SYSTEMIC RISK IN FINANCIAL NETWORKS REVISITED

Jason Roderick Donaldson Giorgia Piacentino Xiaobo Yu

FACTS

Banks' gross debts bigger than net

E.g. HSBC's net position $\left|£24\text{B} - £21.5\text{B}\right| \approx 10\%$ gross

Thought to habor systemic risk $\implies$ Policy makers advocate netting out

Supported by networks models (e.g. Acemoglu–Ozdaglar–Tahbaz-Salehi 15)

Based on one-period debt capturing overnight debts (e.g. repos)

Much interbank debt longer maturity

Germany: Average mat. more than year; frac. overnight less than 10%

QUESTIONS

Do long-term debt networks harbor same systemic risks as short-?

Do the same network structures lead risks to propagate?

Do gross debts serve function that could be undermined by netting out?

THIS PAPER

Model of N banks connected in network of long-term debts

Banks have long-term assets y but could suffer short-term liq. shocks ℓ

THIS PAPER

Model of N banks connected in network of long-term debts

Banks have long-term assets y but could suffer short-term liq. shocks ℓ

Friction: Can pledge only fraction θ of y to borrow to meet shock

THIS PAPER

Model of N banks connected in network of long-term debts

Banks have long-term assets y but could suffer short-term liq. shocks ℓ

Friction: Can pledge only fraction θ of y to borrow to meet shock

Assumption: $y > \ell > \theta y$

LIMITED PLEDGEABILITY

B$_i$'s Balance Sheet

Assets	Liabilities
long-term	short-term
investments	liq. shock
	equity

LIMITED PLEDGEABILITY

$\underline{\text{B}_i\text{'s Balance Sheet}}$

Assets	Liabilities
	short-term
y	liq. shock
	equity

LIMITED PLEDGEABILITY

B_i's Balance Sheet

Assets	Liabilities
y	ℓ
	equity

LIMITED PLEDGEABILITY

$\underline{\text{B}_i\text{'s Balance Sheet}}$

Assets	Liabilities
y	ℓ
debt from B_j	debt to B_j
	equity

RESULTS

High indebtedness and connectedness sources of value and stability

Zero net long-term positions have positive NPV

Embed option to dilute with new debt $\implies$ liquidity insurance

Contingent transfers via plain debt

"Exponential networks" implement optimal transfers for any shocks

RESULTS MATTER FOR POLICY

Policies that help with short-term debt backfire with long-term debt

Decreasing indebtedness/connectedness can decrease efficiency

MODEL

MODEL OVERVIEW

Two dates: Date 1 and Date 2; no discounting; universal risk neutrality

N banks: Assets y at Date 2 and risk of liquidity shock $\ell < y$ at Date 1

Interbank network: Network of long-term debts $\mathbf{F} = [F_{i \to j}]_{ij}$ (due at Date 2)

Friction: Limited pledgeability: Only $\theta y < \ell$ pledgeable

BANKS $B_1, \ldots, B_N$ AND BALANCE SHEETS

BANKS $\mathrm{B}_1, ..., \mathrm{B}_N$ AND BALANCE SHEETS

B_i has total interbank liabilities $F_{i\rightrightarrows} := \sum_j F_{i\to j}$ & claims $F_{i\leftleftarrows} := \sum_j F_{j\to i}$

BANKS $\mathrm{B}_1, ..., \mathrm{B}_N$ AND BALANCE SHEETS

B_i has total interbank liabilities $F_{i\rightrightarrows} := \sum_j F_{i\to j}$ & claims $F_{i\leftleftarrows} := \sum_j F_{j\to i}$

<u>Assumption</u>: Zero net debts: $F_{i\rightrightarrows} = F_{i\leftleftarrows}$ for all B_i

BANKS $B_1, ..., B_N$ AND BALANCE SHEETS

B_i has total interbank liabilities $F_{i\rightrightarrows} := \sum_j F_{i\to j}$ & claims $F_{i\leftleftarrows} := \sum_j F_{j\to i}$

<u>Assumption</u>: Zero net debts: $F_{i\rightrightarrows} = F_{i\leftleftarrows}$ for all B_i

B_i has liquidity needs $\ell\sigma_i$ for $\sigma_i \in \{0, 1\}$

BANKS $\mathrm{B}_1, ..., \mathrm{B}_N$ AND BALANCE SHEETS

B_i has total interbank liabilities $F_{i\rightrightarrows} := \sum_j F_{i\to j}$ & claims $F_{i\leftleftarrows} := \sum_j F_{j\to i}$

Assumption: Zero net debts: $F_{i\rightrightarrows} = F_{i\leftleftarrows}$ for all B_i

B_i has liquidity needs $\ell\sigma_i$ for $\sigma_i \in \{0, 1\}$

Assumption: B_i liquidated if can't pay $\ell\sigma_i$, destroying $(1-\theta)y$

BANKS $\mathrm{B}_1, ..., \mathrm{B}_N$ AND BALANCE SHEETS

B_i has total interbank liabilities $F_{i\rightrightarrows} := \sum_j F_{i\to j}$ & claims $F_{i\leftleftarrows} := \sum_j F_{j\to i}$

<u>Assumption</u>: Zero net debts: $F_{i\rightrightarrows} = F_{i\leftleftarrows}$ for all B_i

B_i has liquidity needs $\ell\sigma_i$ for $\sigma_i \in \{0, 1\}$

<u>Assumption</u>: B_i liquidated if can't pay $\ell\sigma_i$, destroying $(1-\theta)y$

B_i has pledgeable assets $\theta y + \mathrm{PV}[F_{i\leftleftarrows}]$

BANKS $\mathrm{B}_1, \ldots, \mathrm{B}_N$ AND BALANCE SHEETS

B_i has total interbank liabilities $F_{i\rightrightarrows} := \sum_j F_{i\to j}$ & claims $F_{i\leftleftarrows} := \sum_j F_{j\to i}$

<u>Assumption</u>: Zero net debts: $F_{i\rightrightarrows} = F_{i\leftleftarrows}$ for all B_i

B_i has liquidity needs $\ell\sigma_i$ for $\sigma_i \in \{0, 1\}$

<u>Assumption</u>: B_i liquidated if can't pay $\ell\sigma_i$, destroying $(1-\theta)y$

B_i has pledgeable assets $\theta y + \mathrm{PV}[F_{i\leftleftarrows}]$

<u>Assumption</u>: New debt senior (e.g. repo) $\Longrightarrow$ $F_{i\rightrightarrows}$ diluted

REPAYMENTS

Denote B_i's equilibrium repayment to B_j by $R_{i\to j}$

Total repayments: $R_{i\rightrightarrows} := \sum_j R_{i\to j}$ and $R_{i\leftleftarrows} := \sum_j R_{j\to i}$

REPAYMENTS

Denote B_i's equilibrium repayment to B_j by $R_{i\to j}$

Total repayments: $R_{i\rightrightarrows} := \sum_j R_{i\to j}$ and $R_{i\leftleftarrows} := \sum_j R_{j\to i}$

Sequential rationality:

REPAYMENTS

Denote B_i's equilibrium repayment to B_j by $R_{i\to j}$

Total repayments: $R_{i\rightrightarrows} := \sum_j R_{i\to j}$ and $R_{i\leftleftarrows} := \sum_j R_{j\to i}$

<u>Sequential rationality</u>:

B_i liquidated at Date 1 if $\theta y + R_{i\leftleftarrows} < \ell\sigma_i$

REPAYMENTS

Denote B_i's equilibrium repayment to B_j by $R_{i\to j}$

Total repayments: $R_{i\rightrightarrows} := \sum_j R_{i\to j}$ and $R_{i\leftleftarrows} := \sum_j R_{j\to i}$

Sequential rationality:

B_i liquidated at Date 1 if $\theta y + R_{i\leftleftarrows} < \ell\sigma_i \implies R_{i\rightrightarrows} = 0$

REPAYMENTS

Denote B_i's equilibrium repayment to B_j by $R_{i\to j}$

Total repayments: $R_{i\rightrightarrows} := \sum_j R_{i\to j}$ and $R_{i\leftleftarrows} := \sum_j R_{j\to i}$

Sequential rationality:

B_i liquidated at Date 1 if $\theta y + R_{i\leftleftarrows} < \ell\sigma_i \implies R_{i\rightrightarrows} = 0$

Defaults at Date 2 if $\theta y + R_{i\leftleftarrows} < \ell\sigma_i + F_{i\rightrightarrows}$

REPAYMENTS

Denote B_i's equilibrium repayment to B_j by $R_{i\to j}$

Total repayments: $R_{i\rightrightarrows} := \sum_j R_{i\to j}$ and $R_{i\leftleftarrows} := \sum_j R_{j\to i}$

<u>Sequential rationality</u>:

B_i liquidated at Date 1 if $\theta y + R_{i\leftleftarrows} < \ell\sigma_i \implies R_{i\rightrightarrows} = 0$

Defaults at Date 2 if $\theta y + R_{i\leftleftarrows} < \ell\sigma_i + F_{i\rightrightarrows} \implies R_{i\rightrightarrows} = \big[\theta y + R_{i\leftleftarrows} - \ell\sigma_i\big]^+$

REPAYMENTS

Denote B_i's equilibrium repayment to B_j by $R_{i\to j}$

Total repayments: $R_{i\rightrightarrows} := \sum_j R_{i\to j}$ and $R_{i\leftleftarrows} := \sum_j R_{j\to i}$

<u>Sequential rationality</u>:

B_i liquidated at Date 1 if $\theta y + R_{i\leftleftarrows} < \ell\sigma_i \implies R_{i\rightrightarrows} = 0$

Defaults at Date 2 if $\theta y + R_{i\leftleftarrows} < \ell\sigma_i + F_{i\rightrightarrows} \implies R_{i\rightrightarrows} = \left[\theta y + R_{i\leftleftarrows} - \ell\sigma_i\right]^+$

Repays in full at Date 2 if $\theta y + R_{i\leftleftarrows} \geq \ell\sigma_i + F_{i\rightrightarrows}$

REPAYMENTS

Denote B_i's equilibrium repayment to B_j by $R_{i\to j}$

Total repayments: $R_{i\rightrightarrows} := \sum_j R_{i\to j}$ and $R_{i\leftleftarrows} := \sum_j R_{j\to i}$

<u>Sequential rationality</u>:

B_i liquidated at Date 1 if $\theta y + R_{i\leftleftarrows} < \ell\sigma_i \implies R_{i\rightrightarrows} = 0$

Defaults at Date 2 if $\theta y + R_{i\leftleftarrows} < \ell\sigma_i + F_{i\rightrightarrows} \implies R_{i\rightrightarrows} = \left[\theta y + R_{i\leftleftarrows} - \ell\sigma_i\right]^+$

Repays in full at Date 2 if $\theta y + R_{i\leftleftarrows} \geq \ell\sigma_i + F_{i\rightrightarrows} \implies R_{i\rightrightarrows} = F_{i\rightrightarrows}$

REPAYMENTS

Denote B_i's equilibrium repayment to B_j by $R_{i\to j}$

Total repayments: $R_{i\rightrightarrows} := \sum_j R_{i\to j}$ and $R_{i\leftleftarrows} := \sum_j R_{j\to i}$

Sequential rationality:

B_i liquidated at Date 1 if $\theta y + R_{i\leftleftarrows} < \ell\sigma_i \implies R_{i\rightrightarrows} = 0$

Defaults at Date 2 if $\theta y + R_{i\leftleftarrows} < \ell\sigma_i + F_{i\rightrightarrows} \implies R_{i\rightrightarrows} = \big[\theta y + R_{i\leftleftarrows} - \ell\sigma_i\big]^+$

Repays in full at Date 2 if $\theta y + R_{i\leftleftarrows} \geq \ell\sigma_i + F_{i\rightrightarrows} \implies R_{i\rightrightarrows} = F_{i\rightrightarrows}$

NB: Liquidation inefficient (destroys $(1-\theta)y$), default alone is not (transfer)

EQUILIBRIUM

A payment equilibrium is a repayment profile $[R_{i\to j}]_{ij}$ for each $(\sigma_i)_i$ s.t.

Repayments are sequentially rational

Repayments are paid pro rata: $\dfrac{R_{i\to j}}{R_{i\rightrightarrows}} = \dfrac{F_{i\to j}}{F_{i\rightrightarrows}}$

TIMELINE/SUMMARY

Date 1: Shocks realized; banks raise new liq.; banks liquidated/continue

Date 2: Assets y realized; banks repay or default

DEFINITION: EFFICIENCY

DEFINITION: EFFICIENCY

A network more efficient than another if fewer banks liquidated $\forall (\sigma_i)_i$

BENCHMARK: SHORT-TERM DEBT

BENCHMARK: SHORT-TERM DEBT

Suppose interbank liabilities $F_{i \to j}$ are due at Date 1

BENCHMARK: SHORT-TERM DEBT

Suppose interbank liabilities $F_{i \to j}$ are due at Date 1

$\implies$ Interbank liabilities can't be diluted with new debt at Date 1

BENCHMARK: SHORT-TERM DEBT

Suppose interbank liabilities $F_{i\to j}$ are due at Date 1

$\implies$ Interbank liabilities can't be diluted with new debt at Date 1

$\implies$ B_i liquidated if $\theta y + R_{i\leftleftarrows} < \ell\sigma_i + F_{i\rightrightarrows}$ (liquidation $\equiv$ default)

BENCHMARK: SHORT-TERM DEBT

Suppose interbank liabilities $F_{i\to j}$ are due at Date 1

$\implies$ Interbank liabilities can't be diluted with new debt at Date 1

$\implies$ B_i liquidated if $\theta y + R_{i\leftleftarrows} < \ell\sigma_i + F_{i\rightrightarrows}$ (liquidation $\equiv$ default)

Benchmark isomorphic to AOT mutatis mutandis

BM1: INCREASING DEBT DECREASES EFFICIENCY

BM1: DEBT DECREASES EFFICIENCY

Let $\mathbf{F} = [F_{i \to j}]_{ij}$ be regular $(F_{i \rightrightarrows} \equiv F)$

$\alpha \mathbf{F}$ is less efficient than $\mathbf{F}$ whenever $\alpha > 1$

BM1: DEBT DECREASES EFFICIENCY

Let $\mathbf{F} = [F_{i \to j}]_{ij}$ be regular $(F_{i \rightrightarrows} \equiv F)$

$\alpha\mathbf{F}$ is less efficient than $\mathbf{F}$ whenever $\alpha > 1$

$\implies$ No debt $(\alpha = 0)$ is best

BM1: DEBT DECREASES EFFICIENCY

Let $\mathbf{F} = [F_{i \to j}]_{ij}$ be regular $(F_{i \rightrightarrows} \equiv F)$

$\alpha\mathbf{F}$ is less efficient than $\mathbf{F}$ whenever $\alpha > 1$

$\implies$ No debt $(\alpha = 0)$ is best $\implies$ should net out

BM1: $\uparrow$ DEBT $\downarrow$ EFFICIENCY: PROOF (SKETCH)

Say B_i and B_j have offsetting debts $\alpha F_{i \to j} = \alpha F_{j \to i} = \alpha F$ and B_i shocked

BM1: $\uparrow$ DEBT $\downarrow$ EFFICIENCY: PROOF (SKETCH)

Say B_i and B_j have offsetting debts $\alpha F_{i\to j} = \alpha F_{j\to i} = \alpha F$ and B_i shocked

Shocked bank B_i: Liquidated if $\theta y + R_{j\to i} < \ell + \alpha F$

BM1: $\uparrow$ DEBT $\downarrow$ EFFICIENCY: PROOF (SKETCH)

Say B_i and B_j have offsetting debts $\alpha F_{i\to j} = \alpha F_{j\to i} = \alpha F$ and B_i shocked

<u>Shocked bank B_i</u>: Liquidated if $\theta y + R_{j\to i} < \ell + \alpha F \implies$ always

BM1: $\uparrow$ DEBT $\downarrow$ EFFICIENCY: PROOF (SKETCH)

Say B_i and B_j have offsetting debts $\alpha F_{i\to j} = \alpha F_{j\to i} = \alpha F$ and B_i shocked

Shocked bank B_i: Liquidated if $\theta y + R_{j\to i} < \ell + \alpha F \implies$ always

$\theta y < \ell$ by assumption and $R_{j\to i} \leq \alpha F$ given zero-net debts

BM1: $\uparrow$ DEBT $\downarrow$ EFFICIENCY: PROOF (SKETCH)

Say B_i and B_j have offsetting debts $\alpha F_{i\to j} = \alpha F_{j\to i} = \alpha F$ and B_i shocked

Shocked bank B_i: Liquidated if $\theta y + R_{j\to i} < \ell + \alpha F \implies$ always

$\theta y < \ell$ by assumption and $R_{j\to i} \leq \alpha F$ given zero-net debts

Not-shocked B_j: Liquidated if $\theta y + R_{i\to j} < \alpha F$

BM1: $\uparrow$ DEBT $\downarrow$ EFFICIENCY: PROOF (SKETCH)

Say B_i and B_j have offsetting debts $\alpha F_{i \to j} = \alpha F_{j \to i} = \alpha F$ and B_i shocked

<u>Shocked bank B_i</u>: Liquidated if $\theta y + R_{j \to i} < \ell + \alpha F \implies$ always

$\theta y < \ell$ by assumption and $R_{j \to i} \leq \alpha F$ given zero-net debts

<u>Not-shocked B_j</u>: Liquidated if $\theta y + R_{i \to j} < \alpha F \implies$ if α high enough

BM1: $\uparrow$ DEBT $\downarrow$ EFFICIENCY: PROOF (SKETCH)

Say B_i and B_j have offsetting debts $\alpha F_{i\to j} = \alpha F_{j\to i} = \alpha F$ and B_i shocked

Shocked bank B_i: Liquidated if $\theta y + R_{j\to i} < \ell + \alpha F \implies$ always

$\theta y < \ell$ by assumption and $R_{j\to i} \leq \alpha F$ given zero-net debts

Not-shocked B_j: Liquidated if $\theta y + R_{i\to j} < \alpha F \implies$ if α high enough

Face value of liability αF increasing faster than value of claim $R_{i\to j}$

BM1: $\uparrow$ DEBT $\downarrow$ EFFICIENCY: PROOF (SKETCH)

Say B_i and B_j have offsetting debts $\alpha F_{i\to j} = \alpha F_{j\to i} = \alpha F$ and B_i shocked

<u>Shocked bank B_i</u>: Liquidated if $\theta y + R_{j\to i} < \ell + \alpha F \implies$ always

$\theta y < \ell$ by assumption and $R_{j\to i} \leq \alpha F$ given zero-net debts

<u>Not-shocked B_j</u>: Liquidated if $\theta y + R_{i\to j} < \alpha F \implies$ if α high enough

Face value of liability αF increasing faster than value of claim $R_{i\to j}$

<u>Overall</u>: High short-term debt creates claims on the LHS of balance sheet

BM1: $\uparrow$ DEBT $\downarrow$ EFFICIENCY: PROOF (SKETCH)

Say B_i and B_j have offsetting debts $\alpha F_{i\to j} = \alpha F_{j\to i} = \alpha F$ and B_i shocked

Shocked bank B_i: Liquidated if $\theta y + R_{j\to i} < \ell + \alpha F \implies$ always

$\theta y < \ell$ by assumption and $R_{j\to i} \leq \alpha F$ given zero-net debts

Not-shocked B_j: Liquidated if $\theta y + R_{i\to j} < \alpha F \implies$ if α high enough

Face value of liability αF increasing faster than value of claim $R_{i\to j}$

Overall: High short-term debt creates claims on the LHS of balance sheet

But claims more than fully encumbered by liabilities created on RHS

BM2: DEFAULT RADIUS

BM2: DEFAULT RADIUS (STATED INFORMALLY)

All banks “close” enough to shocked banks default

Formalized via “harmonic distance” (captures direct and indirect links)

BM2: DEFAULT RADIUS (STATED INFORMALLY)

All banks “close” enough to shocked banks default

Formalized via “harmonic distance” (captures direct and indirect links)

Intuition: Shocked bank’s neighbors provide it liquidity

BM2: DEFAULT RADIUS (STATED INFORMALLY)

All banks "close" enough to shocked banks default

Formalized via "harmonic distance" (captures direct and indirect links)

Intuition: Shocked bank's neighbors provide it liquidity

Neighbors' neighbors provide them liquidity...

BM2: DEFAULT RADIUS (STATED INFORMALLY)

All banks "close" enough to shocked banks default

Formalized via "harmonic distance" (captures direct and indirect links)

Intuition: Shocked bank's neighbors provide it liquidity

Neighbors' neighbors provide them liquidity...

Overall: Not-shocked near shocked pay out so much that can't meet shocks

BM3: CONNECTEDNESS

BM3: CONNECTEDNESS (STATED INFORMALLY)

Increasing connectedness decreases efficiency

Formalized using “bottleneck parameter”/“delta connectedness”

BM3: CONNECTEDNESS (STATED INFORMALLY)

Increasing connectedness decreases efficiency

Formalized using “bottleneck parameter”/“delta connectedness”

Intuition: Liquidations propagate through network per default radius (BM2)

RESULTS

R1: INCREASING DEBT INCREASES EFFICIENCY

R1: INCREASING DEBT INCREASES EFFICIENCY

Let $\mathbf{F}$ be regular

$\alpha\mathbf{F}$ is <u>more</u> efficient than $\mathbf{F}$ whenever $\alpha > 1$

R1: INCREASING DEBT INCREASES EFFICIENCY

Let $\mathbf{F}$ be regular

$\alpha\mathbf{F}$ is <u>more</u> efficient than $\mathbf{F}$ whenever $\alpha > 1$

$\implies$ Zero debt ($\alpha = 0$) is worst

R1: INCREASING DEBT INCREASES EFFICIENCY

Let $\mathbf{F}$ be regular

$\alpha\mathbf{F}$ is <u>more</u> efficient than $\mathbf{F}$ whenever $\alpha > 1$

$\implies$ Zero debt ($\alpha = 0$) is worst $\implies$ should <u>not</u> net out

R1: $\uparrow$ DEBT $\uparrow$ EFFICIENCY: PROOF (SKETCH)

Say B_i and B_j have offsetting debts $\alpha F_{i \to j} = \alpha F_{j \to i} = \alpha F$ and B_i shocked

R1: $\uparrow$ DEBT $\uparrow$ EFFICIENCY: PROOF (SKETCH)

Say B_i and B_j have offsetting debts $\alpha F_{i\to j} = \alpha F_{j\to i} = \alpha F$ and B_i shocked

Shocked B_i: liquidated if $\theta y + R_{j\to i} < \ell + 0$

R1: $\uparrow$ DEBT $\uparrow$ EFFICIENCY: PROOF (SKETCH)

Say B_i and B_j have offsetting debts $\alpha F_{i \to j} = \alpha F_{j \to i} = \alpha F$ and B_i shocked

Shocked B_i: liquidated if $\theta y + R_{j \to i} < \ell + 0 \implies$ not if α high

R1: $\uparrow$ DEBT $\uparrow$ EFFICIENCY: PROOF (SKETCH)

Say B_i and B_j have offsetting debts $\alpha F_{i \to j} = \alpha F_{j \to i} = \alpha F$ and B_i shocked

Shocked B_i: liquidated if $\theta y + R_{j \to i} < \ell + 0 \implies$ not if α high $(2\theta y > \ell)$

R1: $\uparrow$ DEBT $\uparrow$ EFFICIENCY: PROOF (SKETCH)

Say B_i and B_j have offsetting debts $\alpha F_{i \to j} = \alpha F_{j \to i} = \alpha F$ and B_i shocked

Shocked B_i: liquidated if $\theta y + R_{j \to i} < \ell + 0 \implies$ not if α high $(2\theta y > \ell)$

$\theta y < \ell$ but (i) $R_{j \to i}$ increasing in α and (ii) B_i can dilute liability αF

R1: $\uparrow$ DEBT $\uparrow$ EFFICIENCY: PROOF (SKETCH)

Say B_i and B_j have offsetting debts $\alpha F_{i\to j} = \alpha F_{j\to i} = \alpha F$ and B_i shocked

Shocked B_i: liquidated if $\theta y + R_{j\to i} < \ell + 0 \implies$ not if α high $(2\theta y > \ell)$

$\theta y < \ell$ but (i) $R_{j\to i}$ increasing in α and (ii) B_i can dilute liability αF

Not-shocked B_j: liquidated if $\theta y + R_{i\to j} < 0$

R1: ↑ DEBT ↑ EFFICIENCY: PROOF (SKETCH)

Say B_i and B_j have offsetting debts $\alpha F_{i \to j} = \alpha F_{j \to i} = \alpha F$ and B_i shocked

Shocked B_i: liquidated if $\theta y + R_{j \to i} < \ell + 0 \implies$ not if α high $(2\theta y > \ell)$

$\theta y < \ell$ but (i) $R_{j \to i}$ increasing in α and (ii) B_i can dilute liability αF

Not-shocked B_j: liquidated if $\theta y + R_{i \to j} < 0 \implies$ never

R1: $\uparrow$ DEBT $\uparrow$ EFFICIENCY: PROOF (SKETCH)

Say B_i and B_j have offsetting debts $\alpha F_{i\to j} = \alpha F_{j\to i} = \alpha F$ and B_i shocked

Shocked B_i: liquidated if $\theta y + R_{j\to i} < \ell + 0 \implies$ not if α high $(2\theta y > \ell)$

$\theta y < \ell$ but (i) $R_{j\to i}$ increasing in α and (ii) B_i can dilute liability αF

Not-shocked B_j: liquidated if $\theta y + R_{i\to j} < 0 \implies$ never

B_i can dilute liability αF without causing B_j to be liquidated

R1: $\uparrow$ DEBT $\uparrow$ EFFICIENCY: PROOF (SKETCH)

Say B_i and B_j have offsetting debts $\alpha F_{i\to j} = \alpha F_{j\to i} = \alpha F$ and B_i shocked

Shocked B_i: liquidated if $\theta y + R_{j\to i} < \ell + 0 \implies$ not if α high $(2\theta y > \ell)$

$\theta y < \ell$ but (i) $R_{j\to i}$ increasing in α and (ii) B_i can dilute liability αF

Not-shocked B_j: liquidated if $\theta y + R_{i\to j} < 0 \implies$ never

B_i can dilute liability αF without causing B_j to be liquidated

Overall: High long-term debt creates claims on the LHS of balance sheet

R1: ↑ DEBT ↑ EFFICIENCY: PROOF (SKETCH)

Say B$_i$ and B$_j$ have offsetting debts $\alpha F_{i\to j} = \alpha F_{j\to i} = \alpha F$ and B$_i$ shocked

Shocked B$_i$: liquidated if $\theta y + R_{j\to i} < \ell + 0 \implies$ not if α high $(2\theta y > \ell)$

$\theta y < \ell$ but (i) $R_{j\to i}$ increasing in α and (ii) B$_i$ can dilute liability αF

Not-shocked B$_j$: liquidated if $\theta y + R_{i\to j} < 0 \implies$ never

B$_i$ can dilute liability αF without causing B$_j$ to be liquidated

Overall: High long-term debt creates claims on the LHS of balance sheet

Claims not encumbered by liabilities created on RHS

R1: $\uparrow$ DEBT $\uparrow$ EFFICIENCY: PROOF (SKETCH)

Say B_i and B_j have offsetting debts $\alpha F_{i \to j} = \alpha F_{j \to i} = \alpha F$ and B_i shocked

Shocked B_i: liquidated if $\theta y + R_{j \to i} < \ell + 0 \implies$ not if α high $(2\theta y > \ell)$

$\theta y < \ell$ but (i) $R_{j \to i}$ increasing in α and (ii) B_i can dilute liability αF

Not-shocked B_j: liquidated if $\theta y + R_{i \to j} < 0 \implies$ never

B_i can dilute liability αF without causing B_j to be liquidated

Overall: High long-term debt creates claims on the LHS of balance sheet

Claims not encumbered by liabilities created on RHS (can be diluted)

SHOCKED BANK B_i NEEDS LIQUIDITY

B_i's Balance Sheet

Assets	Liabilities
long-term	short-term
investments	liq. shock
	equity

SHOCKED BANK B_i NEEDS LIQUIDITY

B_i's Balance Sheet

Assets	Liabilities
	short-term
y	liq. shock
	equity

SHOCKED BANK B$_i$ NEEDS LIQUIDITY

B$_i$'s Balance Sheet

Assets	Liabilities
y	ℓ
	equity

SHOCKED BANK B$_i$ NEEDS LIQUIDITY

B$_i$'s Balance Sheet

Assets	Liabilities
y	ℓ
debt from B$_j$	debt to B$_j$
	equity

SHOCKED BANK B_i NEEDS LIQUIDITY

B_i's Balance Sheet

Assets	Liabilities
y	ℓ
αF	αF
	equity

B_i RAISES CASH VIA NEW DEBT AGAINST y & αF

<u>B_i's Balance Sheet</u>

Assets	Liabilities
y	ℓ
αF	αF
	equity

$\rightarrow$

<u>B_i's Balance Sheet</u>

Assets	Liabilities
y	ℓ
αF	αF
cash	new debt
	equity

DILUTES B$_j$

B$_i$'s Balance Sheet

Assets	Liabilities
y	ℓ
αF	αF
	equity

$\rightarrow$

B$_i$'s Balance Sheet

Assets	Liabilities
y	ℓ
αF	$\cancel{\alpha F}$
cash	new debt
	equity

B_j NOT WORSE OFF EX ANTE

Gross debts mean B_j diluted when B_i is shocked

But B_j can also dilute B_i when it is shocked

Gross debt implement transfer from not-shocked to shocked bank

Coinsurance via option to dilute

PRACTICAL IMPLEMENTATION

Banks hold gross long-term dilutable debts

E.g. interbank loans/bonds

Rationalizes why long-maturity

Banks dilute with short-term senior debt

Rationalizes e.g. super-seniority for repos

Explains large interbank positions (quarter of balance sheets)

DILUTION COMPLEMENTS DEFAULT

Banks use the option to default to implement contingencies

- Implements transfer from not-shocked to shocked at Date 2
- Allen–Gale 98, Dubey–Geanakoplos–Shubik 88, and Zame 93

But default not enough here

- Need dilution to prevent liquidation at Date 1

Like defaultable debt, dilutable debt can be good

- Implements transfers before maturity

R2: SALVATION RADIUS

R2: SALVATION RADIUS (STATED INFORMALLY)

Banks close enough to <u>not</u>-shocked bank do <u>not</u> default (via harmonic dist.)

R2: SALVATION RADIUS (STATED INFORMALLY)

Banks close enough to <u>not</u>-shocked bank do <u>not</u> default (via harmonic dist.)

<u>Intuition</u>: Not-shocked bank's neighbors dilute its debt to get liquidity

R2: SALVATION RADIUS (STATED INFORMALLY)

Banks close enough to <u>not</u>-shocked bank do <u>not</u> default (via harmonic dist.)

<u>Intuition</u>: Not-shocked bank's neighbors dilute its debt to get liquidity

Neighbors' neighbors dilute their debt to get liquidity...

R2: SALVATION RADIUS (STATED INFORMALLY)

Banks close enough to not-shocked bank do not default (via harmonic dist.)

Intuition: Not-shocked bank's neighbors dilute its debt to get liquidity

Neighbors' neighbors dilute their debt to get liquidity...

Overall: Banks near not-shocked banks dilute so much that meet shocks

R3: CONNECTEDNESS INCREASES EFFICIENCY

R3: CONNECTEDNESS $\uparrow$ EFF. (INFORMALLY)

Increasing connectedness increases efficiency

Formalized using “bottleneck parameter”/“delta connectedness”

R3: CONNECTEDNESS $\uparrow$ EFF. (INFORMALLY)

Increasing connectedness increases efficiency

Formalized using “bottleneck parameter”/“delta connectedness”

Intuition: Liquidity propagates through network per salvation radius (R2)

TO SUM UP: LT DEBT NETWORKS UNLIKE ST

Indebtedness and connectedness sources of efficiency

Reason: Option to dilute gross debts provides insurance

TO SUM UP: LT DEBT NETWORKS UNLIKE ST

Indebtedness and connectedness sources of efficiency

Reason: Option to dilute gross debts provides insurance

Question: Do high indebtedness and connectedness suffice for efficiency?

TO SUM UP: LT DEBT NETWORKS UNLIKE ST

Indebtedness and connectedness sources of efficiency

Reason: Option to dilute gross debts provides insurance

Question: Do high indebtedness and connectedness suffice for efficiency?

Answer: No!

TO SUM UP: LT DEBT NETWORKS UNLIKE ST

Indebtedness and connectedness sources of efficiency

Reason: Option to dilute gross debts provides insurance

Question: Do high indebtedness and connectedness suffice for efficiency?

Answer: No! Complete network (fully connected) inefficient no matter debt

R4: COMPLETE NETWORK INEFFICIENT

R4: COMPLETE NETWORK INEFFICIENT

Let S be number of shocked banks and suppose $S\ell > N\theta y$

If $\mathbf{F}$ is complete ($F_{i\to j} \equiv F$) then all shocked banks are liquidated

R4: COMPLETE INEFFICIENT: PROOF (SKETCH)

Complete

R4: COMPLETE INEFFICIENT: PROOF (SKETCH)

Complete $\implies$ each not-shocked bank pays at most $\dfrac{\theta y}{S}$ to each shocked

R4: COMPLETE INEFFICIENT: PROOF (SKETCH)

Complete $\implies$ each not-shocked bank pays at most $\dfrac{\theta y}{S}$ to each shocked

$\implies$ shocked liquidated

R4: COMPLETE INEFFICIENT: PROOF (SKETCH)

Complete $\implies$ each not-shocked bank pays at most $\dfrac{\theta y}{S}$ to each shocked

$\implies$ shocked liquidated: $\ell - \theta y > \max R_{i \Leftarrow}$

R4: COMPLETE INEFFICIENT: PROOF (SKETCH)

$$\text{Complete} \implies \text{each not-shocked bank pays at most } \frac{\theta y}{S} \text{ to each shocked}$$

$$\implies \text{shocked liquidated: } \ell - \theta y > \max R_{i\Leftarrow} = (N - S)\frac{\theta y}{S}$$

R4: COMPLETE INEFFICIENT: PROOF (SKETCH)

Complete $\implies$ each not-shocked bank pays at most $\dfrac{\theta y}{S}$ to each shocked

$\implies$ shocked liquidated: $\ell - \theta y > \max R_{i\Leftarrow} = (N-S)\dfrac{\theta y}{S}$ or $S\ell > N\theta y$

R4: COMPLETE INEFFICIENT: INTUITION

Complete network delivers all shocked banks same net payment

- If not enough to save all, each gets same insufficient amount of liquidity
- None saved

R4: COMPLETE INEFFICIENT: INTUITION

Complete network delivers all shocked banks same net payment

If not enough to save all, each gets same insufficient amount of liquidity

None saved

Question: How much better can we do?

DEFINITION: CONSTRAINED EFFICIENCY

DEF: CONSTRAINED EFFICIENCY

A network is constrained efficient if L is minimized for each $(\sigma_i)_i$ s.t.

$$(S - L)(\ell - \theta y) \leq (N - S)\theta y$$

DEF: CONSTRAINED EFFICIENCY

A network is constrained efficient if L is minimized for each $(\sigma_i)_i$ s.t.

$$(S - L)(\ell - \theta y) \leq (N - S)\theta y$$

I.e. liq. provided to shocked not-liquidated $\leq$ available from not-shocked

PRINCIPLES OF EFFICIENCY

Planner should allocate liquidity to save largest number of shocked banks:

(i) Not-shocked banks pay out all liquidity (θy)

(ii) Allocate none to liquidated banks (so all used to save shocked)

PRINCIPLES OF EFFICIENCY

Planner should allocate liquidity to save largest number of shocked banks:

(i) Not-shocked banks pay out all liquidity (θy)

(ii) Allocate none to liquidated banks (so all used to save shocked)

Implementation: Priority

PRINCIPLES OF EFFICIENCY

Planner should allocate liquidity to save largest number of shocked banks:

(i) Not-shocked banks pay out all liquidity (θy)

(ii) Allocate none to liquidated banks (so all used to save shocked)

Implementation: Priority

One bank always gets liquidity needed to survive

PRINCIPLES OF EFFICIENCY

Planner should allocate liquidity to save largest number of shocked banks:

(i) Not-shocked banks pay out all liquidity (θy)

(ii) Allocate none to liquidated banks (so all used to save shocked)

Implementation: Priority

One bank always gets liquidity needed to survive

Next bank does too if enough left in total after saving first...

PRINCIPLES OF EFFICIENCY

Planner should allocate liquidity to save largest number of shocked banks:

(i) Not-shocked banks pay out all liquidity (θy)

(ii) Allocate none to liquidated banks (so all used to save shocked)

<u>Implementation</u>: Priority

One bank always gets liquidity needed to survive

Next bank does too if enough left in total after saving first...

NB: Banks symmetric $\implies$ order need not depend on state

PRINCIPLES OF EFFICIENCY

Planner should allocate liquidity to save largest number of shocked banks:

(i) Not-shocked banks pay out all liquidity (θy)

(ii) Allocate none to liquidated banks (so all used to save shocked)

<u>Implementation</u>: Priority

One bank always gets liquidity needed to survive

Next bank does too if enough left in total after saving first...

NB: Banks symmetric $\implies$ order need not depend on state (cf. extension)

DEFINITION: EXPONENTIAL NETWORK

DEFINITION: EXPONENTIAL NETWORK

Call $\mathbf{F}$ "exponential with base s" if is its fully connected and for all i, j

$$\frac{F_{i \to j+1}}{F_{i \to j}} \leq s < 1$$

DEFINITION: EXPONENTIAL NETWORK

Call $\mathbf{F}$ "exponential with base s" if is its fully connected and for all i, j

$$\frac{F_{i \to j+1}}{F_{i \to j}} \leq s < 1$$

In words: Every bank

DEFINITION: EXPONENTIAL NETWORK

Call $\mathbf{F}$ "exponential with base s" if is its fully connected and for all i, j

$$\frac{F_{i \to j+1}}{F_{i \to j}} \leq s < 1$$

In words: Every bank

is connected to every other one

DEFINITION: EXPONENTIAL NETWORK

Call $\mathbf{F}$ "exponential with base s" if is its fully connected and for all i, j

$$\frac{F_{i \to j+1}}{F_{i \to j}} \leq s < 1$$

In words: Every bank

is connected to every other one

has larger liabilities to those with lower indices ("assortativity")

DEFINITION: EXPONENTIAL NETWORK

Call $\mathbf{F}$ "exponential with base s" if is its fully connected and for all i, j

$$\frac{F_{i \to j+1}}{F_{i \to j}} \leq s < 1$$

<u>In words</u>: Every bank

is connected to every other one

has larger liabilities to those with lower indices ("assortativity")

has liabilities decaying exponentially in indices ("s-dominance")

DEFINITION: EXPONENTIAL NETWORK

Call $\mathbf{F}$ "exponential with base s" if is its fully connected and for all i, j

$$\frac{F_{i \to j+1}}{F_{i \to j}} \leq s < 1$$

In words: Every bank

is connected to every other one

has larger liabilities to those with lower indices ("assortativity")

has liabilities decaying exponentially in indices ("s-dominance")

NB: Ordering by indices arbitrary, can consider permutation

R5: EXPONENTIAL NETWORKS ARE CONSTRAINED EFFICIENT

R5: EXP. NETWORKS CONSTRAINED EFFICIENT

Let $\mathbf{F}$ be an exponential network with base s small enough

For α large enough, $\alpha\mathbf{F}$ is generically constrained efficient

R5: EXP. NETWORKS EFF.: PROOF (SKETCH)

Echoes principles of efficiency

R5: EXP. NETWORKS EFF.: PROOF (SKETCH)

Echoes principles of efficiency

(i) High α

R5: EXP. NETWORKS EFF.: PROOF (SKETCH)

Echoes principles of efficiency

(i) High $\alpha \Longrightarrow$ not-shocked banks' liabilities high

R5: EXP. NETWORKS EFF.: PROOF (SKETCH)

Echoes principles of efficiency

(i) High $\alpha \Longrightarrow$ not-shocked banks' liabilities high

$\Longrightarrow$ pay out (almost) all liquidity

R5: EXP. NETWORKS EFF.: PROOF (SKETCH)

Echoes principles of efficiency

(i) High $\alpha \Longrightarrow$ not-shocked banks' liabilities high

$\Longrightarrow$ pay out (almost) all liquidity

(ii) Low s

R5: EXP. NETWORKS EFF.: PROOF (SKETCH)

Echoes principles of efficiency

(i) High $\alpha \Longrightarrow$ not-shocked banks' liabilities high

$\Longrightarrow$ pay out (almost) all liquidity

(ii) Low $s \Longrightarrow$ shocked with high indices exp. smaller claims on not-shocked

R5: EXP. NETWORKS EFF.: PROOF (SKETCH)

Echoes principles of efficiency

(i) High $\alpha \Longrightarrow$ not-shocked banks' liabilities high

$\Longrightarrow$ pay out (almost) all liquidity

(ii) Low $s \Longrightarrow$ shocked with high indices exp. smaller claims on not-shocked

$\Longrightarrow$ liquidated banks allocated (almost) no liquidity

R5: EXP. NETWORKS EFF.: PROOF (SKETCH)

Echoes principles of efficiency

(i) High $\alpha \Longrightarrow$ not-shocked banks' liabilities high

$\Longrightarrow$ pay out (almost) all liquidity

(ii) Low $s \Longrightarrow$ shocked with high indices exp. smaller claims on not-shocked

$\Longrightarrow$ liquidated banks allocated (almost) no liquidity (all left for saved)

R5: EXP. NETWORKS EFF.: PROOF (SKETCH)

Echoes principles of efficiency

(i) High $\alpha \implies$ not-shocked banks' liabilities high

$\implies$ pay out (almost) all liquidity

(ii) Low $s \implies$ shocked with high indices exp. smaller claims on not-shocked

$\implies$ liquidated banks allocated (almost) no liquidity (all left for saved)

NB: "Almost" is enough except in non-generic cases

R5: EXP. NETWORKS EFF.: PROOF (SKETCH)

Echoes principles of efficiency

(i) High $\alpha \Longrightarrow$ not-shocked banks' liabilities high

$\Longrightarrow$ pay out (almost) all liquidity

(ii) Low $s \Longrightarrow$ shocked with high indices exp. smaller claims on not-shocked

$\Longrightarrow$ liquidated banks allocated (almost) no liquidity (all left for saved)

NB: "Almost" is enough except in non-generic cases (also manageable)

EXTENSIONS

EXTENSIONS

(i) Liquidation can be efficient

EXTENSIONS

(i) Liquidation can be efficient

$\Longrightarrow$ Calibrate debts to avoid bad liquidation but not prevent good

EXTENSIONS

(i) Liquidation can be efficient

$\Longrightarrow$ Calibrate debts to avoid bad liquidation but not prevent good

(ii) Default can be costly

EXTENSIONS

(i) Liquidation can be efficient

$\Longrightarrow$ Calibrate debts to avoid bad liquidation but not prevent good

(ii) Default can be costly

$\Longrightarrow$ Calibrate debts to avoid liquidation without inducing default

EXTENSIONS

(i) Liquidation can be efficient

$\Longrightarrow$ Calibrate debts to avoid bad liquidation but not prevent good

(ii) Default can be costly

$\Longrightarrow$ Calibrate debts to avoid liquidation without inducing default

(iii) Banks can be heterogeneous

EXTENSIONS

(i) Liquidation can be efficient

$\implies$ Calibrate debts to avoid bad liquidation but not prevent good

(ii) Default can be costly

$\implies$ Calibrate debts to avoid liquidation without inducing default

(iii) Banks can be heterogeneous

$\implies$ Exp. network imperfect as ranking ind. of state

EXTENSIONS

(i) Liquidation can be efficient

$\implies$ Calibrate debts to avoid bad liquidation but not prevent good

(ii) Default can be costly

$\implies$ Calibrate debts to avoid liquidation without inducing default

(iii) Banks can be heterogeneous

$\implies$ Exp. network imperfect as ranking ind. of state (but not that bad)

CONCLUSION

CONCLUSION

Off-setting long-term debts provide insurance

Indebtedness and connectedness sources of efficiency

Contrary to conclusions based on short-term debt

Indebtedness and connectedness implement efficiency if network exponential

Minimize number of liquidations no matter realization of shocks

"Robust but never fragile"

SYSTEMIC RISK IN FINANCIAL NETWORKS REVISITED

APPENDIX

NON-CONTINGENT LIQUIDTY

If transfer ℓ to all banks at Date 0 at rate $R = \dfrac{L - \pi\theta y}{(1-\pi)L}$

All banks meet their shocks

Shocked banks repay θy, not-shocked banks repay RL

Outside lender breaks even

Works, but requires outside liquidity $NL \gg ML$ at Date 0

OUTSIDE CREDIT LINES

Extend credit line to all banks to borrow ℓ at Date 1 at rate ϵ

For non-contingent repayment $F = \dfrac{L - \pi\theta y}{1 - \pi}$

Shocked banks draw down, not shocked banks don't

Outside lender breaks even

Works, but requires commitment from outside lender