Leverage Stacks and the Financial System

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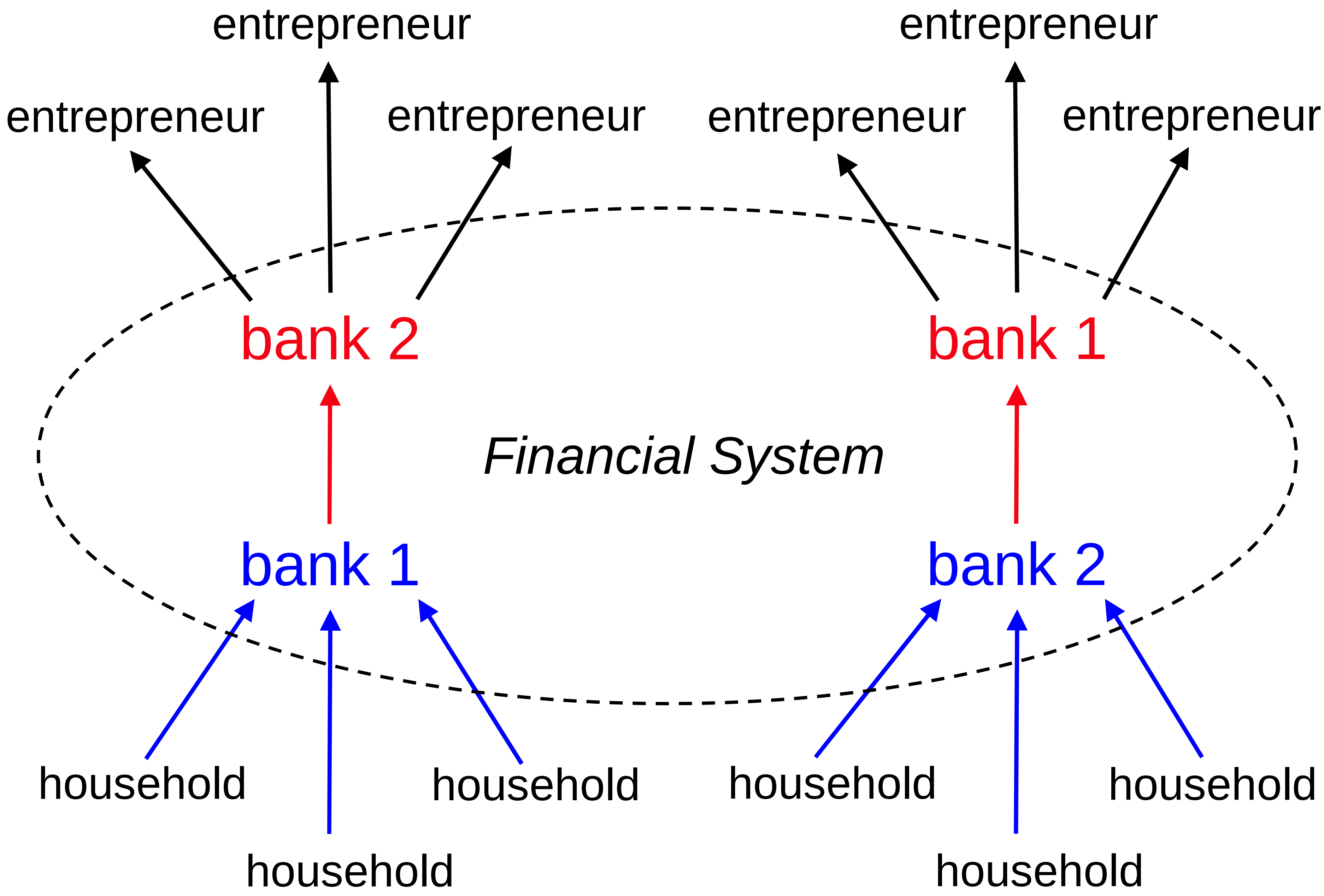
entrepreneur
entrepreneur
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bank 2
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Financial System
bank 1
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household
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household

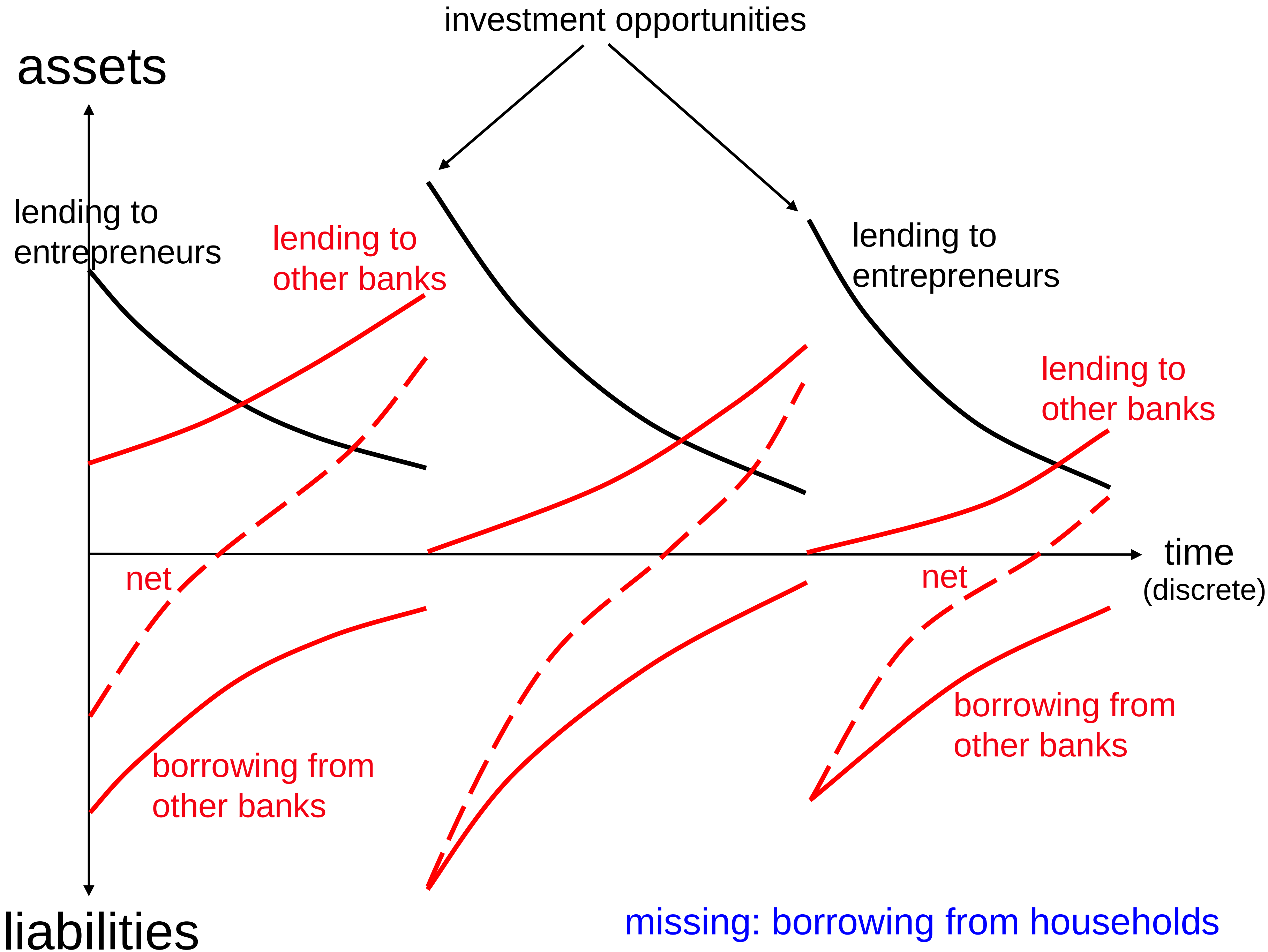
Two questions:

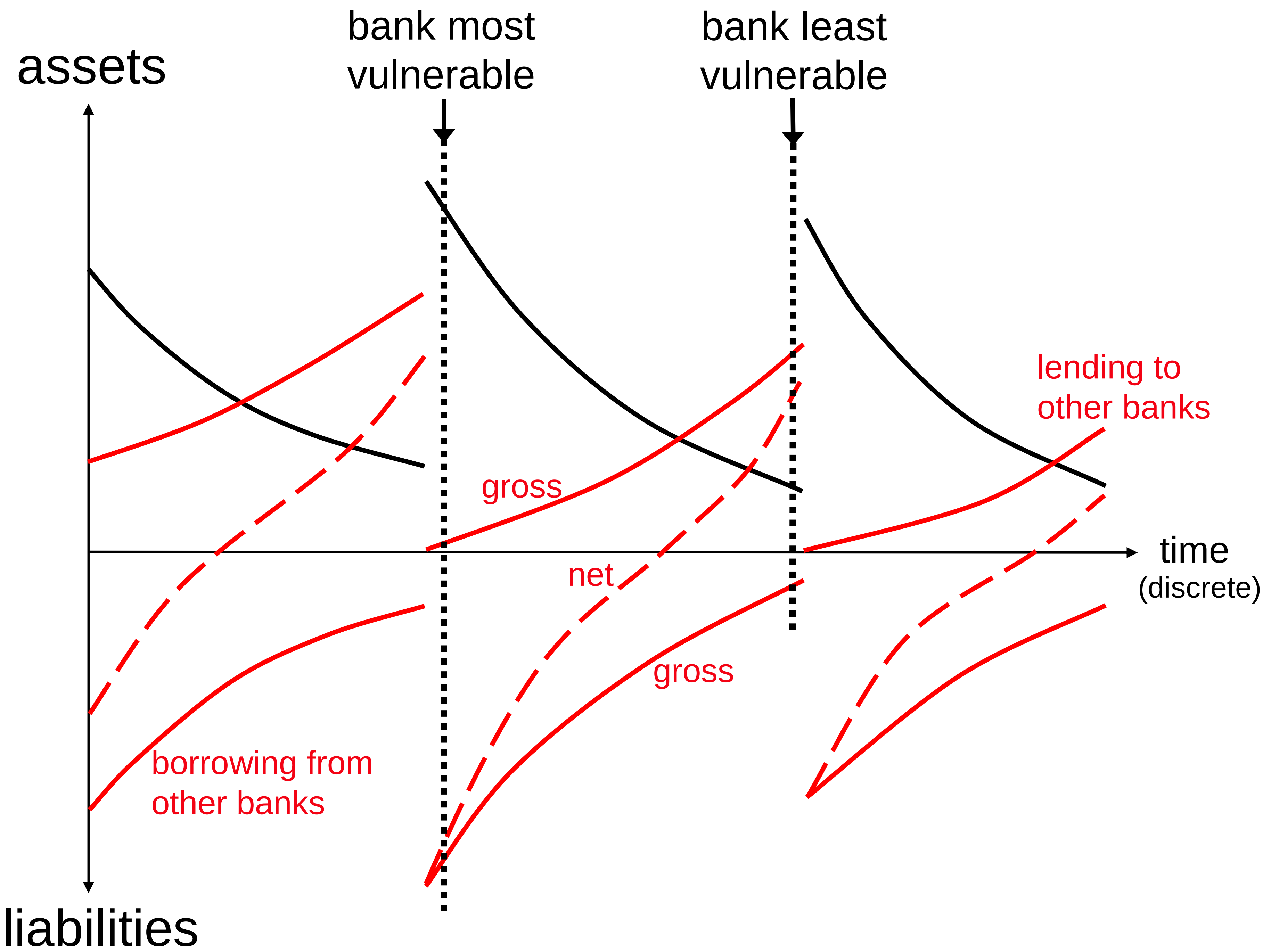
Q1 “Why hold mutual gross positions?”

Why should a bank borrow from another bank and simultaneously lend to that other bank (or to a third bank), even at the same rate of interest?   Is there a social benefit?

Q2 “Do gross positions create systemic risk?”

Is a financial system without netting – where banks lend to and borrow from each other (as well as to and from outsiders) – more fragile than a financial system with netting?


investment opportunities
assets
lending to entrepreneurs
lending to other banks
lending to entrepreneurs
lending to other banks
time
(discrete)
net
net
borrowing from other banks
borrowing from other banks
liabilities
missing: borrowing from households


assets
bank most
vulnerable
bank least
vulnerable
lending to
other banks
gross
net
time
(discrete)
gross
borrowing from
other banks
liabilities

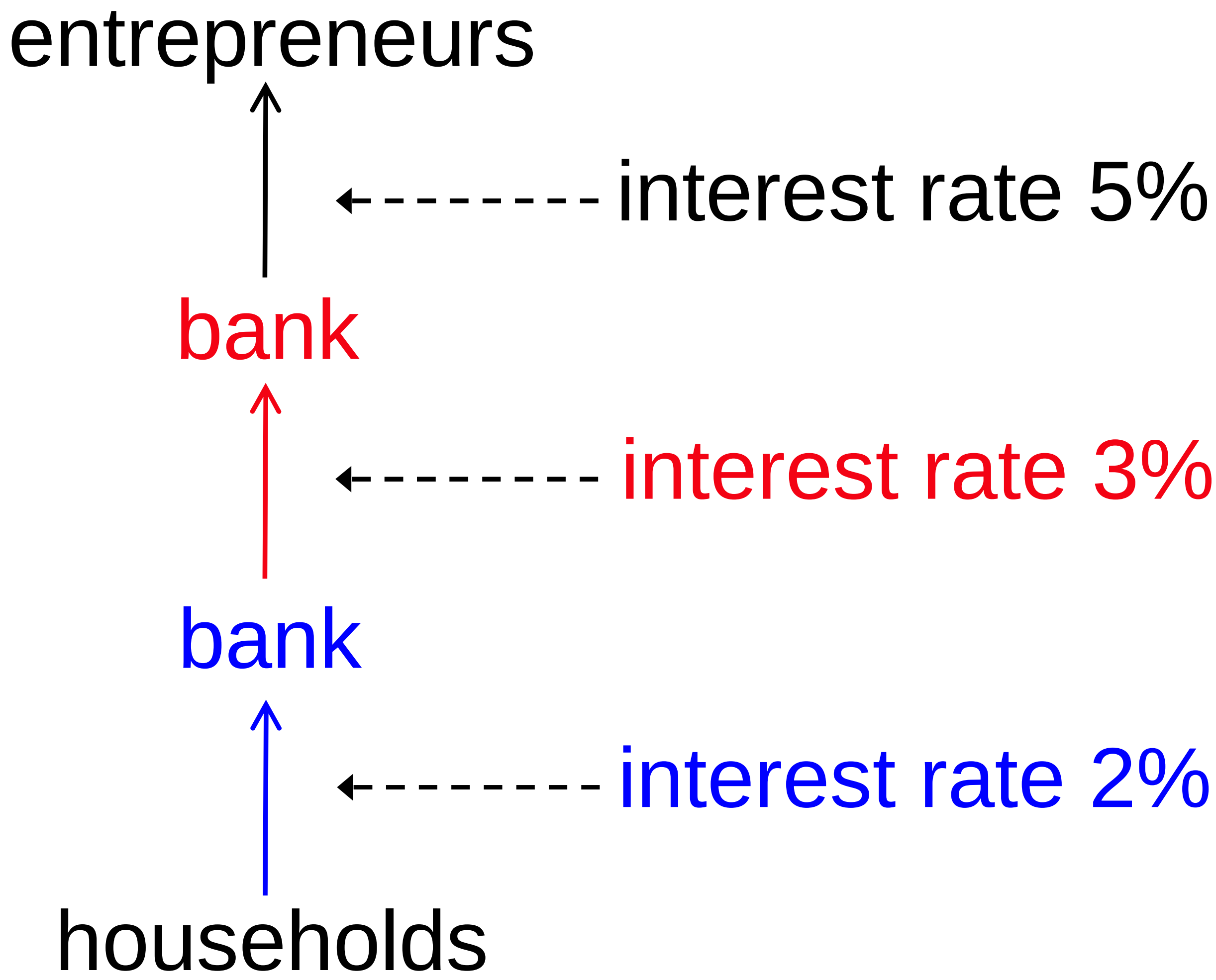
Proposition: If economy with mutual gross positions is hit by a productivity shock just big enough to cause the most vulnerable banks to fail, then, under plausible parameter restrictions, all banks fail.

cf. with netting, no other banks would fail

This answers Q2: gross positions do create systemic risk

But what about Q1? Why hold gross positions?

Numerical Example of Leverage Stack:



where, at each level, borrower can credibly pledge at most 9/10 of return

A bank has two feasible strategies:

Lend to entrepreneurs,
levered by borrowing from another bank:

> lend at 5%,
> 9/10 levered by borrowing at 3%,
> yields net return $\approx$ 23% (see Appendix)

Lend to another bank,
levered by borrowing from households:

> lend at 3%,
> 9/10 levered by borrowing at 2%,
> yields net return $\approx$ 12% (see Appendix)

Crucial assumption: it is *not* feasible to lend to entrepreneurs, levered by borrowing from households:

> lend at 5%,
> 9/10 levered by borrowing at 2%,
> would yield net return $\approx$ 32%

Why not? When lending to bank 1, say, a householder can't rely on entrepreneurs' bonds as security, because she does not know enough to judge them. But she can rely on a bond sold to bank 1 by bank 2 that is itself secured against entrepreneurs' bonds *which bank 1 is able to judge* (and bank 1 has "skin in the game").

levered lending to entrepreneurs (@ 23%)

> levered lending to banks (@ 12%)

$\Rightarrow$ all banks should adopt 23% strategy

But, in formal model, not all banks can do so:

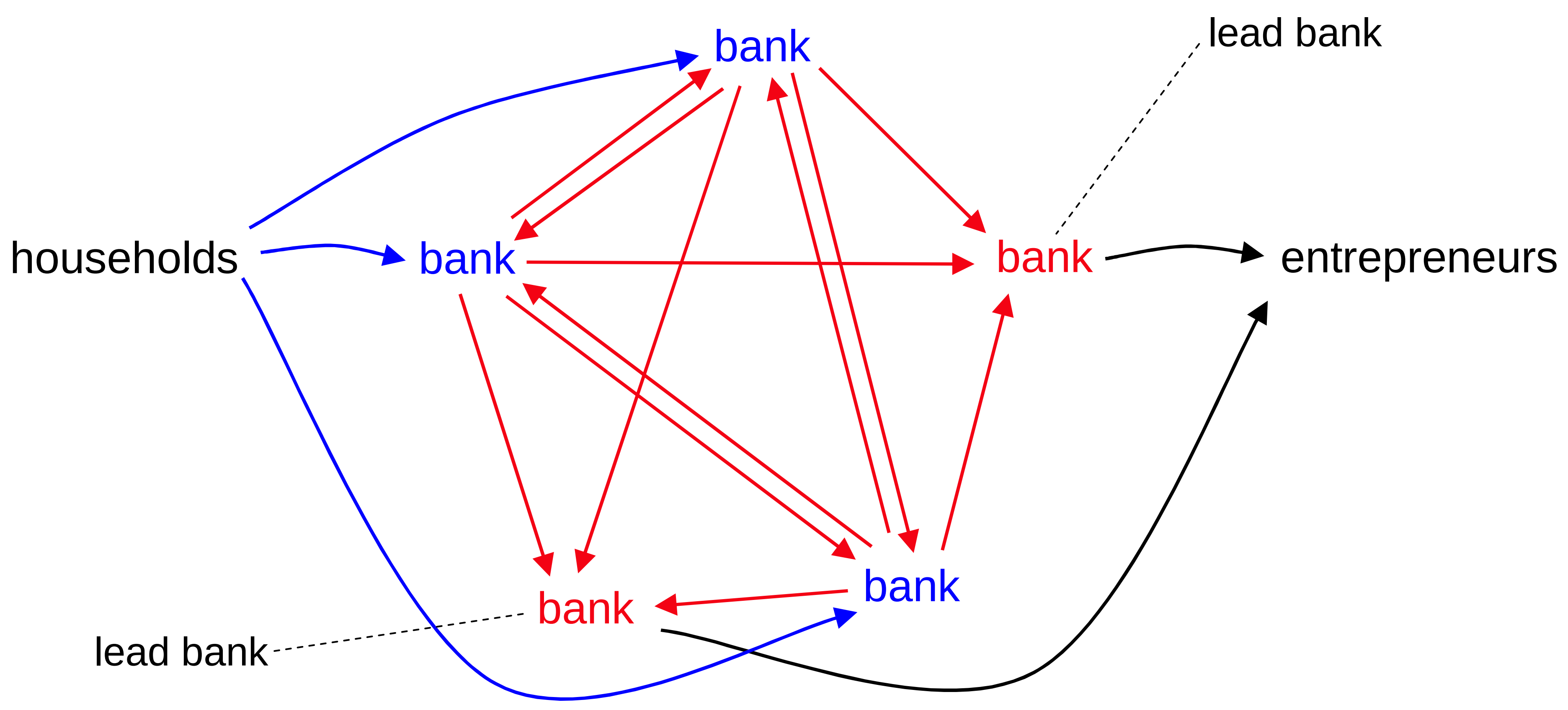
entrepreneurial lending opportunities are periodic

specifically, we assume:

at each date, with probability $\pi < 1$ a bank
has an opportunity to lend to entrepreneurs

In effect, banks take turns to be “lead banks”:

e.g. five banks and π = 2/5:



crucial: ∃ mutual gross positions
among non-lead banks

Why do non-lead banks *privately* choose to hold mutual gross positions? (Q1 again)

assume loans to entrepreneurs are long-term (though depreciating)

$\Rightarrow$ every bank has some of these old assets on its balance sheet

(from when, in the past, it was a lead bank)

Should non-lead bank spend its marginal dollar

on paying down ($\equiv$ not rolling over) old interbank debt secured against these old assets

$\Rightarrow$ return of 3%

or

on buying new interbank debt @ 3%, levered by borrowing from households @ 2%

$\Rightarrow$ effective return of 12% ✓

This answers Q1

socially, mutual gross positions among non-lead banks “certify” each others’ entrepreneurial loans and thus offer additional security to households

$\Rightarrow$ more funds flow in to the banking system, from households

$\Rightarrow$ more funds flow out of the banking system, to entrepreneurs

$\Rightarrow$ greater investment & aggregate activity

but though the economy operates at a higher average level, it is susceptible to systemic failure

MODEL

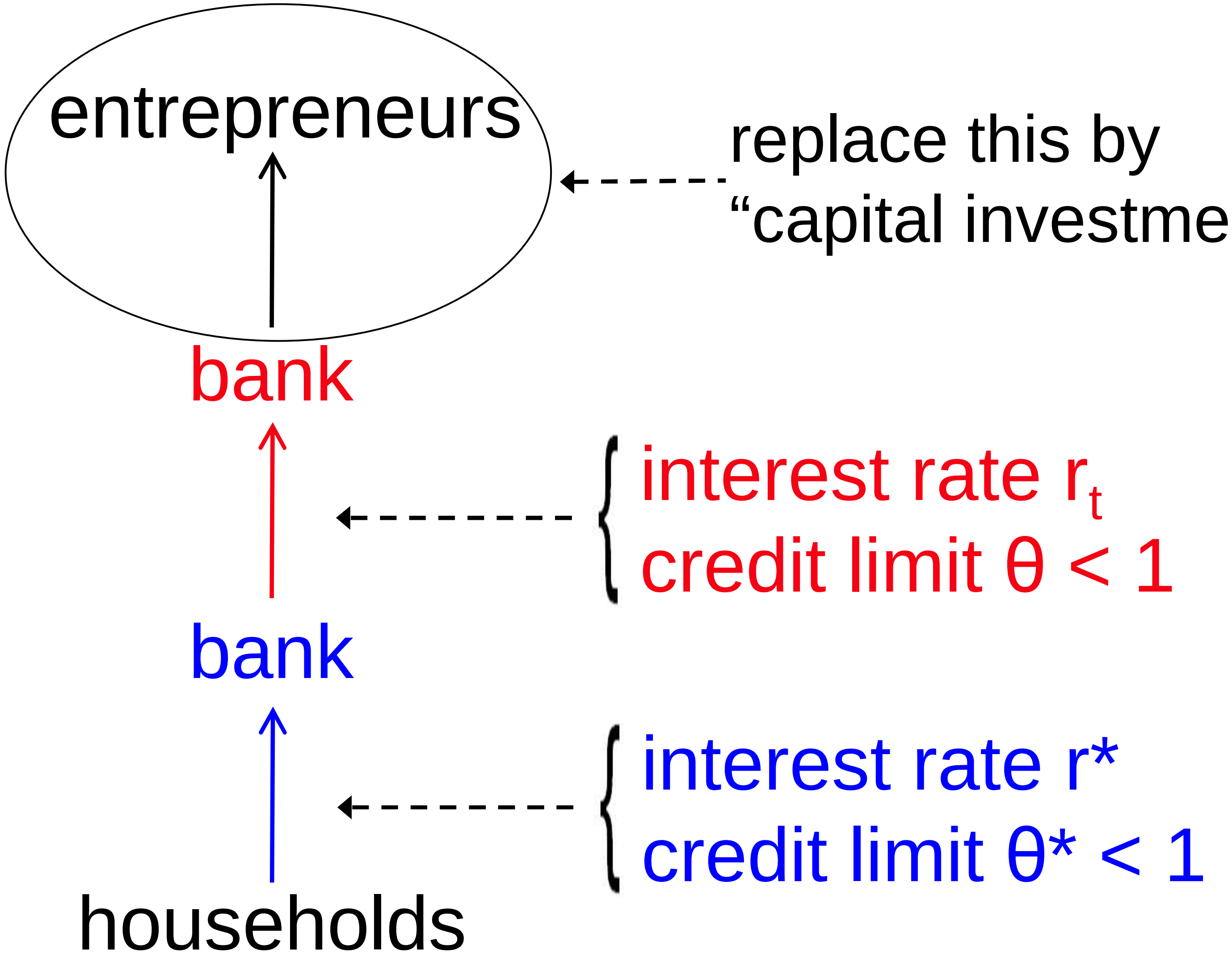
discrete time, dates $t = 0, 1, 2, \ldots$

at each date, single good (numeraire)

fixed set of agents ("inside" banks)

in background: outside suppliers of funds
(households; "outside" banks)

Apply Occam's Razor to top of leverage stack:



Capital investment

constant returns to scale; per unit of project:

date t	date t+1	date t+2	date t+3	…
-1	a_{t+1}	λa_{t+2}	$\lambda^2 a_{t+3}$	…
unit cost		depreciation factor $\lambda < 1$		

where the economy-wide productivity shock $\{a_{t+s}\}$ follows stationary stochastic process

Investment opportunities arise with probability π
(i.i.d. across banks and through time)

to simplify the presentation, let's suppose banks derive utility from their scale of investment

$\Rightarrow$ a bank invests maximally if opportunity arises

in full model, banks consume (pay dividends)

Capital investment is illiquid: projects are specific & succeed only with expertise of investing bank

However, the bank can issue "*interbank bonds*" (i.e. borrow from other banks) against its capital investment:

per unit of project, bank can issue

$$\theta < 1 \quad \text{interbank bonds}$$

price path of interbank bonds: $\{q_t, q_{t+1}, q_{t+2}, \ldots\}$

an interbank bond issued at date t+s promises

$$[\, E_{t+s} a_{t+s+1} \;+\; \lambda E_{t+s} q_{t+s+1} \,] \quad \text{at date } t+s+1$$

(expectations conditional on
no default at t+s+1)

i.e., bonds are short-term & creditor is promised (a fraction θ of) expected project return next period + expected price of a new bond issued next period against residual flow of returns

collateral securing old bond

= expected project return
+ expected sale price of new bond

from the price path $\{q_t, q_{t+1}, q_{t+2}, \ldots\}$ we can compute the interbank interest rates:

effective risk-free interbank interest rate, r_{t+s},

between date t+s and date t+s+1 solves:

$$q_{t+s} = \frac{1 - \delta_{t+s+1}}{1 + r_{t+s}} \left[E_{t+s} a_{t+s+1} + \lambda E_{t+s} q_{t+s+1} \right]$$

where δ_{t+s+1} = probability of default at date t+s+1 (endogenous)

NB in principle δ_{t+s+1} is bank-specific – but see Corollary to Proposition below

A bank can issue "*household bonds*" (i.e. borrow from households) against its holding of interbank bonds. Household bonds mimic interbank bonds:

– a household bond issued at date t+s promises

to pay $\left[E_{t+s}a_{t+s+1} + \lambda E_{t+s}q_{t+s+1} \right]$ at date t+s+1

per interbank bond, bank can issue

$\theta^* < 1$ household bonds

at price

$$q_{t+s}^* = \frac{1 - \delta_{t+s+1}}{1 + r^*} \left[E_{t+s}a_{t+s+1} + \lambda E_{t+s}q_{t+s+1} \right]$$

households lend at r*

Critical assumption: these promised payments – on interbank & household bonds – are *fixed* at issue, date t+s, using that date's expectation (E_{t+s}) of future returns & bond prices

$\Rightarrow$ bonds are unconditional,
without any state-dependence

In the event of, say, a fall in returns, or
a fall in bond prices,

the debtor bank must honour its fixed payment obligations, or risk default & bankruptcy

Assume bankruptcy $\Rightarrow$ creditors receive nothing typical bank’s balance sheet at start of date t

assets	liabilities
	secured against
capital investment holdings (k_t)	interbank bonds issued ($\leq \theta k_t$)
	secured against
interbank bond holdings (b_t)	household bonds issued ($\leq \theta^* b_t$)
	own equity

lead bank's flow-of-funds

rollover

$$i_t \leq a_t k_t - [\,E_{t-1}a_t + \lambda E_{t-1}q_t\,]\,\theta k_t$$

capital investment   returns   payments to other banks

$$+ [\,E_{t-1}a_t + \lambda E_{t-1}q_t\,]\,b_t - [\,E_{t-1}a_t + \lambda E_{t-1}q_t\,]\,\theta^* b_t$$

payments from other banks   payments to households

rollover

$$+ \; q_t\theta\,(\lambda k_t + i_t)$$

sale of new interbank bonds

Hence, for a lead bank starting date t with (k_t, b_t),

$$b_{t+1} = 0$$

$$\text{and} \quad k_{t+1} = \lambda k_t + i_t$$

where i_t is given by

$$\frac{(a_t - \theta E_{t-1}a_t)k_t + (1-\theta^*)\left[E_{t-1}a_t + \lambda E_{t-1}q_t \right]b_t + \theta(q_t - E_{t-1}q_t)\lambda k_t}{1 - \theta q_t}$$

non-lead bank's flow-of-funds

rollover

$$q_t\, b_{t+1} \leq a_t k_t - [\, E_{t-1} a_t + \lambda E_{t-1} q_t \,]\, \theta k_t$$

purchase of other banks' bonds | returns | payments to other banks

$$+ [\, E_{t-1} a_t + \lambda E_{t-1} q_t \,]\, b_t - [\, E_{t-1} a_t + \lambda E_{t-1} q_t \,]\, \theta^* b_t$$

payments from other banks | payments to households

rollover

$$+ \quad q_t\, \theta \lambda k_t \quad + \quad q_t^* \theta^* b_{t+1}$$

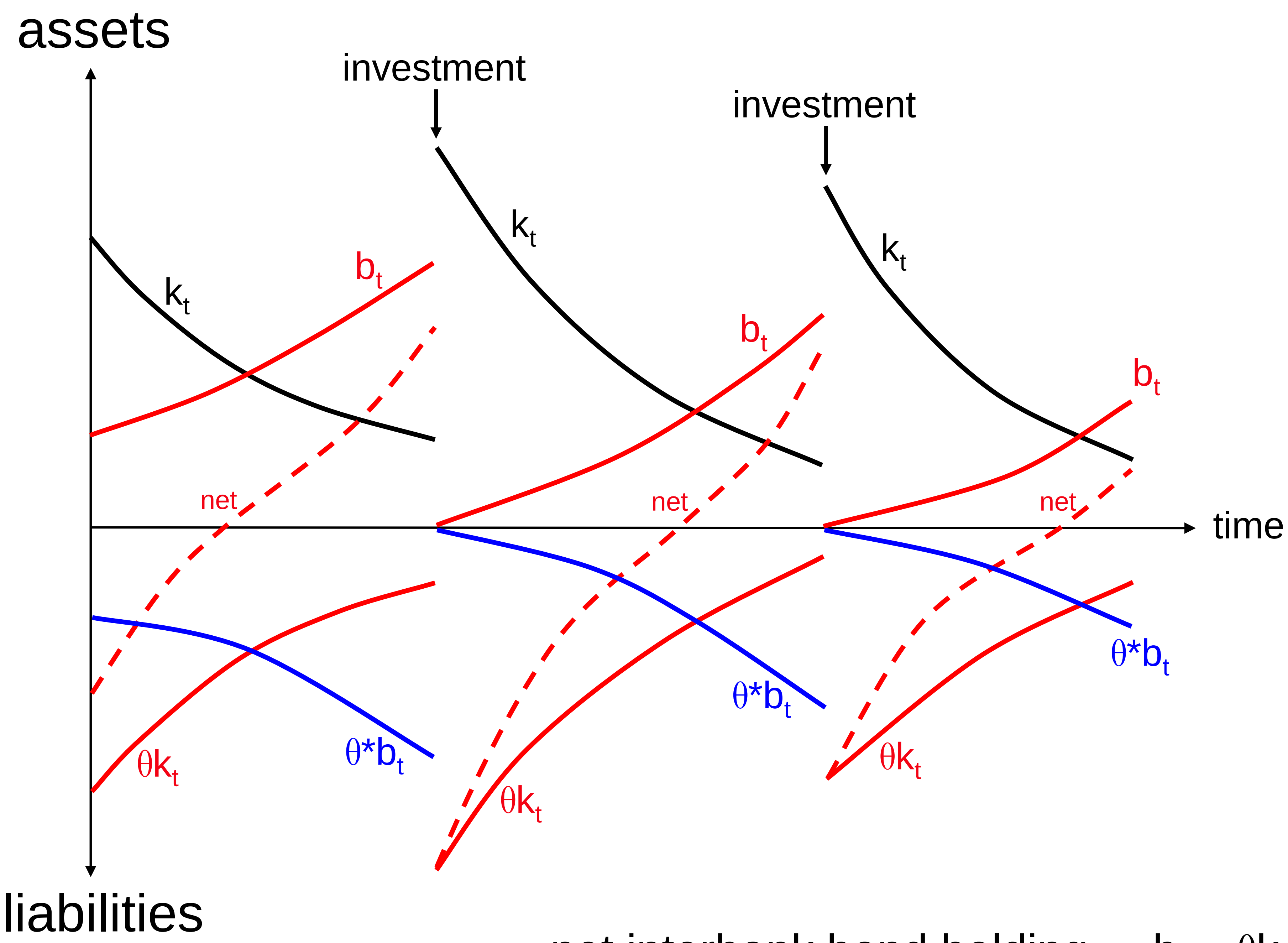
sale of new interbank bonds | sale of new household bonds

Hence, for a non-lead bank starting date t with (k_t, b_t),

$$k_{t+1} = \lambda k_t$$

and b_{t+1} is given by

$$b_{t+1} = \frac{(a_t - \theta E_{t-1}a_t)k_t + (1-\theta^*)\left[\, E_{t-1}a_t + \lambda E_{t-1}q_t \,\right]b_t + \theta(q_t - E_{t-1}q_t)\lambda k_t}{q_t - \theta^* q_t^*}$$



net interbank bond holding = $b_t - \theta k_t$

each bank has its personal history of, at each past date, being either a lead or a non-lead bank

$\Rightarrow$ in principle we should keep track of how the distribution of $\{k_t, b_t\}$'s evolves (hard)

however, the great virtue of our expressions for k_{t+1} and b_{t+1} is that they are linear in k_t and b_t

$\Rightarrow$ aggregation is easy

At the start of date t, let

K_t = banks' stock of capital investment

B_t = banks' stock of interbank bonds

$K_{t+1} = \lambda K_t + I_t$ where

I_t = banks' capital investment =

Investment is v sensitive to falls in the bond price

$$\frac{\pi \left\{ (a_t - \theta E_{t-1} a_t) K_t + (1-\theta^*)\left[E_{t-1} a_t + \lambda E_{t-1} q_t \right] B_t + \theta(q_t - E_{t-1} q_t)\lambda K_t \right\}}{1 - \theta q_t}$$

and B_{t+1} is given by

$$B_{t+1} = \frac{(1-\pi)\left\{(a_t - \theta E_{t-1}a_t)K_t + (1-\theta^*)\left[E_{t-1}a_t + \lambda E_{t-1}q_t\right]B_t + \theta(q_t - E_{t-1}q_t)\lambda K_t\right\}}{q_t - \theta^* q_t^*}$$

Market clearing

Price q_t clears the market for interbank bonds at each date t:

interbank banks' bond demand $= B_{t+1}$

interbank banks' bond supply $= \theta K_{t+1}$

Posit additional demand from outside banks:

$$\underbrace{D(\overset{+}{r_t})}_{\uparrow} = q_t \left(\theta K_{t+1} - B_{t+1} \right)$$

outside banks' supply of loanable funds
is increasing in risk-free interest rate r_t

The following results hold near to steady-state

Assume that most interbank loans come from the other inside banks, not from outside banks:

$$q_t B_{t+1} \quad \gg \quad D(r_t)$$

We need to confirm that inside (non-lead) banks *will* choose to lever their interbank lending with borrowing from households:

<u>Lemma 1</u> $r_t > r^*$ iff (A.1):

$$\theta \;>\; \pi\theta\theta^* \;+\; (1-\pi)(1-\lambda+\lambda\theta) \;+\; (1-\pi)(1-\theta\theta^*)r^*$$

Lemma 2a

A fall in a_t raises the current interest rate r_t

Intuition: $a_t \downarrow$ raises bond supply/demand ratio:

$$\frac{\text{inside banks' bond supply}}{\text{inside banks' bond demand}} = \frac{\theta\left(\lambda K_t + \frac{\pi}{1-\theta q_t} W_t\right)}{\frac{1-\pi}{q_t - \theta^* q_t^*} W_t}$$

which implies $r_t \uparrow$

where

$$W_t = \Big\{ (a_t - \theta E_{t-1} a_t) K_t + (1-\theta^*)\big[E_{t-1} a_t + \lambda E_{t-1} q_t \big] B_t + \theta(q_t - E_{t-1} q_t)\lambda K_t \Big\}$$

Lemma 2b

For $s \geq 0$, a rise in r_{t+s} raises r_{t+s+1}

Intuition: $r_{t+s} \uparrow \;\Rightarrow\; (1 + r_{t+s})D(r_{t+s}) \uparrow$

debt (inclusive of interest) owed
by inside banks to outside banks
at date t+s+1

$\Rightarrow\; W_{t+s+1} \downarrow$ (debt overhang)

$\Rightarrow\; r_{t+s+1} \uparrow$ (cf. Lemma 2a)

Lemma 2c

A rise in future interest rates raises the current interest rate if (A.2): $\theta^*\pi > (1 - \lambda + \lambda\pi)^2$

Intuition: a rise in any of $E_t r_{t+1}$, $E_t r_{t+2}$, $E_t r_{t+3}$, …

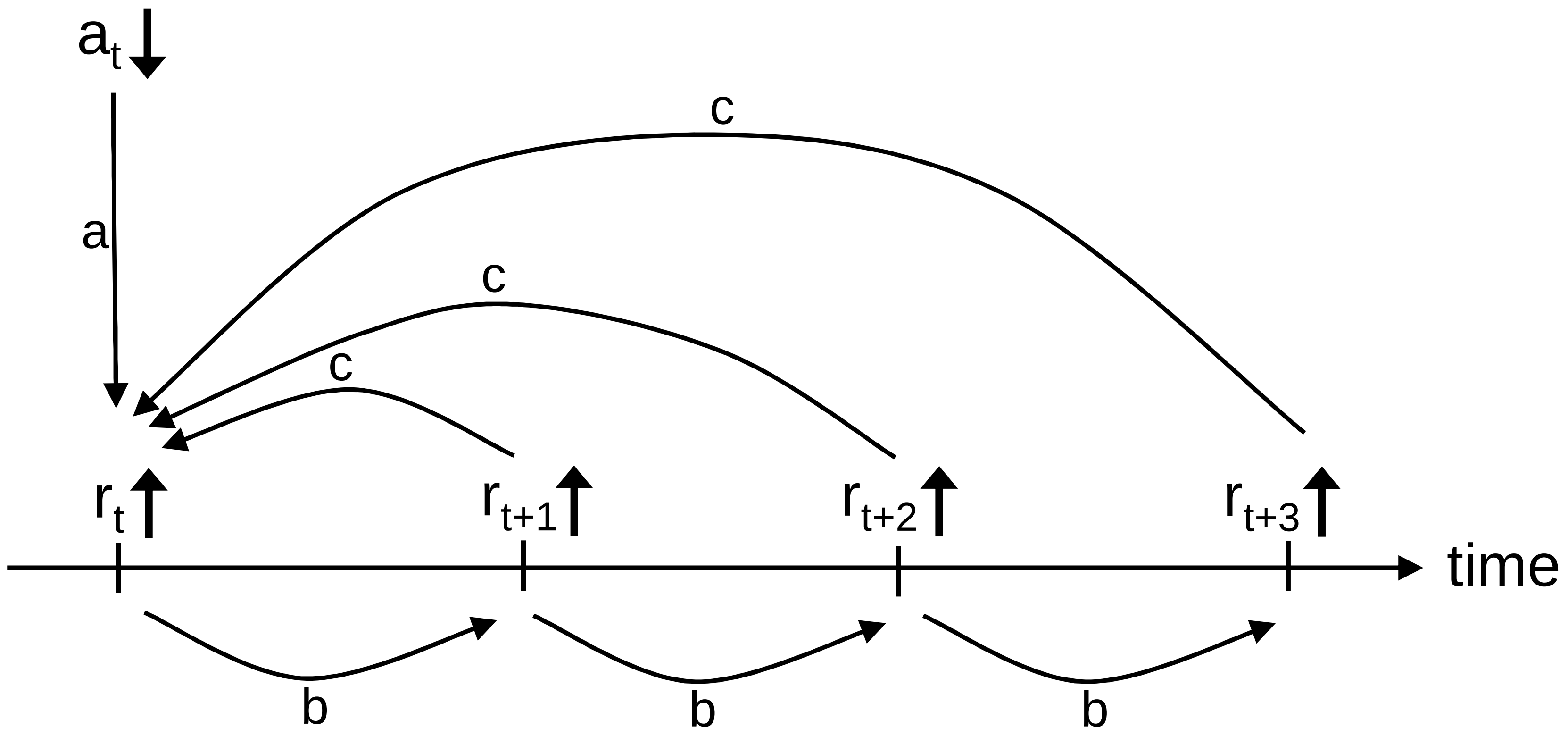
$$\Rightarrow E_t q_{t+1} \downarrow \quad \Rightarrow \quad q_t^* = \frac{1-\delta_{t+1}}{1+r^*}\left\{ E_t a_{t+1} + \lambda E_t q_{t+1} \right\} \downarrow$$

$\Rightarrow$ ratio of inside banks' bond supply/demand

$$= \frac{\theta\left(\lambda K_t + \frac{\pi}{1-\theta q_t} W_t\right)}{\frac{1-\pi}{q_t - \theta^* q_t^*} W_t} \uparrow \qquad \Rightarrow \quad r_t \uparrow$$

under (A.2), this channel dominates
(borrowing from households ↓)

amplification through interest rate cascades:



$\Rightarrow q_t \downarrow$

$\Rightarrow I_t \downarrow\downarrow$

collateral-value multiplier:

interbank bond prices ↓

collateral values ↓

borrowing from households ↓

net interbank lending by non-lead banks ↓

interbank interest rates ↑

broad intuition:

negative shock

$\Rightarrow$ interbank interest rates $\uparrow$ and bond prices $\downarrow$

$\Rightarrow$ banks' household borrowing limits tighten

$\Rightarrow$ funds are taken *from* banking system, just as they are most needed fall in interbank bond prices

$\Rightarrow$ banks may have difficulty rolling over their debt, and so be vulnerable to failure

"most vulnerable" banks:

banks that have just made maximal capital investment (because they hold no cushion of interbank bonds)

Failure of these banks can precipitate a failure of the entire banking system:

Proposition (systemic failure)

In addition to Assumption (A.1), assume

$$\text{(A.3):} \qquad \theta^* \; > \; (1-\pi)\,\lambda$$

If the aggregate shock is enough to cause the most vulnerable banks to fail, then *all* banks fail (in the order of the ratio of their capital stock to their holding of other banks' bonds).

NB In proving this Proposition, use is made of the steady-state (ergodic) distribution of the $\{k_t, b_t\}$'s across banks

Corollary

At each date t, the probability of default, δ_t, is the same for all inside banks

We implicitly assumed this earlier – in effect, we have been using a guess-and-verify approach

Banks make no attempt to self-insure – e.g. by lending to “less risky” banks (because there are none: all banks are equally risky)

Parameter consistency?

Assumptions (A.1), (A.2) and (A.3) are mutually consistent:

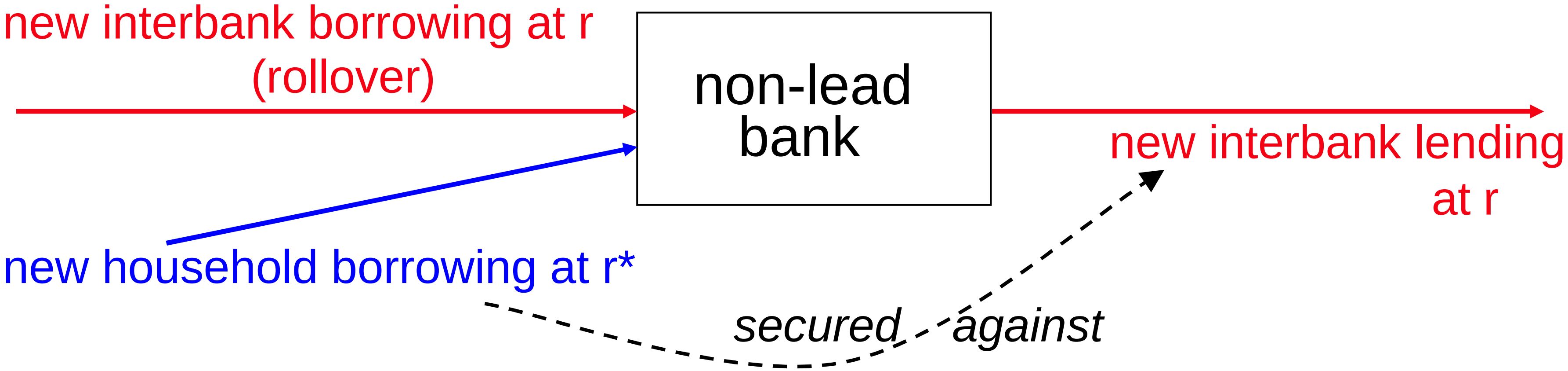
e.g. $\pi = 0.1$

$\lambda = 0.975$

$\theta = \theta^* = 0.9$

$r^* = 0.02$

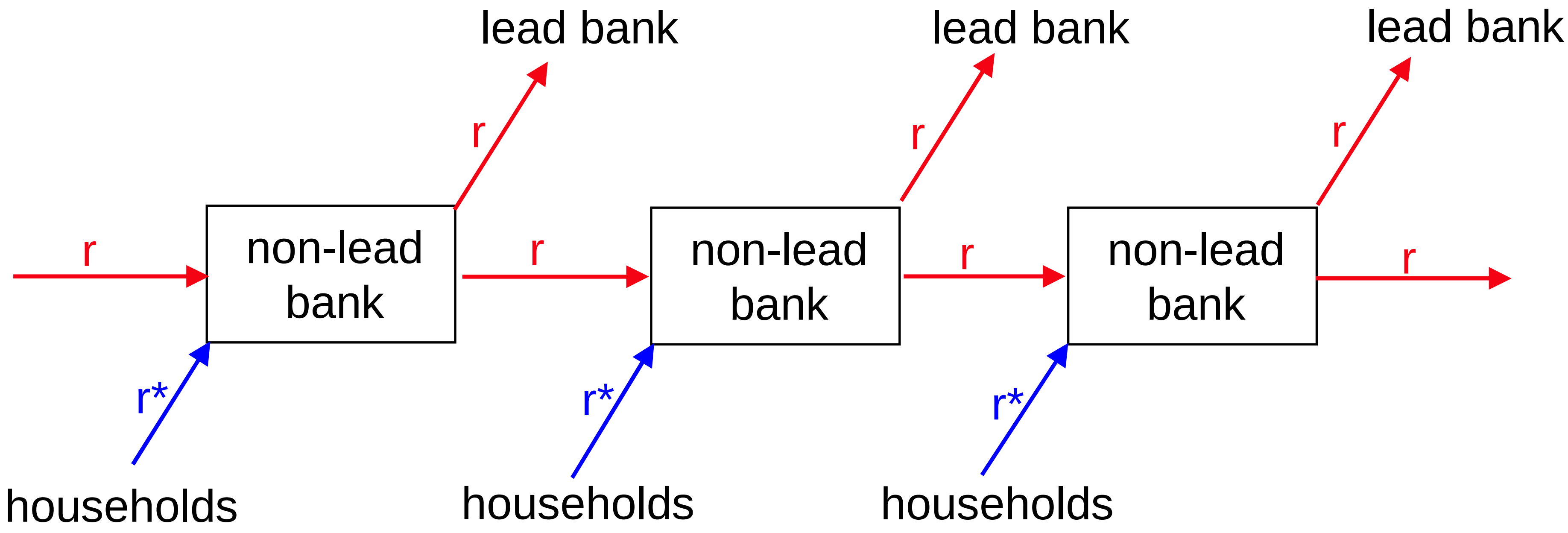
key point: non-lead banks are both borrowers and lenders in the interbank market



notice multiplier effect: if for some reason bank's value of new interbank borrowing ↓ (by x dollars, say)

⇒ bank's value of new interbank lending ↓↓ (by >> x dollars, because of household leverage)

⇒ bank's *net* interbank lending ↓



if the “household-leverage multiplier”

exceeds the “leakage” to lead banks

then we get amplification along the chain

APPENDIX

borrower has net worth w

and has constant-returns investment opportunity:

net rate of return on investment $= r$

lender has lower opportunity cost of funds:

net rate of interest on loans $= r^* < r$

but only lends against $\theta^* < \dfrac{1+r^*}{1+r}$ of gross return

e.g. $r = 3\%$, $r^* = 2\%$, $\theta^* = 9/10$

borrower's flow-of-funds:

$$i \leq w + \left(\frac{1}{1 + r^*}\right) d$$

investment borrowing

s.t.

$$d \leq \theta^*(1 + r)i$$

debt pledgable return

with maximal levered investment:

$$i = \frac{w}{\left(1 - \frac{\theta^*(1+r)}{1+r^*}\right)}$$

net rate of return on *levered* investment equals

$$\frac{(1 - \theta^*)(1 + r)i \quad - \quad w}{w}$$

$$= \quad r \quad + \quad \frac{\dfrac{\theta^*(1+r)}{1+r^*}}{\left(1 - \dfrac{\theta^*(1+r)}{1+r^*}\right)}(r - r^*)$$

$\approx$ 12% when $r = 3\%$, $r^* = 2\%$, $\theta^* = 9/10$

Double check: suppose net worth $w = 100$

$\theta^* = 9/10 \Rightarrow$ borrow $b = 900$ approx

$\Rightarrow$ invest $i = 1000$

$r = 3\% \Rightarrow$ gross return = 1030

$r^* = 2\% \Rightarrow$ gross debt repayment = 918

$\Rightarrow$ net return = 112

ie. net rate of return on levered investment = 12%