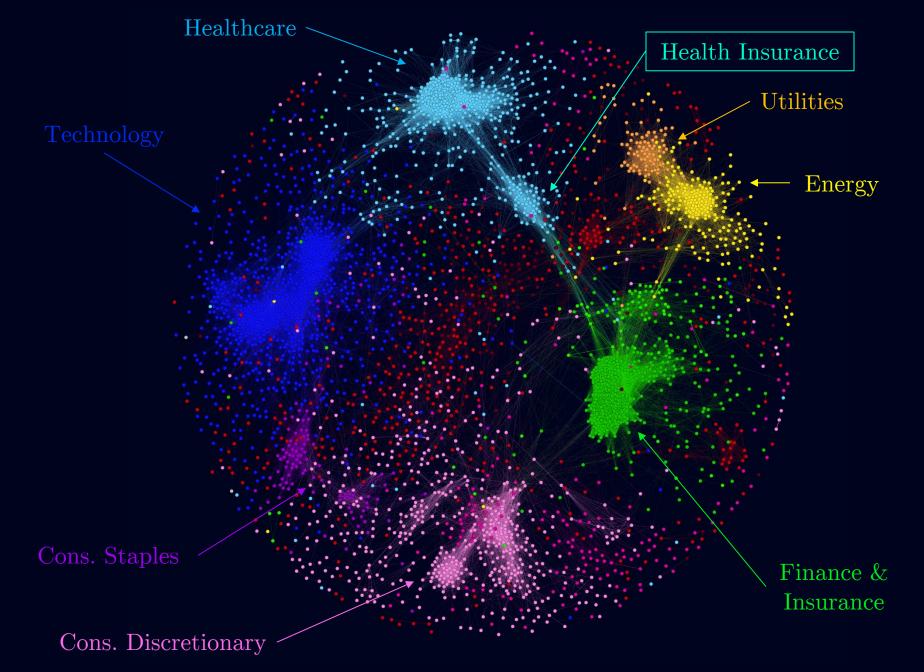


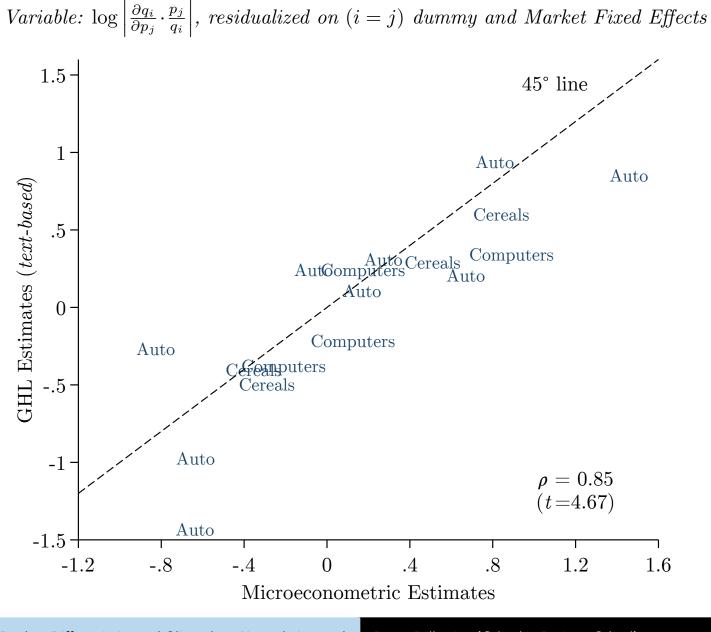
Linear Demand

- 1. Allows to write demand in terms of cosine similarity
- 2. Already standard in literature (see Syverson 2019 JEP review)
- 3. Data is begging you to use it

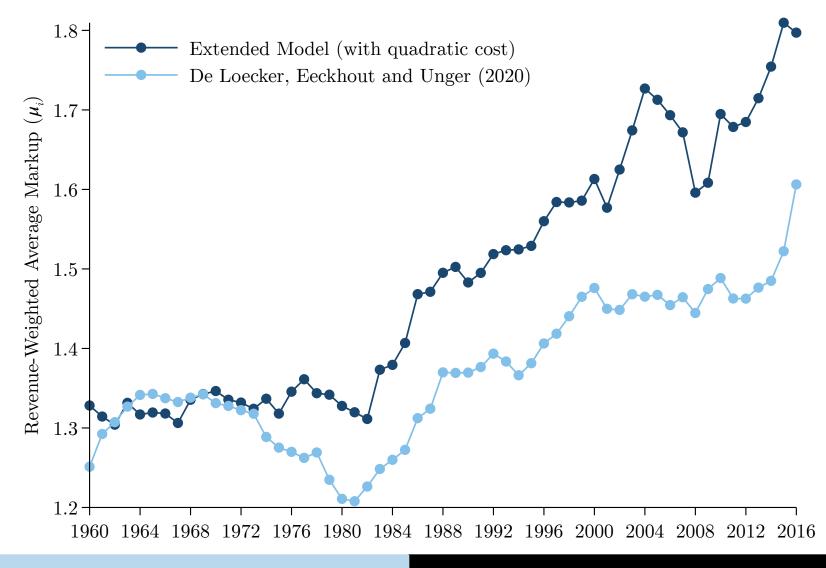








Markups: Time Series



Profits, Potential and Welfare

$$\Pi (\mathbf{q}) = \mathbf{q}' (\mathbf{b} - \mathbf{c}^{0}) - \frac{1}{2} \cdot \mathbf{q}' (2\mathbf{I} + \mathbf{\Delta} + 2\mathbf{\Sigma}) \mathbf{q} - F$$

$$\Phi (\mathbf{q}) = \mathbf{q}' (\mathbf{b} - \mathbf{c}^{0}) - \frac{1}{2} \cdot \mathbf{q}' (2\mathbf{I} + \mathbf{\Delta} + \mathbf{\Sigma}) \mathbf{q} - F$$

$$W (\mathbf{q}) = \mathbf{q}' (\mathbf{b} - \mathbf{c}^{0}) - \frac{1}{2} \cdot \mathbf{q}' (\mathbf{I} + \mathbf{\Delta} + \mathbf{\Sigma}) \mathbf{q} - F$$

$$\text{where} \quad \mathbf{\Delta} \stackrel{\text{def}}{=} \begin{bmatrix} \delta_{1} & 0 & \cdots & 0 \\ 0 & \delta_{2} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \delta_{n} \end{bmatrix} \quad \text{and} \quad F \stackrel{\text{def}}{=} \sum_{i=1}^{n} f_{i}$$

• Compustat: Revenues (p_iq_i) , COGS (TVC_i), SG&A (f_i) .

- Compustat: Revenues (p_iq_i) , COGS (TVC_i), SG&A (f_i) .
- Assume $\delta_i = 0$ (later relaxed). Only one free parameter: α .

- Compustat: Revenues (p_iq_i) , COGS (TVC_i), SG&A (f_i) .
- Assume $\delta_i = 0$ (later relaxed). Only one free parameter: α .
- **Proposition**: $\partial \log p_i / \partial \log q_j$ is observed for firm pair (K,Q):

- Compustat: Revenues (p_iq_i) , COGS (TVC_i), SG&A (f_i) .
- Assume $\delta_i = 0$ (later relaxed). Only one free parameter: α .
- **Proposition**: $\partial \log p_i / \partial \log q_j$ is observed for firm pair (K,Q):

$$\alpha = -\frac{\varepsilon_{\rm KQ} \cdot p_{\rm K} q_{\rm K} + \varepsilon_{\rm QK} \cdot p_{\rm Q} q_{\rm Q}}{2 \cdot \cos_{\rm KQ}^{\rm HP} \cdot \sqrt{p_{\rm K} q_{\rm K} - \rm{TVC}_{\rm K}} \cdot \sqrt{p_{\rm Q} q_{\rm Q} - \rm{TVC}_{\rm Q}}}$$

- Compustat: Revenues (p_iq_i) , COGS (TVC_i), SG&A (f_i) .
- Assume $\delta_i = 0$ (later relaxed). Only one free parameter: α .
- **Proposition**: $\partial \log p_i / \partial \log q_j$ is observed for firm pair (K,Q):

$$\alpha = -\frac{\varepsilon_{\rm KQ} \cdot p_{\rm K} q_{\rm K} + \varepsilon_{\rm QK} \cdot p_{\rm Q} q_{\rm Q}}{2 \cdot \cos_{\rm KQ}^{\rm HP} \cdot \sqrt{p_{\rm K} q_{\rm K} - \rm{TVC}_{\rm K}} \cdot \sqrt{p_{\rm Q} q_{\rm Q} - \rm{TVC}_{\rm Q}}}$$

• Every other object is identified in closed form (correct units).

 $q_i = \sqrt{\pi_i}$ $c_i = \frac{\text{TVC}_i}{q_i}$

$\mathbf{b} = (2\mathbf{I} + \boldsymbol{\Sigma}) \, \mathbf{q} + \mathbf{c}$

Entry and Exit

The paper takes into account entry and exit in two ways.

- <u>Atomistic Firms</u> with quadratic cost and Pareto-distributed productivity that enter/exit endogenously, modelled through a representative firm. New aggregation result that allows for intensive and extensive margin. Results are virtually unchanged under this extension.
- Granular Firms have a choke price: when the social planner forces firms to price at marginal cost (Perfect Competition) some exit. Fewer firms compete much more aggressively (TS ↑)

Adding a representative competitive firm

Proposition 9. Assume that there is a continuum of potential entrants that are indexed by a productivity parameter $\zeta \in (\underline{\zeta}, \infty)$, with $\underline{\zeta} > 0$, and that produce a homogeneous good using the following quadratic cost function:

$$h(\zeta) = \frac{1}{2\zeta} \cdot q^2(\zeta) \tag{2.75}$$

Assume also that the firms face cost of entry equal to one unit of labor and that the probability density of type- ζ potential entrants is given by

$$pdf\left(\zeta\right) = \frac{\beta - 1}{\zeta^{\beta + 1}} \tag{2.76}$$

implying that ζ follows a Pareto distribution with shape parameter β and scale parameter $\underline{\zeta} \stackrel{\text{def}}{=} [(\beta - 1) / \beta]^{\frac{1}{\beta}}.^9$ Then, as the parameter β converges down to 1, the cost function of the corresponding aggregate representative firm is approximated by

$$h_{n+1} = \frac{q_{n+1}^2}{2} \tag{2.77}$$

where and h_{n+1} and q_{n+1} are, respectively, the labor input and the output of the representative firm, and the productivity cutoff for entry converges to $\zeta_{\min} = \frac{1}{q_{n+1}}$.

Because employment and revenues are proportional to ζ , it follows that, if the assumptions above are respected, both the revenue and employment distribution of firms also approximate a Pareto distribution with shape parameter $\beta = 1$, sometimes called a Zipf Law.

Input-Output Linkages

• Leontief production function links intermediate/final output

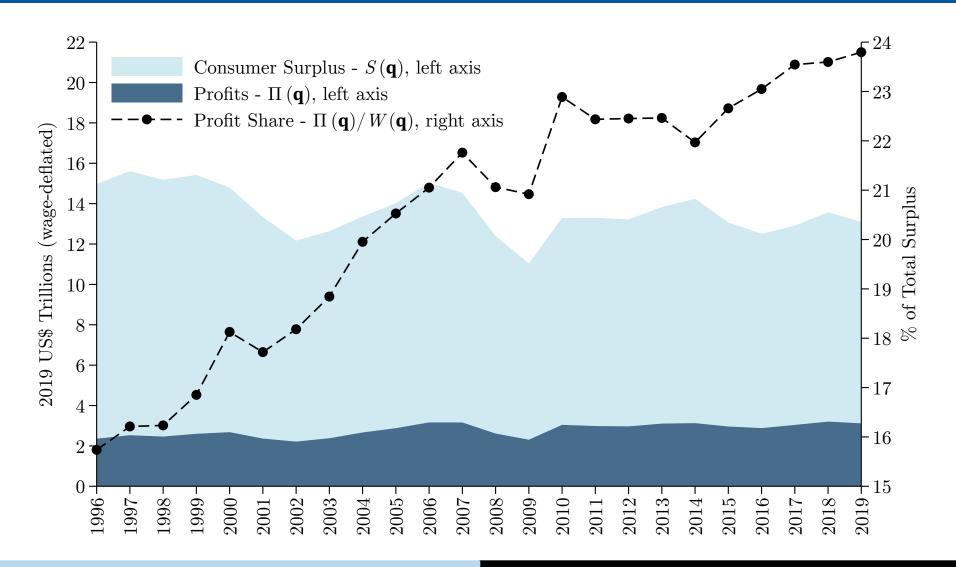
$$\mathbf{q}^{\mathrm{I}} = \mathbb{F}'\mathbf{q}$$
 and $\mathbf{q}^{\mathrm{C}} = (\mathbf{I} - \mathbb{F})'\mathbf{q}$

• Firms are price-takers in input markets - profit vector:

$$\boldsymbol{\pi} = \operatorname{diag}(\mathbf{q}) \left(\mathbf{p} - \mathbf{c}^0 - \mathbb{F}\mathbf{p} \right) - \mathbf{f}$$

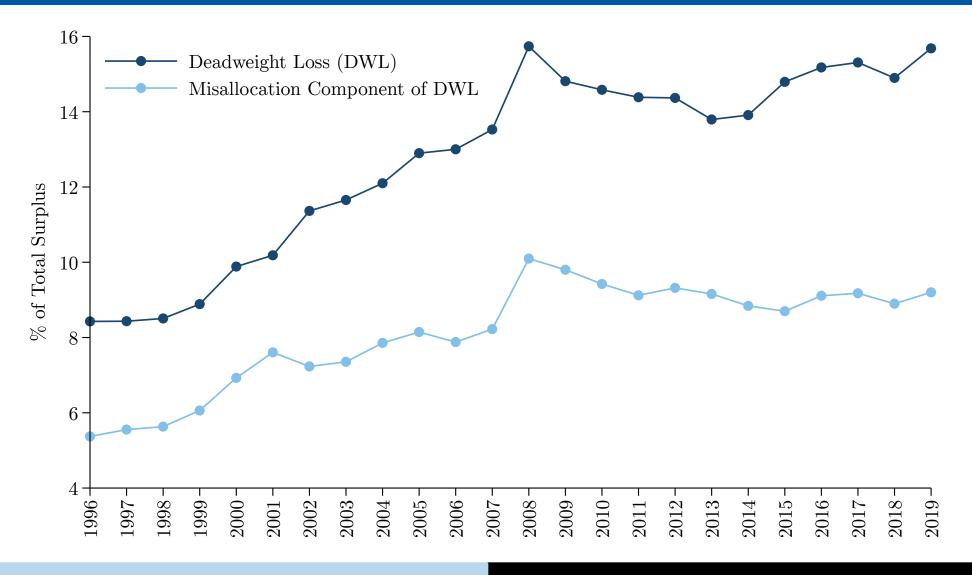
$$\mathbf{q} \; = \; \left\{ \left(\mathbf{I} + \mathbf{1}\mathbf{1}'\right) \circ \left[\left(\mathbf{I} + \boldsymbol{\Sigma}\right) \left(\mathbf{I} - \mathbb{F}\right)' \right] \right\}^{-1} \left[\left(\mathbf{I} - \mathbb{F}\right)\mathbf{b} - \mathbf{c}^0 \right]$$

Total Surplus and its Breakdown (input-output)



Product Differentiation and Oligopoly: a Network Approach Bruno Pellegrino (Columbia Business School)

Deadweight Loss (Input-Output)



Multi-Product Firms and Mergers

Company z maximizes the sum of profits over all product lines i where $o_{iz} = 1$ if company z produces product i:

 \mathbf{n}

$$\varpi_z = \sum_{i=1}^n o_{iz} \pi_i$$

$$\mathbf{K} \equiv \begin{bmatrix} \kappa_{11} & \kappa_{21} & \cdots & \kappa_{1n} \\ \kappa_{12} & \kappa_{22} & \cdots & \kappa_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \kappa_{n1} & \kappa_{n2} & \cdots & \kappa_{nn} \end{bmatrix} \stackrel{\text{def}}{=} \mathbf{O}'\mathbf{O}$$

$$\mathbf{q}^{\Phi} = (2\mathbf{I} + \boldsymbol{\Delta} + \boldsymbol{\Sigma} + \mathbf{K} \circ \boldsymbol{\Sigma})^{-1} (\mathbf{b} - \mathbf{c}^{0})$$

Construction of Product Cosine Similarities

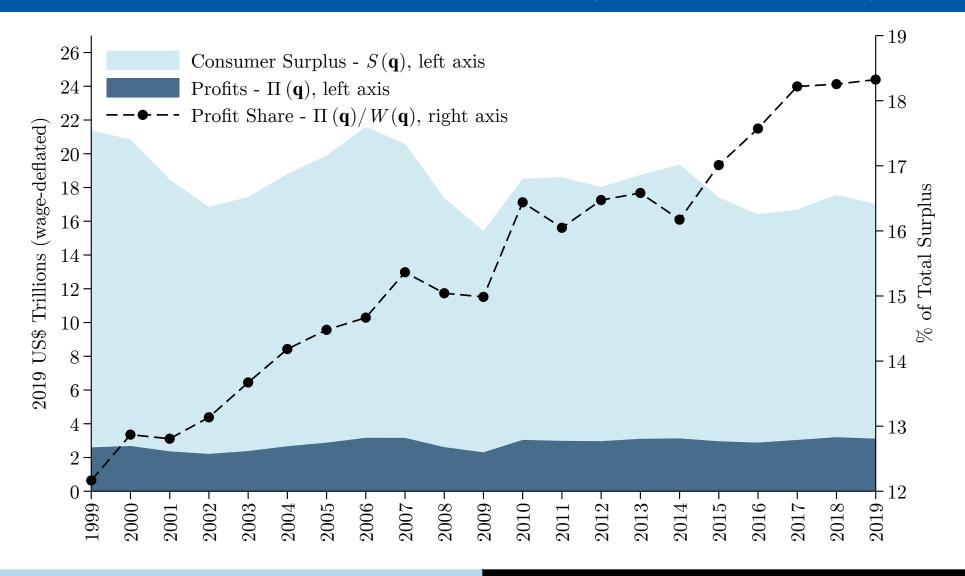
Company z maximizes the sum of profits over all product lines i where $[\mathbf{O}]_{iz} = 1$ if company z produces product i:

$$\left[\mathbb{Q}\right]_{i\mathcal{S}} = \begin{cases} 1 & \text{if } i \in \mathcal{S} \\ 0 & \text{if } i \notin \mathcal{S} \end{cases}$$

 $[S]_{zS} = z's$ share of SIC code S sales

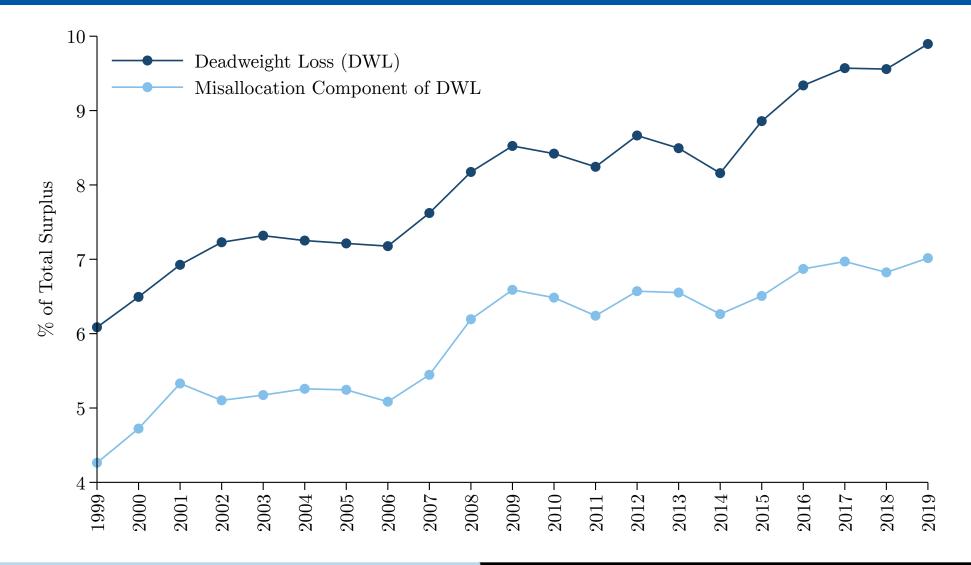
$$\left(\mathbf{A}'\mathbf{A}\right)_{\mathrm{P}} = \frac{1}{2} \left[\mathbf{O} \left(\mathbf{A}'\mathbf{A}\right)_{\mathrm{F}} \mathbf{O}' + \mathbb{Q}' \mathbb{S}' \left(\mathbf{A}'\mathbf{A}\right)_{\mathrm{F}} \mathbb{S} \mathbb{Q}\right]$$

Total Surplus and breakdown (Multi-Product)



Product Differentiation and Oligopoly: a Network Approach Bruno Pellegrino (Columbia Business School)

Deadweight Loss (Multi-Product)

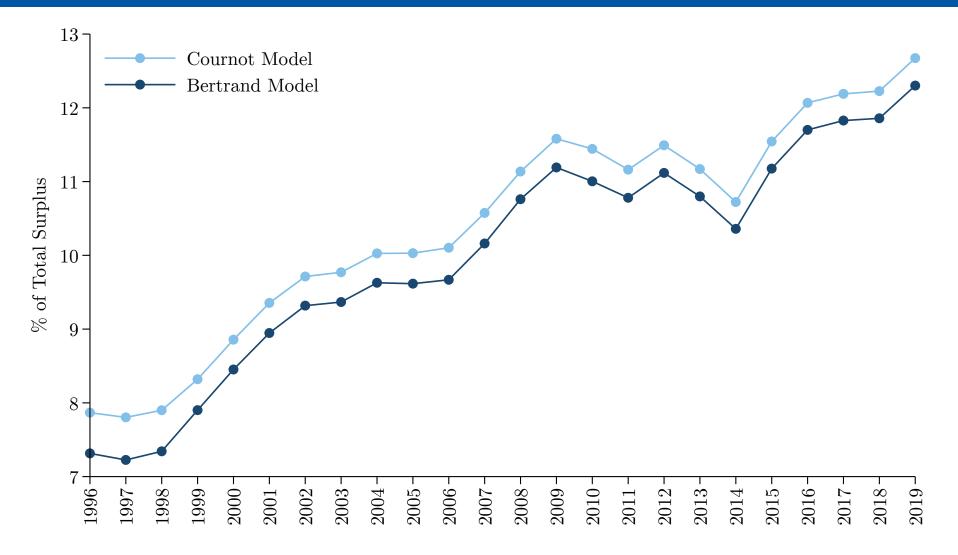


Product Differentiation and Oligopoly: a Network Approach Bruno Pellegrino (Columbia Business School)

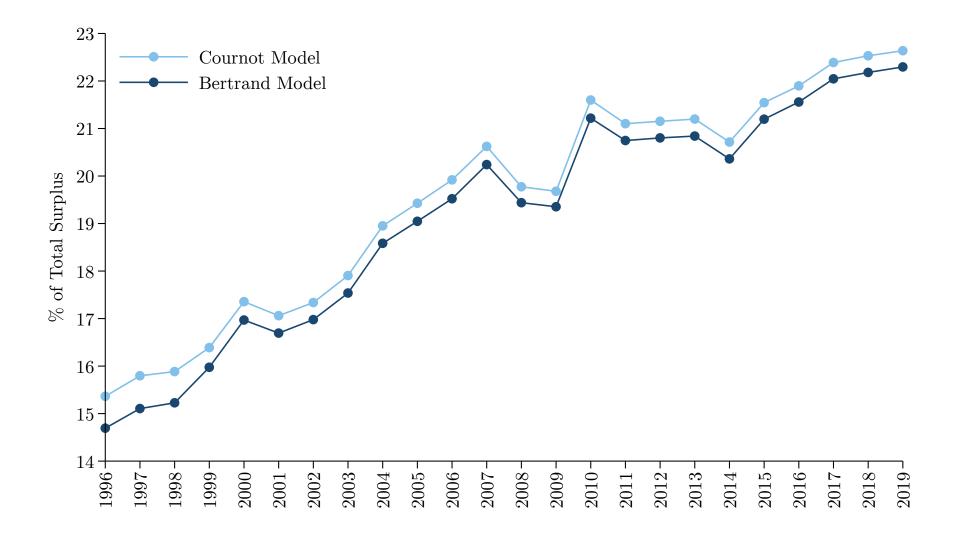
Bertrand Equilibrium (flat marginal cost)

$\mathbf{q}^{\Psi} = \left(\mathbf{I} + \mathbb{D}^{-1} + \mathbf{\Sigma}\right)^{-1} (\mathbf{b} - \mathbf{c})$

Deadweight Loss (Cournot v/s Bertrand)



Profit Share of Surplus (Cournot v/s Bertrand)

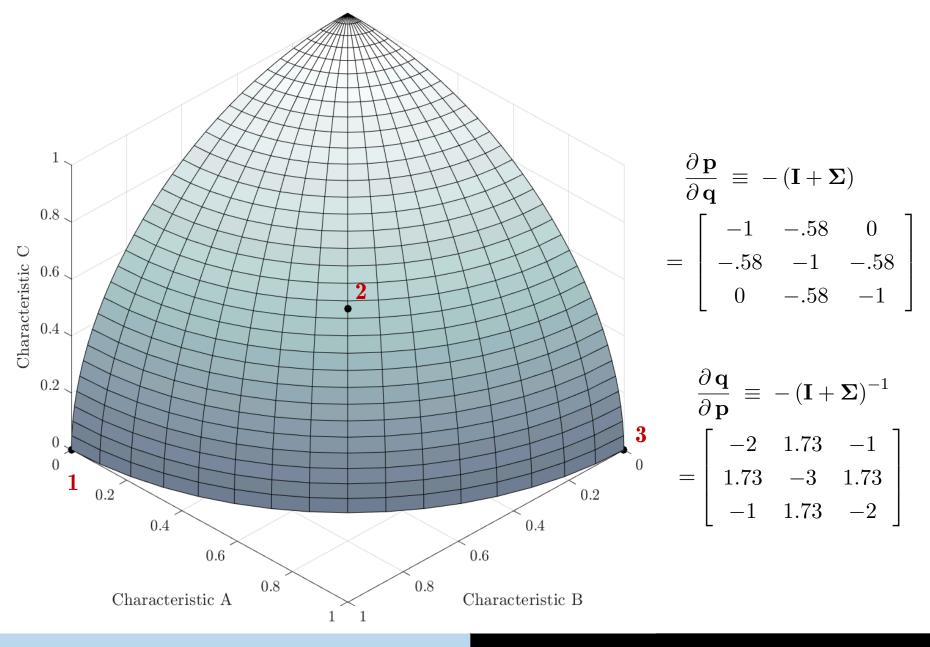


• A new GE theory of oligopoly with hedonic demand.

- A new GE theory of oligopoly with hedonic demand.
- Estimated for Compustat using 10-K product similarities.

- A new GE theory of oligopoly with hedonic demand.
- Estimated for Compustat using 10-K product similarities.
- Distribution of markups is jointly determined by productivity and product market centrality.
 - Both have undergone significant changes

- A new GE theory of oligopoly with hedonic demand.
- Estimated for Compustat using 10-K product similarities.
- Distribution of markups is jointly determined by productivity and product market centrality.
 - Both have undergone significant changes
- Rising Oligopoly Power
 - increasing deadweight loss
 - lower consumer surplus share.



Product Differentiation and Oligopoly: a Network Approach Bruno Pellegrino (Columbia Business School)