The Ecology of Financial Markets

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London School of Economics
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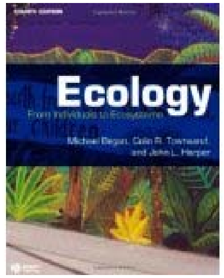
Content

My understanding of ecology



Begon, Townsend, Harper (2012): »Ecology is the scientific study of the distribution and abundance of organisms an the interactions that determine distribution and abundance.»

- Conditions (temperature, water, sun, …)
- Resources (Nytrogen, Oxygen, Carbon, …)
- Genetic drift and the definition of species
- Diversity as a measure for risk management
- Interactions (predator-prey, competition, symbiosis, parasitism, …)

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- My understanding of Ecology
- **Ecology as a Paradigm for Finance**
- Principles and Assumptions in Economics
- Our Evolutionary Finance Model
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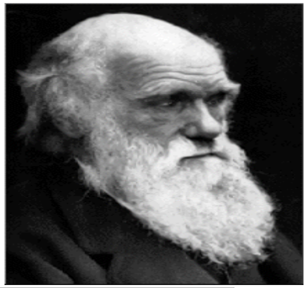
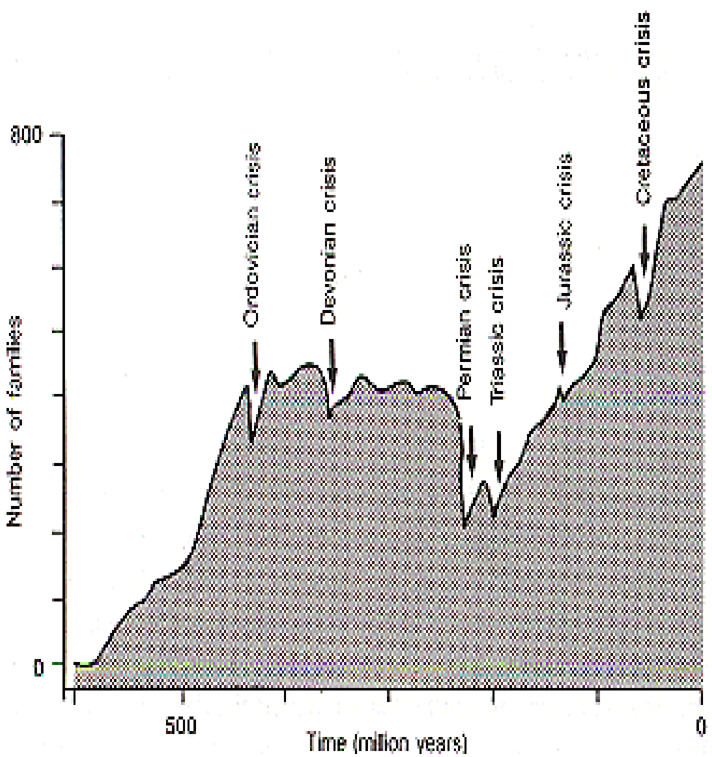
Ecology as a New Paradigm for Finance

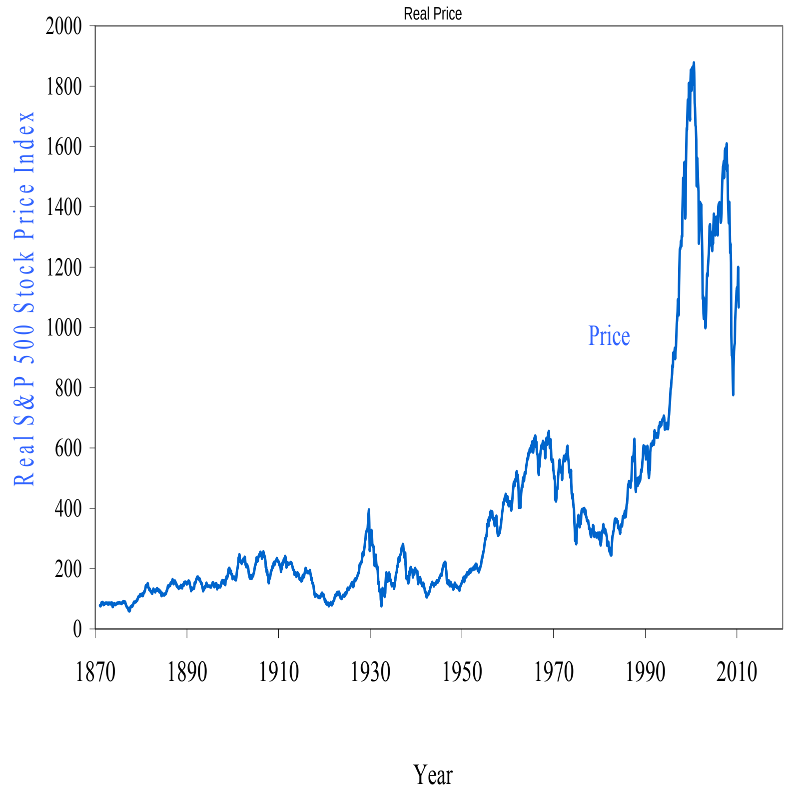


„Survival of the Fittest on Wall Street."

Biology:	**Finance:**
Conditions	Economic Growth
Resources	Market capital
Species	Portfolio rule
Selection	Gains / losses
Mutation	Innovation
Mass extinctions	Crashes

Mass Extinctions and Crashes on S&P 500





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Principles and Assumptions in Economics

Principles

- Walras Law
- No perfect foresight
- Time runs from t to t+1
- …

Assumptions

- Market (Micro-)Structure
- Dividend Process
- Ecology of Strategies
- …

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Ecology of Strategies

Lucas (1978) Tree Model:

$\lambda_t^{i,k}$ $i =$ 1,...,I strategies/agents

$k =$ 1,...,K assets

$k =$ c consumption good

Value, Growth, Momentum & Reversal,…
Long Only, Rebalance, Long/Short, Volatility Pumping,..
Relative Dividend Yield, Dogs of the Dow, Junk Bonds
L/S-Equity, Statistical Arbitrage, M&A-Arbitrage,
Global Macro,…

→ Model allows for any strategy **adapted to information!**

Our Evolutionary Model

Lucas (1978) Tree Model: $\omega_t \in \Omega,\quad \omega^t = (\omega_0, \ldots, \omega_t)$

$\lambda_t^{i,k}$ $i =$ 1,...,I strategies/agents P prob measure

$k =$ 1,...,K assets

$k =$ c consumption good $\sum_{k=1}^{K} \lambda_t^{i,k} = 1 - \lambda_t^{i,c}$

Evolution of Wealth:

$$W_{t+1}^{i}(\omega^{t+1}) = \left\{ \sum_{k=1}^{K} \left[\frac{D_{t+1}^{k}(\omega_{t+1}) + q_{t+1}^{k}(\omega^{t+1})}{q_t^k(\omega^t)} \right] \lambda_t^{i,k}(\omega^t) \right\} W_t^i(\omega^t)$$

Market Interaction

Short Run Equilibrium in Period t:

Demand = Supply (normalized to 1)

$$\sum_{i=1}^{I} \frac{\lambda_t^{i,k}(\omega^t) W_t^i(\omega^t)}{q_t^k(\omega^t)} = 1$$

Note: Interpretation of q_t^k

is market capitalization of firm k!

Market Interaction

Equilibrium Prices:

$$q_t^k(\omega^t) = \sum_{i=1}^{I} \lambda_t^{i,k}(\omega^t) W_t^i(\omega^t)$$

„The price of asset k is the wealth-average

of the strategies` portfolio share for asset k.“

Evolution of **relative** wealth:

$$r_{t+1}^{i}(\omega^{t+1}) = \left\{ \sum_{k=1}^{K} \left[\frac{\lambda^{c} d_{t+1}^{k}(\omega_{t+1}) + \hat{q}_{t+1}^{k}(\omega^{t+1})}{\hat{q}_{t}^{k}(\omega^{t})} \right] \lambda_{t}^{i,k}(\omega^{t}) \right\} r_{t}^{i}(\omega^{t})$$

where

$$\hat{q}_{t}^{k}(\omega^{t}) = \frac{q_{t}^{k}(\omega^{t})}{\sum_{i} W_{t}^{i}(\omega^{t})} \quad d_{t+1}^{k}(\omega_{t+1}) = \frac{D_{t+1}^{k}(\omega^{t+1})}{\sum_{j} D_{t+1}^{j}(\omega_{t+1})}$$

$$r_{t}^{i}(\omega^{t}) = \frac{W_{t}^{i}(\omega^{t})}{\sum_{i} W_{t}^{i}(\omega^{t})}$$

Assuming $\lambda_{t}^{i,c}(\omega^{t}) = \lambda^{c}$, we get $\lambda^{c} \sum_{i} W_{t}^{i}(\omega^{t}) = \sum_{k} D_{t}^{k}(\omega^{t})$, since

$W_{t}^{i}(\omega^{t}) = \sum_{k} \left(D_{t}^{k}(\omega^{t}) + q_{t}^{k}(\omega^{t}) \right) \theta_{t-1}^{i,k}(\omega^{t-1})$, so that $\sum_{k} q_{t}^{k}(\omega^{t}) = (1-\lambda^{c}) \sum_{i} W_{t}^{i}(\omega^{t})$.

Market Interaction

Equilibrium Prices:

$$\hat{q}_t^k(\omega^t) = \sum_{i=1}^{I} \lambda_t^{i,k}(\omega^t) r_t^i(\omega^t)$$

„The price of asset k is the **relative** wealth-average of the strategies` portfolio share for asset k.“

Deriving a Dynamical System: $r_t \rightarrow r_{t+1} \rightarrow r_{t+2} \rightarrow \ldots$

$$r_{t+1}^{i}(\omega^{t+1}) = \left\{ \sum_{k=1}^{K} \left[\frac{\lambda^{c} d_{t+1}^{k}(\omega_{t+1}) + \hat{q}_{t+1}^{k}(\omega^{t+1})}{\hat{q}_{t}^{k}(\omega^{t})} \right] \lambda_{t}^{i,k}(\omega^{t}) \right\} r_{t}^{i}(\omega^{t})$$

$$\text{where } \hat{q}_{t+1}^{k}(\omega^{t+1}) = \sum_{i=1}^{I} \lambda_{t+1}^{i,k}(\omega^{t+1}) r_{t+1}^{i}(\omega^{t+1})$$

Deriving a Dynamical System: $r_t \rightarrow r_{t+1} \rightarrow r_{t+2} \rightarrow \ldots$

$$r_{t+1}^{i}(\omega^{t+1}) = \left\{ \sum_{k=1}^{K} \left[\frac{\lambda^{c} d_{t+1}^{k}(\omega_{t+1}) + \hat{q}_{t+1}^{k}(\omega^{t+1})}{\hat{q}_{t}^{k}(\omega^{t})} \right] \lambda_{t}^{i,k}(\omega^{t}) \right\} r_{t}^{i}(\omega^{t})$$

Circular reference!

$$\text{where } \hat{q}_{t+1}^{k}(\omega^{t+1}) = \sum_{i=1}^{I} \lambda_{t+1}^{i,k}(\omega^{t+1}) r_{t+1}^{i}(\omega^{t+1})$$

Deriving a Dynamical System: $r_t \rightarrow r_{t+1} \rightarrow r_{t+2} \rightarrow \ldots$

$$r^i_{t+1}(\omega^{t+1}) = \left\{ \sum_{k=1}^{K} \left[\frac{\lambda^c d^k_{t+1}(\omega_{t+1}) + \hat{q}^k_{t+1}(\omega^{t+1})}{\hat{q}^k_t(\omega^t)} \right] \lambda_t^{i,k}(\omega^t) \right\}_{\lambda^c} r^i_t(\omega^t)$$

Circular reference!

$$\text{where } \hat{q}^k_{t+1}(\omega^{t+1}) = \sum_{i=1}^{I} \lambda_{t+1}^{i,k}(\omega^{t+1}) r^i_{t+1}(\omega^{t+1})$$

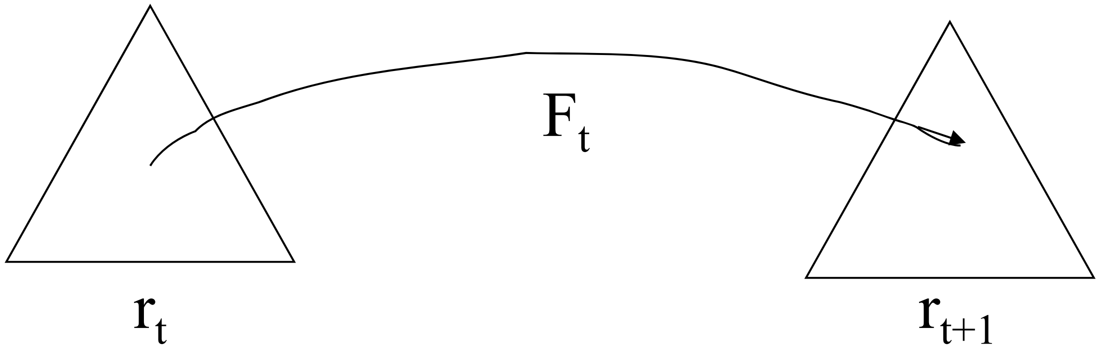
Solution:

$$r_{t+1} = \left(\mathrm{Id} - \left[\frac{\hat{\lambda}^i_{t,k}(\omega^t)\, r^i_t}{\hat{\lambda}_{t,k}(\omega^t)\, r_t} \right]_i^k \Lambda_{t+1}(\omega^{t+1}) \right)^{-1} \left[\sum_{k=1}^{K} d^k_{t+1}(\omega^{t+1}) \frac{\hat{\lambda}^i_{t,k}(\omega^t)\, r^i_t}{\hat{\lambda}_{t,k}(\omega^t)\, r_t} \right]_i$$

$$\text{where } \hat{\lambda}_t^{i,k} = \lambda_t^{i,k} / (1 - \lambda^c) \text{ so that } \sum_{k=1}^{K} \hat{\lambda}_t^{i,k} = 1.$$

Random Dynamical System

$$r_{t+1}(\omega^{t+1}) = F_t\,(\omega^{t+1}, r_t)$$



Principals: Walras Law, no perfect foresight, dynamics

Assumptions :

So far: batch auction, common consumption rate
Further assumptions: Dividend Process , Set of Strategies

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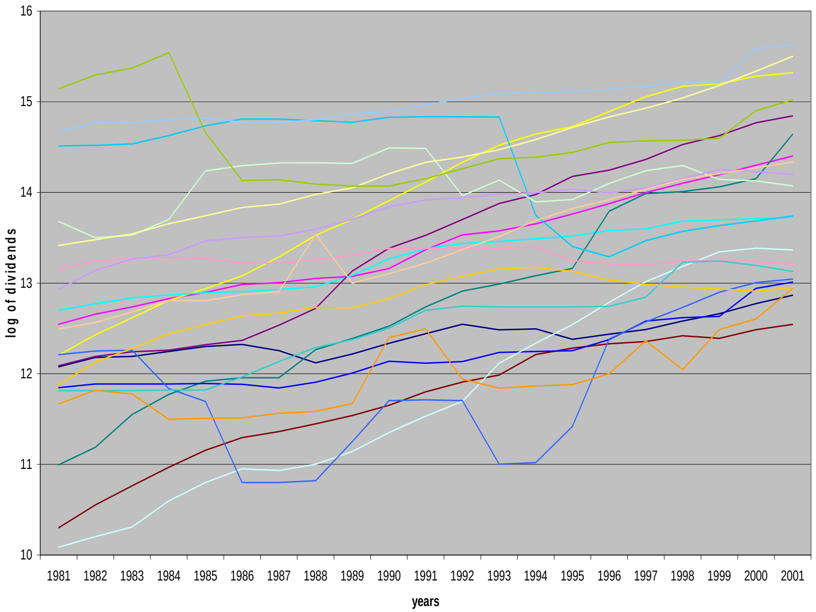
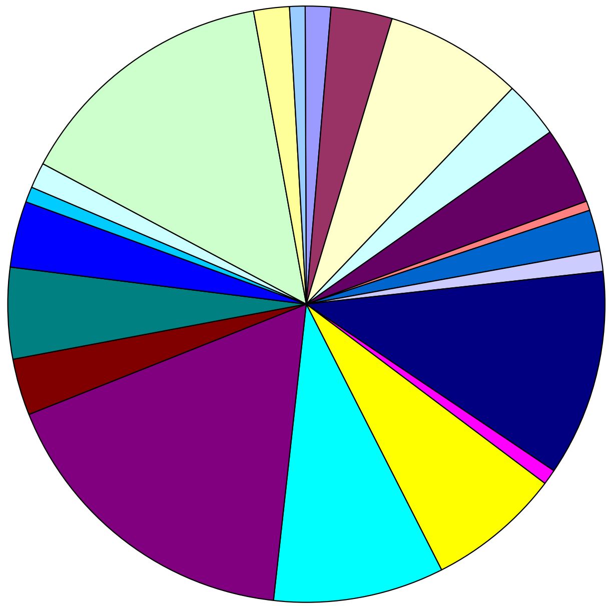
The Portfolio Rule $\overset{*}{\lambda}$ (The Kelly Rule)

$$\overset{*}{\lambda}{}^{k} = (1-\lambda^{c}) E_p d_{(\omega)}^{k}$$

Dividends DJIA 1981-2001



Expected Dividends Portfolio



Result (1)

Theorem (Evstigneev, Hens, Schenck-Hoppe, JET 2008)

λ* is **unique global survivor** with **i.i.d.** dividends and **simple** strategies!

Note: Simple strategies are constant rebalancing strategies: $\lambda_t^{i,k}(\omega^t)$

Result (2)

Theorem (Evstigneev, Hens, Schenk-Hoppé, Economic Theory, 2005)

Suppose dividends d follow an **i.i.d.** process and consider λ **stationary adapted.**

Then $$\overset{*}{\lambda}{}^{k} = (1-\lambda^{c})E_{p}d^{k}_{(\omega)}$$

is the **unique evolutionary stable strategy**.

stationary adapted $\lambda_{\cancel{t}}^{i,k}(\omega^{t})$

INCUMBENT

MUTANT: growth	$\overset{*}{\lambda}$	$\hat{\lambda}$	$\overset{\approx}{\lambda}$
$\overset{*}{\lambda}$	0	< 0	> 0
$\hat{\lambda}$	< 0	0	> 0
$\overset{\approx}{\lambda}$	< 0	> 0	0

The Markovian Case $\lambda^{*} = \lambda^{c}\sum_{n=1}^{\infty}(1-\lambda^{c})^{n}P^{n}d$

$\lambda^{*} = \lambda^{c}\sum_{n=1}^{\infty}(1-\lambda^{c})^{n}P^{n}d$ is expected discounted dividends

where

$$\lambda^{*} = \begin{bmatrix} \lambda_1^1 & & \lambda_1^K \\ & \lambda_s^k & \\ \lambda_S^1 & & \lambda_S^K \end{bmatrix}, P = \begin{bmatrix} P_1^1 & & P_1^S \\ & P_s^{s'} & \\ P_S^1 & & P_S^{S'} \end{bmatrix}, d = \begin{bmatrix} d_1^1 & & d_1^K \\ & d_s^k & \\ d_S^1 & & d_S^K \end{bmatrix}$$

in particular if p i.i.d then

$$\lambda^{*,k} = (1-\lambda^{c})E_P d^{k}$$

because $\sum_{\mathrm{n}=1}^{\infty}(1\text{-}\lambda^{c})^{\mathrm{n}} = \frac{1-\lambda^{c}}{\lambda^{c}}$

More Recent Result

Theorem (Amir, Evstigneev, Hens, Xu, 2012, MAFE))

With **general dividends** and **general strategies,** i.e. $\lambda_t^{i,k}(\omega^t, q_{t-\tau}, \lambda_{t-\tau})$

λ* is a **surviving strategy,** i.e. $\liminf_{t\to\infty} r^{\lambda^*} > 0$

Moreover, one can show that any other surviving **basic** strategy must almost surely coincide with λ* , i.e. $\sum_{t=0}^{\infty} \left\| \lambda_t - \lambda_t^* \right\|^2 < \infty$ a.s.

Note:
Every equilibrium path can also be generated with basic strategies only.

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Practical Applications

Questions

- Active or Passive?
- Fundamental or Chartist?
- Concentrate or Diversify?
- Maximize Expected Utility?

Answers

- Semi Active: Rebalance!
- Fundamental!
- Diversify!
- Follow simple rules!

The Evolutionary test of a seemingly good investment strategy
Backtesting versus reflecting who would pay my returns

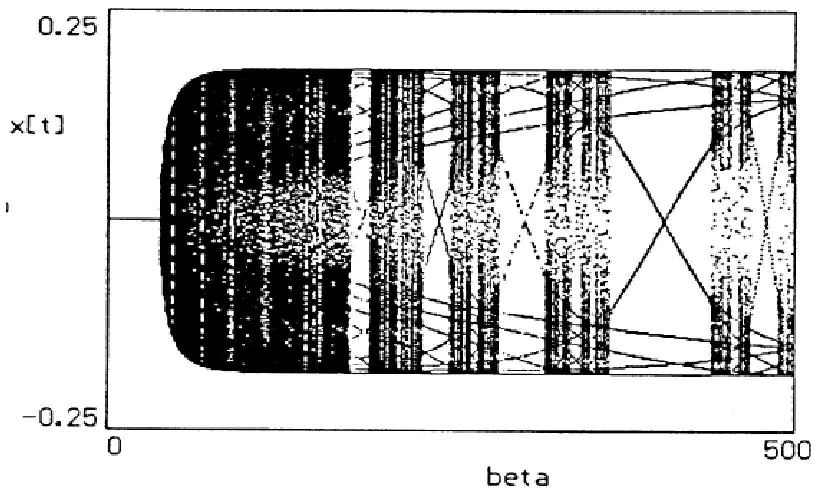
Content

Switching Model

- Brock&Hommes, Lux, etc suggest that

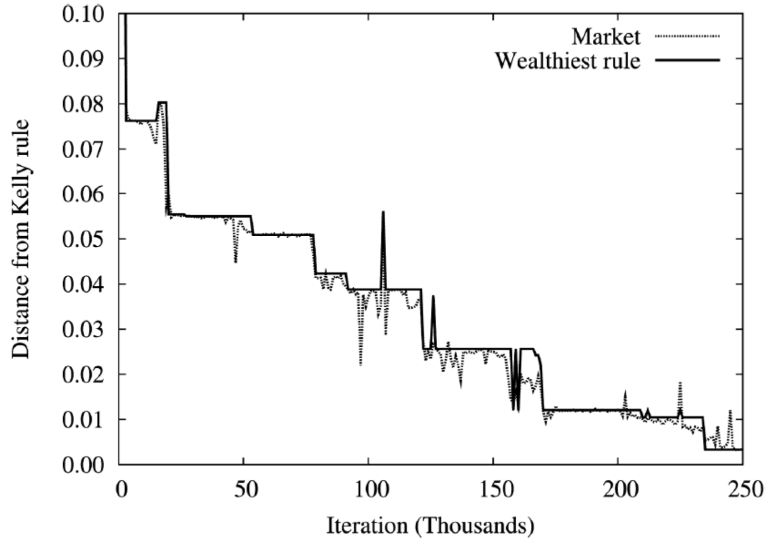
  investors may change their type (e.g. imitation,…)

- Result including switching in our model (Elmiger and Wang (2014))
  - If all investors switch then choatic behavior is possible
  - **But if one investor stolidly plays λ* then only this one survives!**



Genetic Programming

- Santa Fe Institute Model suggests to generate strategies by genetic programming.
  - Tournament
  - Reproduction
  - Mutation
  - Crossover
  - Noise



- Result

Lensberg and Schenk-Hoppe (2008)

λ^* evolves as best survivor.

Market Microstructure

- Santa Fe Institute Model and others also suggests to use a realistic market microstructure e.g. double auction with oderbook
- Result of Ladley, Lensberg and Schenk-Hoppe (2014)
- Rich ecology of investment styles:

Investment styles (% of traders,

Variable	Base case
Liquidity suppliers	18.7%
	(0.000)
Value traders	24.6%
	(0.000)
News traders / arbs.	41.9%
	(0.000)
Informed traders	83.3%
	(0.000)

Summary

- Structure of a model matters more than behavioral assumptions
- Nature will find its way!
- You might personalize this structure by interpreting the result in a way you act (maximize expected utility) – but that can only be a metaphor!

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References (our approach) (1)

- Amir, Evstigneev, Hens, Xu (2012): “Evolutionary Finance and Dynamic Games”, *Mathematics and Financial Economics*, forthcoming.
- Evstigneev, Hens, Schenk-Hoppe (2012): “Local Stability Analysis of a Stochastic Evolutionary Financial Market Model with a Risk-Free Asset”, *Mathematics and Financial Economics*, .
- Hens, Lensberg, Schenk-Hoppe, Wöhrmann (2009): “An Evolutionary Explanation of the Value Premium Puzzle”, *Journal of Evolutionary Economics*, Vol. 21(5), pp. 803-815.
- Evstigneev, Hens, Schenk-Hoppe (2008): “Globally Evolutionarily Stable Portfolio Rules”, J*ournal of Economic Theory*, May 08, Vol. 140(1), pp. 197-228.
- Evstigneev, Hens, Schenk-Hoppe (2006): “Evolutionary Stable Markets”, *Economic Theory*, February 2006, Vol. 27(2), pp. 449 – 468.

References (our approach) (2)

- Amir, Evstigneev, Hens, Schenk-Hoppe (2005): “Market selection and survival of investment strategies”, *Journal of Mathematical Economics*, Vol. 41(1-2), pp. 105-122.
- Hens and Schenk-Hoppe (2005):“Evolutionary Stability of Portfolio Rules”, *Journal of Mathematical Economics*, Vol. 41(1-2), pp. 43-66.
- Lensberg and Schenk-Hoppe (2007):»On the Evolution of Investment Strategies and the Kelly Rule – A Darwinian Approach», Review of Finance, Vol 11, pp.25-50.
- Lensberg, Schenk-Hoppe and Ladley (2014):»Costs and Benefits of Financial Regulation: Short Selling Ban and Transaction Tax» WP.

Further reading all approaches

- **Hens, Schenk-Hoppe (2009): *Handbook of Finance on Evolution and Dynamics in Financial Markets*, North-Holland.**