

**Strategic Network Formation and the
Emergence of Equilibrium Network Dynamics
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Objectives:

Networks are pervasive. We routinely communicate over the internet, advance our careers by networking, travel to conferences over the transportation network and pay for the trip using the banking network. Doing this utilizes networks in our brain. The list could go on. While network models have had a long history in sociology, the natural sciences, and engineering (e.g., in modeling social organizations, brain architecture, and electrical circuits), the rise of the network paradigm in economics is relatively recent. Economists are now beginning to think of political, economic, and financial interactions as network phenomena and to model everything from terrorist activities to asset market micro structures as games of network formation. This trend in economics, which began with the seminal paper by Myerson (1977) on graphs and cooperation and accelerated with the publication of the paper by Jackson and Wolinsky (1996) on social and economic networks, is likely to continue with the development of new algorithms, the expansion of computational capacity and the broad application of network theories to political, economic, and financial phenomena.

What economists bring to the study of networks that is new is game theory. For the most part sociologists, natural scientists and engineers have used networks descriptively and have focused on the design of networks from the perspective of a single designer or on the random evolution of networks from the perspective of nature. This singularity of perspective is a consequence of the nonstrategic nature of the phenomena being explained or the problem being solved (e.g., the spread of a disease through a given population, the transmission of electrical impulses in the brain, or the optimal design of an integrated circuit). In economics the perspective is often times strategic. In particular, in many economic situations, several individuals, guided by their own self interest, behave strategically in putting into place pieces of the network of economic, political, or financial interactions under their control and in so doing generate payoffs and externalities that determine the network of economic interactions that eventually emerges in equilibrium. Thus in economics, pieces of the network are the strategies and the network that ultimately prevails is the result of strategic competition rather than the design of a single individual or nature. Conversely, what networks bring to the study of economics is a way of modeling the structure economic interactions and externalities that makes possible a game-theoretic analysis of how these structures influence individual payoffs and the economic equilibrium that emerges from competition.

In all social and economic interactions, individuals or coalitions choose not only with whom to interact but how to interact, and over time both the structure (the “with whom”) and the strategy (“the how”) of interactions change. Our objectives will be to model the structure and strategy of interactions prevailing at any point in time as a directed network and to address the following questions in the theory of social and economic network formation: (i) Given the rules of network formation, the preferences of individuals over networks, and the strategic behavior of individuals and coalitions in forming networks, what networks are likely to emerge and persist. Thus, we propose to study the emergence of **endogenous networks** from the rules of network formation and the strategic behavior of individuals and coalitions in forming networks. (ii) Given the rules of network formation, the preferences of individuals over networks, the strategic behavior of coalitions in forming networks, and the trembles of nature through time, what network and coalitional dynamics are likely to emerge and persist. Thus, we propose to study the emergence of **equilibrium network dynamics** from the rules of network formation, the preferences of individuals over networks, the strategic behavior of coalitions in forming networks, and the randomness in nature through time.

Outline:

Part 0: What is a Network: A Formal Definition

Part 1: Abstract Games of Network Formation

1.1. Primitives and Assumptions

- 1.1.1 Feasible Networks
- 1.1.2 Players' Preferences
- 1.1.3 The Rules of Network Formation
- 1.1.4 Supernetworks
- 1.1.5 Dominance Relations

1.2. Stability

- 1.2.1 Network Formation Games with Respect to Irreflexive Dominance
- 1.2.2 Network Formation Games with Respect to Path Dominance: Basins of Attraction, Stable Sets, and the Path Dominance Core

1.3 Financial Networks

1.4 Strong Stability, Pairwise Stability, Nash Stability, and Farsighted Consistency

- 1.3.1 Strongly Stable Networks
- 1.3.2 Pairwise Stable Networks
- 1.3.3 Nash Networks
- 1.3.4 Farsightedly Consistent Networks

1.5 Singleton Basins of Attraction

- 1.4.1 Network Formation Games with a Potential Function: Club Networks
- 1.4.2 Jackson-Wolinsky Network Formation Games

Part 2: Discounted Stochastic Games of Network Formation and

Endogenous Network Dynamics

2.1. Primitives and Assumptions

2.2. Discounted Stochastic Games of Network Formation

2.3 Stationary Markov Equilibria

2.4 Endogenous Network Dynamics and Equilibrium Markov Supernetworks

2.5 Dynamic Basins of Attraction, Ergodic Measures, and Invariant Measures

2.5 Long Run Stability and Dynamic Consistency: : The Dynamic Path Dominance Core, Dynamic Strong Stability, and Dynamic Pairwise Stability

2.6 Endogenous Financial Networks

Some Basic Texts:

Social and Economic Networks, by Matthew O. Jackson, Princeton University Press, 2008

Networks, Crowds, and Markets, by David Easley and Jon Kleinberg, Cambridge University Press, 2010

Group Formation Economics: Networks, Clubs, and Coalitions, edited by Gabrielle Demange and Myrna Wooders, Cambridge University Press, 2005

Basic Probability Theory, by Robert B. Ash, Dover Publications Inc., Mineola, New York, 2008 (republication of the work originally published by John Wiley & Sons, New York in 1970)

The Theory of Graphs, by Claude Berge, Dover Publications Inc., Mineola, New York, 2001 (translated from the French edition published by Dunod, Paris, in 1958)

An Introduction to Markov Processes, by Daniel W. Strook, Springer Graduate Texts in Mathematics, 2005

Some Basic Papers:

Bloch F, Jackson MO (2006) Definitions of equilibrium in network formation games. *International Journal of Game Theory* 34: 305-318

Bloch F, Jackson MO (2007) The formation of networks with transfers among players. *Journal of Economic Theory* 133: 83-110

Blume L (1993) The statistical mechanics of strategic interaction. *Games and Economic Behavior* 5: 387-424

Calvo-Armengol A, Ilklic, R (2009) Pairwise-stability and Nash equilibria in network formation. *International Journal of Game Theory* 38:51-79

Chvatal V, Lovasz L (1972) Every directed graph has a semi-kernel. In: *Hypergraph Seminar, Lecture Notes in Mathematics* 411, Springer-Verlag, Berlin

Chwe M. (1994) Farsighted coalitional stability. *Journal of Economic Theory* 63: 299-325

Chwe M (2000) Communication and coordination in social networks. *Review of Economic Studies* 67: 1-16

Galeana-Sanchez H, Xueliang Li (1998) Semikernels and (k, l) -kernels in digraphs. *SIAM Journal of Discrete Mathematics* 11: 340-346

Galeotti A, Goyal S, Kamphorst J (2006) Network formation with heterogeneous players. *Games and Economic Behavior* 54: 353-372.

- Gillies DB (1959) Solutions to general non-zero-sum games. In: Tucker AW, Luce RD (eds) Contributions to the Theory of Games, Volume 4, Princeton University Press, Princeton, 47-85
- Hollard G (2000) On the existence of a pure strategy equilibrium in group formation games. *Economic Letters* 66: 283-287
- Inarra E, Kuipers J, Olaizola N (2005) Absorbing and generalized stable sets. *Social Choice and Welfare* 24: 433-437
- Inarra E, Larrea C, Molis E (2008) Random paths to P-stability in the roommate problem. *International Journal of Game Theory* 36: 461-471
- Inarra E, Larrea C, Molis E (2013) Absorbing sets in roommate problems. *Games and Economic Behavior* *forthcoming*
- Jackson MO, van den Nouweland A (2005) Strongly stable networks. *Games and Economic Behavior* 51: 420-444
- Jackson MO, Watts A (2002) The evolution of social and economic networks. *Journal of Economic Theory* 106: 265-295
- Jackson MO, Wolinsky A (1996) A strategic model of social and economic networks. *Journal of Economic Theory* 71: 44-74
- Johari R, Mannor S, Tsitsiklis JN (2006) A contract-based model for directed network formation, *Games and Economic Behavior* 56: 201-224
- Kalai E, Pazner A, Schmeidler D (1976) Collective choice correspondences as admissible outcomes of social bargaining processes. *Econometrica* 44: 233-240
- Kalai E, Schmeidler D (1977) An admissible set occurring in various bargaining situations. *Journal of Economic Theory* 14: 402-411
- Konishi H, Ray D (2003) Coalition formation as a dynamic process. *Journal of Economic Theory* 110: 1-41
- Monderer D, Shapley LS (1996) Potential games. *Games and Economic Behavior* 14: 124-143
- Mutuswami S, Winter E (2002) Subscription mechanisms for network formation. *Journal of Economic Theory* 106: 242-264
- Myerson RB (1977) Graphs and cooperation in games. *Mathematics of Operations Research* 2: 225-229
- Page Jr FH, Wooders M (2010) Club Networks with Multiple Memberships and Noncooperative Stability. *Games and Economic Behavior* 20: 12-20.
- Page Jr FH, Wooders M (2009) Strategic basins of attraction, the path dominance core, and network formation games. *Games and Economic Behavior* 66: 462-487
- Page Jr FH, Wooders M (2009) Networks and stability. *Encyclopedia of Complexity and System Science*, Springer, Heidelberg

- Page Jr FH, Wooders M (2007) Networks and clubs. *Journal of Economic Behavior and Organization* 64: 406-425
- Page Jr FH, Wooders M, Kamat S (2005) Networks and farsighted stability. *Journal of Economic Theory* 120: 257-269
- Page Jr FH, Kamat S (2005) Farsighted stability in network formation. In: Demange G, Wooders M (eds) *Group formation in economics: networks, clubs, and coalitions*, Cambridge University Press, Cambridge, 89-121
- Richardson M (1953) Solutions of irreflexive relations. *Annals of Mathematics* 58: 573-590
- Rosenthal RW (1973) A class of games possessing pure-strategy Nash equilibria. *International Journal of Game Theory* 2: 65-67
- Shenoy PP (1979) On coalition formation: a game theoretic approach. *International Journal of Game Theory* 8: 133-164
- Shenoy PP (1980) A dynamic solution concept for abstract games. *Journal of Optimization Theory and Applications* 32: 151-169
- Vartiainen Hannu (2011) Dynamic Coalition Equilibrium. *Journal of Economic Theory* 146: 672-698

Manuscripts on Strategic Network Formation and

Endogenous Network Dynamics:

- Page F, Resende J (2013) Strategic Network Formation and the Emergence of Equilibrium Network Dynamics. CAEPR Working Paper 2013, Indiana University
- Fique, J., Page F, Resende J (2013) Equilibrium Markov Supernetworks. CAEPR Working Paper 2013, Indiana University
- Page F, Wooders M (2009) Endogenous network dynamics. CAEPR Working Paper 2009-02, Indiana University
- Page F, Resende J (2013) Noise, Correlation, and Stationarity in Dynamic Games of Network Formation. CAEPR Working Paper 2013, Indiana University