Network Risk and Key Players: A Structural Analysis of Interbank Liquidity*

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Abstract

This paper studies whether the structural properties of interbank networks affect individual banks' liquidity holding decisions. In a simultaneous game of liquidity provision on a network, we show that, at the Nash equilibrium, the contributions of each bank to the network liquidity level and liquidity risk are distinct functions of its indegree and outdegree Katz-Bonacich centrality measures. The equilibrium system liquidity level and volatility are determined by the network links, risk aversion of banks, and the availability of collateralized and uncollateralized borrowing. Using a sterling interbank network database from January 2006 to September 2010, we estimate the model in a spatial error framework, and find evidence for a substantial, and time varying, network risk: in the period before the Lehman crisis, the network is cohesive and liquidity holding decisions are complementary and there is a large network liquidity multiplier; during the 2007-08 crisis, the network becomes less clustered and liquidity holding less dependent on the network; after the crisis, during Quantitative Easing, the network liquidity multiplier becomes negative, implying a lower network potential for generating liquidity. The network impulse-response functions indicate that the risk key players during these periods vary, and are not necessarily the largest borrowers.

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I Introduction

The collapse of Lehman Brothers and the subsequent great recession have stressed the need of understanding endogenous systematic risk in the banking sector. Traditional regulatory tools have tended to examine banks' risk exposure in isolation, and focus on bank-specific risk variables (e.g. capital ratios). However, the recent events have shown that banks are interconnected and decisions by individual banks in the banking network could have ripple effects leading to increased risk across the financial system. Academics and policy-makers alike realise the importance of macro-prudential perspectives, that assess the systemic implications of individual bank's behaviour in interbank networks, and put more stringent requirements upon banks that are considered to pose greater systemic risks.[1]

In this paper we model the role that the network externalities of the interbank money market play in banks' liquidity holding decisions. Since banks are interlinked through the interbank lending network, their liquidity holding decisions are not only dependent on their own balance sheet characteristics, but also on their location in the interbank network. Using a linear-quadratic model, we outline an amplification mechanism for liquidity shocks which originate in individual banks, and show the implications for aggregate liquidity level and risk. Based on this amplification mechanism, we estimate each bank's multiplier effect and identify the liquidity level key players (the banks whose removal would result in the largest liquidity reduction in the overnight interbank system) and the liquidity risk key players (the banks whose idiosyncratic shocks have the largest aggregate effect) in the network. Based on the estimation of the network effect, we characterise the social optimum and contrast that with the equilibrium level of systemic liquidity level and risk. This analysis allows us to identify ways for planner's intervention to achieve social optimum.

In our model, all banks decide simultaneously how many liquid assets to hold at the beginning of the day as a buffer stock for liquidity shocks that need to be absorbed intraday. By holding liquidity reserves, banks are able to respond immediately to calls on their assets without relying on liquidizing less liquid securities. Furthermore, banks are linked in a network of borrowing and lending relations that can be used to smooth daily shocks – it is this network that gives rise to the network externality. When banks decide how large their liquidity buffers should be, they do not take into account the impact that their decisions have on the overall system through the linkages in the interbank money markets. This implies that, in equilibrium, the network itself can act as an amplification mechanism of bank specific shocks.

Banks in our model derive utility benefits from holding a liquidity buffer stock to meet their unique daily liquidity needs, but are averse to the volatility of their available liquidity (directly or via borrowing on the network). Moreover, the availability of collateralized borrowing might reduce their overall borrowing costs. The aversion to risk leads them to make liquidity holding decisions that are less correlated with their neighbours, hence resulting in a strategic substitution effect. The collateralized borrowing technology leads instead to strategic complementarity in liquidity holding decisions among banks: a higher liquidity holding by a bank allows it to signal to neighbouring banks its ability to pay back its borrowing, and this signal is more valuable when neighbouring banks provide more to borrow from. This ef-

[1] Basel III is putting in place a framework for G-SIFI (Globally Systemically Important Financial Institutions). This will increase capital requirements for those banks which are deemed to pose a systemic risk. (See `http://www.bis.org/publ/bcbs207cn.pdf`).

fect is similar to the leverage stack phenomenon of Moore (2012). This effect is stronger if the extent of collateralized borrowing is larger (for example, when the haircuts on the collaterals are smaller). The equilibrium outcome depends on the tradeoff of strategic complementarity or substitutability in the liquidity holding decisions. The lower (higher) the risk aversion, the higher (lower) the availability of collateralized borrowing, and the lower (higher) the availability of uncollateralized borrowing, the more the equilibrium will be characterised by strategy complementarity (substitutability). The existing theoretical literature has mostly modelled the liquidity holding decisions among banks as strategic substitutes. The pioneering work by Bhattacharya and Gale (1987), for example, shows that the substitution effect arises from the free-riding incentive of banks in holding liquidity. That is, an individual bank has a lower incentive to set aside liquid assets when neighbouring lending banks have high liquidity levels which it can draw upon. Our structural model is flexible enough to incorporate both strategic substitution and strategic complementtarity and, when taken to the data, is able to identify when one or the other effect dominates. The combination of these two opposing network externalities is summarised, in equilibrium, by a network decay factor ϕ.

At the Nash equilibrium, the liquidity holding of each individual bank embedded in the network is proportional to its *indegree* Katz-Bonacich centrality measure. That is, the liquidity holding decision of a bank is related to how it is affected by the choices and shocks of its neighours, of neighbours of neighbours, etc, weighted by the distance between banks in the network and the network attenuation factor, ϕ^k, where k is the length of the path.[2] When banks are less (more) risk averse, liquidity collateral/signal value is larger (smaller), the network attenuation factor ϕ is larger (smaller), and liquidity multiplies faster (slower), resulting in larger (smaller) aggregate liquidity level and systemic liquidity risk. We also find that the contribution by each bank to the network risk is related to its (analogously defined) *outdegree* Katz-Bonacich centrality measure – that is, it depends upon how the individual bank's shocks propagate to its (direct and indirect) neighbours. These two centrality measures identify the key players in the determination of both liquidity levels and liquidity risk in the network.

We show also that, due to the network externality, there is a wedge between the central planner's optimum and the market equilibrium outcome. Individual banks maximise their own liquidity benefit and risk tradeoff taking into account how this is affected by the other agents' behaviour, but disregarding how their own behaviour affects other agents' payoff function. The planner instead is also concerned about how a bank's liquidity choice affects other banks' liquidity benefits – via the bank's outdegree links – and other banks' liquidity risk exposures – via the bank's outdegree volatility contribution. The central planner might desire a lower liquidity level when she is more concerned about the level of systemic liquidity risk in the network, while individual banks are only concerned with liquidity risk specific to their respective network location.

We apply the model to study the central bank reserves holding decisions of banks who are members of the sterling large value payment system, CHAPS. We consider a network of 11 member banks. These banks play a key role in the sterling payment system since they make

[2]This centrality measure takes into account the number of both immediate and distant connections in a network. For more on the Bonacich centrality measure, see Bonacich ((1987),) and Jackson ((2003)). For other economic applications, see Ballester, Calvo-Armengol, and Zenou (2006) and Acemoglu, Carvalho, Ozdaglar, and Tahbaz-Salehi (2005)

payments both on their own behalf and on behalf of banks that are not direct members of CHAPS. We consider the banks' liquidity holding decisions in terms of the amount of central bank reserves that they hold along with assets that are used to generate intraday liquidity from the Bank of England (BoE).[3] These reserve holdings are the ultimate settlement asset for interbank payments, fund intraday liquidity needs, and can also act as a buffer to protect the bank against unexpected liquidity shocks. They are the most liquid form of assets on a bank's balance sheet. The UK monetary framework allows individual banks to choose their own level of reserve holdings. However, post Quantitative Easing (QE) the BoE has targeted the purchase of assets, and so has largely determined the aggregate supply of bank reserves.[4] The network that we consider between these banks is the sterling unsecured overnight interbank money market. This is where banks lend central bank reserves to each other, unsecured, for repayment the following day. As an unsecured market it is sensitive to changes in risk perception. The strength of the link between any two banks in our network is measured using the fraction of borrowing by one bank from the other. Hence, our network is weighted and directional. As well as relying on their own liquidity buffers, banks can also rely on their borrowing relationship within the network to meet unexpected liquidity shocks. Using daily data from January 2006 to September 2010, we cast the theoretical model in a spatial error framework and estimate the network effect. Our parametrization is flexible and allows the network to exhibit either substitutabilities or complementarities, to change its role over time. The empirical estimations decompose the risk into exogenous risk and network generated risk which allow us to assess the role and relevance of the network. We also construct network impulse-response functions to individual banks' shocks to further understand the shock amplification mechanism in the interbank network and sources of systematic risks.

The empirical estimation sheds light on network effects in the liquidity holding decision of the banks over this period. Our work shows that this effect is time varying: a multiplier effect during the credit boom, close to zero in the aftermath of the Bear Stearns collapse and during the Lehman crisis, and a dampening effect during the Quantitative Easing (QE) period. That is, liquidity holding decisions among banks are a strategic complement during a credit boom but a strategic substitute during the QE period. As the first paper that structurally estimates the network effect, our finding of this time-varying network effect is empirically significant. The long standing notion in the theoretical interbank literature has assumed that banks have incentives to free ride on other banks in holding liquidity and liquidity is a strategic substitute (Bhattacharya and Gale (1987)). Our finding that liquidity holding decisions among banks sometimes exhibit strategic complementarity indicates this notion is an incomplete understanding of the interbank market. We interpret this finding as supportive of the "leverage stack" view of the interbank network in Moore (2012). During booms, banks use liquid assets (implicitly or explicitly) as collateral to borrow more. In our case, as we are looking at the unsecured market and central bank reserve holdings, we interpret this as meaning that banks which are more liquid have greater access to borrowing from other banks, some of which is then held as reserves as the balance sheet expands. The multiplier effect amplifies shocks from each individual banks, creating excessive aggregate

[3]In addition to central bank reserves, payment system participants may also repo government bonds to the BoE to provide extra intraday liquidity.

[4]In Appendix A.1, we provide some background information on the monetary framework (i.e. reserve regimes) including QE, the payment system, and the overnight interbank money markets.

liquidity. As crises unfold, banks, as rational agents, decide to lower their exposure to network risk by reducing the correlation of their liquidity decision with their neighbouring banks. This has a dampening effect on shocks between banks but also results in lower aggregate liquidity generated through the network interaction. This unique finding enriches our understanding of the interbank market and poses new questions to the corresponding theoretical literature. Since the network effect can be considered as a liquidity multiplier in our model, our empirical finding implies that the liquidity multiplier is positive during the boom prior to 2007, close to zero during the crisis of 2007, and turns negative during the QE period. We find these results to be robust to various specifications and controls.

Moreover, using the estimated network effects, we identify risk key players, that is, the banks that contribute the most to the aggregate liquidity risk, through these three periods. We find that although the network risk is dominated by a small number of banks during the majority of the sample period, there are substantial time varying differences among their contribution to the network risk. In fact during the last period (the QE period), the largest banks are seen as absorbents rather than contributors to the network risk. We also find that the key players in the network are not necessarily the largest borrowers. In fact, during the credit boom, large lenders and borrowers are equally likely to be key players. This set of findings is of policy relevance, and give guidance on how to effectively inject liquidity, to reduce the network risk, if the government decides to intervene.

Related Literature: There is a large theoretical literature on liquidity formation in interbank markets. The seminal paper by Bhattacharya and Gale (1987) shows that banks in interbank networks face free-riding problems when choosing their liquidity holdings. Other papers also explore the externality in the interbank network. For example, Allen and Gale (2000) build a model of coinsurance against uncertain liquidity shocks through bank cross-holdings. Freixas, Parigi, and Rochet (2000) show that counterparty risk could cause a gridlock equilibrium in the interbank payment system even when all banks are solvent. Allen, Carletti and Gale (2008) model liquidity hoarding among banks being driven by an increase in aggregate uncertainty. Afonso and Lagos (2012) use a search theoretical framework to study the interbank market and banks' trading behaviour. More recently, Moore (2012) show that the externality of banks' risk-taking can act as a multiplier when banks use interbank loans as collaterals to leverage. Our paper contributes to the theoretical literature on the interbank market by modeling banks' liquidity holding decision as the outcome of a network game and estimating the impact of the externality taking into account of network topology. Our empirical finding of time-varying strategic interactions among banks' liquidity holding decisions in the interbank market potentially add the theoretical understanding of this market. [5]

Our paper is also related to the network literature that utilizes the concept of Katz-Bonacich centrality measure, see Katz (1953), Bonacich (1987), Jackson (2003), and Ballester, Calvó-Armengol, and Zenou (2006). We depart from this literature by analyzing how bank-specific shocks translate into (larger or smaller) aggregate network risks, relating not only key level players but also key risk players to the Bonacich centrality measure. In this way, we are more related to the recent works on aggregate fluctuation generated by networks (Ace-

[5] Our paper is also related to the theoretical literature on financial networks that studies contagion and systematic risks. The papers in this area include but not limited to: Allen and Gale (2000), Freixas, Parigi, and Rochet (2000), Furfine (2000), Leitner (2005) , Babus (2009), Zawadowski (2012). Babus and Allen (2009) gives a comprehensive survey of this literature.

moglu, Carvalho, Ozdaglar, and Tahbaz-Salehi (2005); Acemoglu, Ozdaglar, and Tahbaz-Salehi (2012), and Kelly, Lustig, and Nieuwerburgh (2013)).

There is a limited empirical literature that has studied the liquidity formation in interbank markets due to limitations on data availabilities. Furfine (2000) was one of the first to do an empirical analysis using Fed fund market data, finding that daily Fed fund rate variability is related to the volatility payments flows. More recently, Afonso, Kovner, and Schoar (2010) show that counterparty risk plays a role in the fed fund market condition during the financial crisis in 2008. Ashcraft, McAndrews and Skeie (2010) find theoretically and empirically that, in response to heightened payment uncertainty, banks hold excess reserves and become reluctant to lend in the Fed fund market. Afonso and Shin (2011) calibrate a payment system based on the US Fedwire system and find a multiplier effect. By examining large Sterling settlement banks during the subprime crisis of 2007-08, Acharya and Merrouche (2010) find evidence of precautionary liquidity demands among the U.K. banks.[6] Fecht, Nyborg and Rocholl (2010) study the German banks' behaviour in ECB's repo auctions during June 2000 to December 2001 and find that the rate a bank pays for liquidity depends on other banks' liquidity and not just its own. We follow this line of literature by empirically relating a bank's reserve holding decision to both its payment characteristics and the decisions of its neighbouring banks in the overnight money market. To the best of our knowledge, we are the first to estimate the spatial (network) effect of liquidity holding decisions.

The reminder of the paper is organized as follows. In Section II, we present and solve a liquidity holding decision game in a network, and define key players in terms of level and risk. Section III casts the equilibrium of the liquidity network game in the spatial econometric framework, and outlines the estimation methodology. In Section IV, we describe the data, the construction of the network, and the basic network characteristics throughout the sample period. In Section V, we present and discuss the estimation results, and Section VI concludes.

II The Network Model

In this section, we present a network model of interbank liquidity holdings, where the network reflects the collection of bilateral effects. Additionally, to study the risk in the network, we model the game with individual banks facing shocks.

The network: a finite set of n banks. The network g is endowed with a n-square adjacency matrix $\mathbf{G}$ where $g_{ii} = 0$. The term g_{ij} is the fraction of borrowing by bank i from bank j. Hence the network g is weighted and directed.[7] Banks i and j are directly connected (in other words, they have a direct lending or borrowing relationship) if g_{ij} or $g_{ji} \neq 0$.

[6] There is also extensive policy related research in the BoE on the Sterling payment systems and the money market. For example, Wetherilt, Zimmerman, and Soramaki (2010) document the network characteristics during the recent crisis. Benos, Garratt, and Zimmerman (2010) find that banks make payments at a slower pace after the Lehman failure. Ball, Denbee, Manning and Wetherilt (2011) examine the risks that intraday liquidity pose and suggest ways to ensure that regulation doesn't lead banks to a bad equilibrium of delayed payments.

[7] We also explore other definition of the adjacency matrix where g_{ij} is either the sterling amount of borrowing by bank i from bank j, or 1 (0) if there is (not) borrowing or lending between Bank i and j. Note that, in this latter case, the adjacency matrix is unweighed and undirected. In the theoretical part of the paper, we provide results and intuitions when $\mathbf{G}$ is right stochastic matrix. However, the results should be easily extended with other forms of adjacency matrices with some restrictions on parameter values which we will highlight when needed.

The matrix $\mathbf{G}$ is a (right) stochastic matrix by construction, is not symmetric, and keeps track of all direct connections – links of order one – among network players. That is, it summarizes all the paths of length one between any pair of banks in the network. Similarly the matrix $\mathbf{G}^k$, for any positive integer k, encodes all links of order k between banks, that is the paths of length k between any pair of banks in the network. For example, the coefficient in the (i, j) cell of $\mathbf{G}^k$ – i.e. $\left\{\mathbf{G}^k\right\}_{ij}$ – gives the amount of exposure of bank i to bank j in k steps. Since, in our baseline construction, $\mathbf{G}$ is a right stochastic matrix, $\mathbf{G}$ can also be interpreted as a Markov chain transition Kernel, implying that $\mathbf{G}^k$ can be thought of as the k step transition probability matrix, i.e. the matrix with elements given by the probabilities of reaching bank j from bank i in k steps.

As in Katz (1953), direct and indirect links can be aggregated using an attenuation factor, ϕ, that penalizes the contribution of links between distant nodes at the rate ϕ^k, where k is the length of the path between nodes. That is, we can summarize all direct and indirect links among banks using the aggregator

$$\mathbf{M}(\phi, \mathbf{G}) := \mathbf{I} + \phi\mathbf{G} + \phi^2\mathbf{G}^2 + \phi^3\mathbf{G}^3 + ... = \sum_{k=0}^{\infty} \phi^k \mathbf{G}^k \tag{1}$$

where $\mathbf{I}$ is the $n \times n$ identity matrix and captures the (implicit) link of each bank with itself, the second term in the sum captures all the direct links between banks, the third term in the sum captures all the indirect links corresponding to paths of length two, and so on. Since $\mathbf{G}$ is a stochastic matrix, its largest eigenvalue is equal to 1, hence a sufficient condition for the above sum to converge[8] is that $|\phi| < 1$, and in this case we have $\mathbf{M}(\phi, \mathbf{G}) \equiv (\mathbf{I} - \phi\mathbf{G})^{-1}$ (Debreu and Herstein (1953)). The elements of the matrix $\mathbf{M}(\phi, \mathbf{G})$, given by $m_{ij}(\phi, \mathbf{G}) := \sum_{k=0}^{+\infty} \phi^k \left\{\mathbf{G}^k\right\}_{ij}$, count the exposure in the network from i to j, when the contribution of the kth step is weighted by ϕ^k.

As we will show below, in equilibrium, the matrix $\mathbf{M}(\phi, \mathbf{G})$, contains the relevant information needed to characterize the centrality of the players in the network. That is, it provides a metric from which the relevant centrality of the network players can be recovered. In particular, multiplying the rows (columns) of $\mathbf{M}(\phi, \mathbf{G})$ by a vector of appropriate dimensions, we recover the indegree (outdegree) Katz-Bonacich centrality measure.[9] The indegree centrality measure provides the weighted count of the number of ties directed to the nodes, while the outdegree centrality measure provides the weighted count of ties that each node directs to the other nodes. That is, the i-th column of $\mathbf{M}(\phi, \mathbf{G})$ captures how the network as a whole loads on bank i, while the i-th row of $\mathbf{M}(\phi, \mathbf{G}))$ captures how bank i loads on the network as whole.

Banks and their liquidity preference in a network: We define the total liquidity holding by bank i, denoted by l_i, as the sum of two components: bank i's liquidity holding absent of any bilateral effects (i.e., the level of liquidity that a bank would be holding if it were not part of a network), and bank i's liquidity holding level made available to the network, and

[8]When $\mathbf{G}$ is not a stochastic matrix, its Jordan canonical form implies that a sufficient condition for the infinite sum to converge is that $|\phi\lambda^{max}(\mathbf{G})| < 1$, where the function $\lambda^{max}(\cdot)$ returns the largest (in modulus) eigenvalue of $\mathbf{G}$.

[9]Newman (2004) shows that weighted networks can in many cases be analyzed using a simple mapping from a weighted network to an unweighted multigraph. Therefore, the centrality measures developed for unweighted networks apply also to the weighted cases.

that depends on its neighbouring banks' liquidity contribution to the network. We use q_i and z_i to denote these two components respectively, and $l_i = q_i + z_i$.

A bank's liquidity holding in absence of any bilateral effects is related to its bank-specific as well as macro variables, which we specify as:

$$q_i = \alpha_i + \sum_{m=1}^{M} \beta_m x_i^m + \sum_{p=1}^{P} \beta_p x^p \qquad (2)$$

where α_i is bank fixed effect, x_i^m is a set of M variables accounting for observable differences in individual bank i, x_p is a set of P variables controlling for time-series variation in systematic risks. That is, q_i captures the liquidity need specific to each individual bank due to its balance sheet and fundamental characteristics (e.g. leverage ratio, lending and borrowing rate), and its exposure to macroeconomic shocks (e.g. aggregate economic activity, monetary policy etc.).

To study a bank's endogenous choice of z_i, that is, its liquidity holding in a banking network, however, we need to model the various sources of bilateral effects. To do so, we assume that banks are situated in different locations in a borrowing-lending network g, where g_{ij} is the fraction of borrowing by bank i from bank j. Each bank decides how much liquid capital z to set aside simultaneously on g.

We assume that banks derive utility benefits from having an accessible buffer stock of liquidity, but at the same time they dislike the variability of this quantity. The accessible network liquidity for bank i has two components: direct holdings, z_i, and what can be borrowed from other banks connected through the network. This second component is proportional to the neighbouring banks direct holdings, z_j, weighted by the borrowing intensities, g_{ij}, and a technological parameter ψ, that is, $\psi \sum_j g_{ij} z_j$, which can be thought as unsecured borrowing. The direct utility benefit of this buffer stock of accessible liquidity for bank i is $\hat{\mu}_i$ per unit.

We also assume that bank i derives an indirect benefit from its own holding of liquidity, arising from a reduction of opportunity costs. Typically, setting liquidity aside implies that banks have to forgo more high-interest-yielding long-term investments. However, for each unit of liquidity it sets aside, bank i can also use it as a signal of its trustworthiness to borrow from its neighbouring banks' liquidity holdings z_j for long-term investments. This collateral/signal benefit is proportional to the borrowing capacity of the bank from its neighbours and a technological parameter δ. The parameter δ reflects the reduction in the collateral value as travelling in the network. This reduction could be due to transaction costs such as haircut treatments etc. We hence parametrize the additional benefit of liquidity holding as: $\delta z_i \sum_j g_{ij} z_j$, which can be thought as collaterallized borrowing.[10] We treat this collateralized liquidity differently as it can potentially be used for long-term investments and hence lowers the opportunity cost of liquidity holding. In summary, the benefit of network liquidity for

[10]This effect is reminiscent of the leverage stack phenomenon in Moore (2012), where the interbank lending market is used by individual banks to generate collateral that can then be used to raise more funds from households. By comparison, banks in our paper are engaged in unsecured borrowing and lending. Banks, which are more liquid, have greater access to borrowing from other banks, some of which is then held as reserves as the balance sheet expands.

bank i is modelled as:

$$\underbrace{\hat{\mu}_i \left(z_i + \psi \sum_j g_{ij} z_j \right)}_{\text{Accesible Liquidity}} + \underbrace{z_i \delta \sum_j g_{ij} z_j}_{\text{Collateralized Liquidity}}$$

The term $\hat{\mu}_i$ captures the benefit (not necessarily positive) of a unit of bank i's accessible buffer stock of liquidity. It is specified as a random variable with bank-specific mean and shock. The shock is independent across banks, but it is assumed to be common knowledge. The formulation of $\hat{\mu}_i$ is intuitive. The bank-specific mean represents the average bank i's valuation of a unit of liquidity, while the bank-specific shock captures how the bank's valuation changes as a consequence of unexpected changes in the market condition.

However, by establishing bilateral relationships in the banking network g, a bank is also exposed itself to the shocks from its neighbouring banks. We assume that the bank dislikes the volatility of it's own liquidity and of the liquidity it can access given it's links. Hence the network liquidity risk faced by bank i is:

$$\left(z_i + \psi z_i \sum_{j \neq i} g_{ij} z_j \right)^2 .$$

Denoting the risk aversion parameter as γ, we now can fully characterise bank i's utility from holding liquidity as:

$$u_i(z_i|g) = \hat{\mu}_i \left(z_i + \psi \sum_j g_{ij} z_j \right) - \frac{1}{2}\gamma \left(z_i + \psi \sum_{j \neq i} g_{ij} z_j \right)^2 + z_i \delta \sum_j g_{ij} z_j \tag{3}$$

The above has the same spirit as a mean-variance utility representation (with the addition of a first order externality term). The bilateral network influences are captured by the following cross derivatives for $i \neq j$:

$$\frac{\partial^2 u_i (z|g)}{\partial z_i \partial z_j} = (\delta - \gamma\psi)\, g_{ij}$$

If $\delta > \gamma\psi$, the liquidity holding decision among neighbouring banks is a strategic complement. That is, an individual bank sets aside a large amount of liquid assets when neighbouring lending banks have high liquidity levels. If $\delta < \gamma\psi$, the liquidity holding decision among neighbouring banks is a strategic substitute. That is, an individual bank sets aside a smaller amount of liquid assets when its neighbouring lending banks have high liquidity level which it can draw upon.

The bilateral network effect nests these two strategic effects and depends on the difference of the parameter δ and the (network liquidity) risk aversion parameter γ. When γ is relatively large, that is, when banks are averse to liquidity risks in the network, it is likely that $\delta < \gamma\psi$ and the network exhibits strategic substitute behavior. When δ is relatively large, the haircut is small and money velocity is large and collateral chain is long, it is likely that $\delta > \gamma\psi$ and the network exhibits strategic complementary behavior. In our paper, we are agnostic about the the sign of $\delta - \gamma\psi$ and estimate it empirically.[11]

[11] Bhattacharya and Gale (1987) show that banks' liquidity holdings are strategic substitutes for a different reason. In their paper, setting liquidity aside comes at a cost of forgoing higher interest revenue from long-term investments. Banks would like to free-ride their neighbouring banks for liquidity rather than conducting precautionary liquidity saving themselves.

Equilibrium behaviour: We now characterize the Nash equilibrium of the game where banks choose their liquidity level z simultaneously. Each bank i maximizes (3) and we obtain the following best response function for each bank:

$$z_i^* = \frac{\hat{\mu}_i}{\gamma} + \left(\frac{\delta}{\gamma} - \psi\right) \sum_j g_{ij} z_j = \mu_i + \phi \sum_j g_{ij} z_j \tag{4}$$

where $\phi := \delta/\gamma - \psi$ and $\mu_i := \hat{\mu}_i/\gamma =: \bar{\mu}_i + \nu_i$. The parameter $\bar{\mu}_i$ denotes the average valuation of liquidity by bank i scaled by γ, and ν_i denotes the i.i.d. shock of this normalized valuation, and its variance is denoted by σ_i^2.

Proposition 1 *The matrix $(\mathbf{I} - \phi \mathbf{G})^{-1}$ is well defined and nonnegative if and only if $|\phi \lambda^{max}(\mathbf{G})| < 1$ where the function $\lambda^{max}(\cdot)$ returns the largest eigenvalue. Then, the individual equilibrium outcome is uniquely defined and given by*

$$z_i^*(\phi, g) = \{\mathbf{M}(\phi, \mathbf{G})\}_{i.} \, \mu, \tag{5}$$

where $\{\}_{i.}$ is the operator that returns the i-th row of its argument, $\mu := [\mu_1, ..., \mu_n]'$, and z_i denotes the bilateral liquidity holding by bank i.

Proof. Apply Theorem 1, part b, in Calvo-Armengol, Patacchini, and Zenou (2009) to our problem. Since $\mathbf{G}$ is a stochastic matrix, the largest eigenvalue of $\mathbf{G}$ is 1. The uniqueness condition is hence $|\phi| < 1$. ■

To roughly reproduce the proof, note that a Nash equilibrium in pure strategies $\mathbf{z}^* \in \mathbb{R}^n$, where $\mathbf{z} := [z_1, ..., z_n]'$, is such that equation (4) holds for for all $i = 1, 2, ..., n$. Hence, if such an equilibrium exists, it solves

$$(\mathbf{I} - \phi \mathbf{G}) \, \mathbf{z} = \mu.$$

Inverting the matrix, we obtain $\mathbf{z}^* = (\mathbf{I} - \phi \mathbf{G})^{-1} \mu \equiv \mathbf{M}(\phi, \mathbf{G}) \, \mu$. The rest follows by simple algebra. The condition $|\phi| < 1$ in the above proposition states that network complementarities must be small enough in order to prevent the positive feedback triggered by such complementarities to escalate without bounds.

Equilibrium properties: We now examine the properties of this equilibrium. In our setup, since μ is subject to random shocks, defining $\bar{\mu} \equiv [\bar{\mu}_1, ..., \bar{\mu}_n]'$, we have that

$$b_i^{in}(\phi, \mathbf{G}) := \mathbb{E}\left[\sum_{j=1}^n m_{ij} \mu_j\right] = \mathbb{E}\left[\{\mathbf{M}(\phi, \mathbf{G})\}_{i.} \, \mu\right] = \{\mathbf{M}(\phi, \mathbf{G})\}_{i.} \, \bar{\mu},$$

which is analogous to the traditional *indegree* Katz-Bonacich centrality measure for player i, except that the adjacency matrix used in the Katz-Bonacich measure is typically unweighted and undirected. Here $b_i^{in}(\phi, \mathbf{G})$ measures the expected contribution of bank i to the total liquidity in the network.

Furthermore, in our setup an individual bank can affect the volatility of aggregate level of liquidity in the economy through its own shock. More precisely, bank i's shock on the volatility of the aggregate level of network liquidity, $Z^* \equiv \mathbf{1}' [z_1^*, ..., z_n^*]'$, can be summarized as:

$$\frac{\partial Z^*}{\partial \nu_i} \sigma_i = \mathbf{1}' \{\mathbf{M}(\phi, \mathbf{G})\}_{.i} \, \sigma_i =: b_i^{out}(\phi, \mathbf{G}) \tag{6}$$

The above expression is the *outdegree* centrality for bank i. Moreover, the volatility of the aggregate liquidity level is:

$$Var(Z^* (\phi, \mathbf{G})) = vec\left(\left\{b_i^{out} (\phi, \mathbf{G})\right\}_{i=1}^n\right) vec\left(\left\{b_i^{out} (\phi, \mathbf{G})\right\}_{i=1}^n\right)' \tag{7}$$

$$= \mathbf{1}'\mathbf{M} (\phi, \mathbf{G})\, diag(\left\{\sigma_i^2\right\}_{i=1}^n)\mathbf{M} (\phi, \mathbf{G})' \,\mathbf{1}. \tag{8}$$

Therefore, we can define a risk key player as the bank that contributes most to the volatility of aggregated liquidity in the economy.

Definition 1 *[**Risk key player**] The risk key player is the player that contributes most to the volatility in the overall level of bilateral liquidity.*

$$\max_i \quad \left(b_i^{out} (\phi, \mathbf{G}) \right) \mid i = 1, ..., n\right)$$

Furthermore, we can decompose the network contribution to the total bilateral liquidity into level and risk effect. To see this note that the total bilateral liquidity can be written as:

$$Z^* = \underbrace{\mathbf{1}'\mathbf{M} (\phi, \mathbf{G})\, \bar{\mu}}_{\text{level effect}} + \underbrace{\mathbf{1}'\mathbf{M} (\phi, \mathbf{G})\, \nu}_{\text{risk effect}}$$

where $\nu \equiv [\nu_1, ..., \nu_n]'$ and the first components captures the network level effect, while the second captures the network risk effect. It is clear that both the network liquidity level and liquidity risk are increasing in ϕ. That is, a higher network multiplier leads the interbank network to produce more liquidity but also generate more risk.

Given the properties of this network equilibrium, we can identify a bank that may cause the expected maximum level of reduction in the network liquidity when removed from the banking network.[12]

Definition 2 *[**Level key player**] The level key player is the player that, when removed, causes the maximum expected reduction in the overall level of bilateral liquidity. We use $\mathbf{G}_{\setminus\tau}$ to denote the new adjacency matrix by setting to zero all of $\mathbf{G}$'s τ-th row and column coefficients. The resulting network is $g_{\setminus\tau}$.*

In this definition, we assume that when the level key player i is removed, the remaining other banks do not form new links. In this definition, the level key player is the one with the largest impact on the total expected bilateral liquidity. Therefore, the level key player τ is found by solving

$$\max_\tau \quad E\left[\sum_i z_i^*(\phi, g) - \sum_{i\neq\tau} z^*(\phi, g_{\setminus\tau}) \mid i = 1, ..., n\right] \tag{9}$$

An immediate application of Proposition 1 yields the following approach to identify the level key player.

[12]This definition is in the same spirit as the concept of the key player in the crime network literature as defined in Ballester, Calvo-Armengol, and Zenou (2006). There, it is important to target the key player for maximum crime reduction. Here, it is useful to consider the ripple effect on the network liquidity when a bank fails. Bailouts for key level players might be necessary to avoid major disruptions to the banking network.

Corollary 1 *A player τ is the level key player that solves (9) if and only if removing τ results in the largest of the following:*

$$\max_{\tau} \quad \mathbb{E}\left[\mathbf{M}(\phi,\mathbf{G})\}_{\tau.}\mu + \mathbf{1}'\{\mathbf{M}\left(\phi,\mathbf{G}\right)\}_{.\tau}\mu_\tau - m_{\tau\tau}(\phi,\mathbf{G})\mu_\tau \mid \tau = 1,...,n\right].$$

To see this, note that if bank τ is removed, the expected reduction in the total bilateral liquidity can be written as:

$$\mathbb{E}\left[\sum_i z_i^*(\phi,g) - \sum_{i\neq\tau} z^*(\phi,g_{\setminus\tau})\right] = \{\mathbf{M}(\phi,G)\}_{\tau.}\bar{\mu} + \sum_{i\neq\tau} m_{i\tau}(\phi,\mathbf{G})\bar{\mu}_\tau$$
$$= \underbrace{\{\mathbf{M}(\phi,\mathbf{G})\}_{\tau.}\bar{\mu}}_{Indegree} + \underbrace{\mathbf{1}'\{\mathbf{M}\left(\phi,\mathbf{G}\right)\}_{.\tau}\bar{\mu}_\tau}_{Outdegree} - \underbrace{m_{\tau\tau}(\phi,\mathbf{G})\bar{\mu}_\tau}_{double\ count\ correction} \tag{10}$$

Basically, a removal of the level key player results in a direct effect on its own (the first term in (10)) and an indirect bilateral effect on other banks' liquidity reduction (the second term in (10)).

In Proposition 1, we have shown that the bank with the largest indegree centrality has the largest network liquidity holding (z_i^*) in the decentralized equilibrium. The level key bank, however, according to Corollary 1 is in fact the bank with the largest indegree and outdegree combined contribution to the bilateral liquidity (correcting for the double counting of own liquidity contribution). This discrepancy exists because, in the decentralized equilibrium, each bank does not internalize the effect of its own liquidity holding level on the utilities of other banks in the network, that is, does not internalize its impact on other bank's indegree centrality. A relevant metric for a planner to use when deciding whether to bail out a failing bank, therefore, should not be based on the size of the bank's own liquidity solely, but also include its indirect network impact on other banks' liquidity.

This discussion leads us to analyze formally a planner's problem in this networked economy. A planner chooses $z_i, i = 1,,...n$ to maximize the total payoff of all n banks:

$$\max_{z_1,...,z_i,...z_n} \sum_i \left[\hat{\mu}_i\left(z_i + \psi\sum_j g_{ij}z_j\right) + z_i\delta\sum_j g_{ij}z_j - \frac{1}{2}\gamma\left(z_i + \psi\sum_{j\neq i} g_{ij}z_j\right)^2\right]. \tag{11}$$

The first order condition with respect to z_i yields:

$$z_i = \left(1 + \psi\sum_j g_{ji}\right)\mu_i + \underbrace{\phi\sum_j g_{ij}z_j}_{\text{own indegree}} + \underbrace{\phi\sum_j g_{ji}z_j}_{\substack{\text{neighbors' indegree} \\ \text{i.e. own outdegree}}} - \underbrace{\psi^2\sum_{j\neq i}\sum_{m\neq j} g_{ji}g_{jm}z_m}_{\substack{\text{volatility of neighbors'} \\ \text{accessible network liquidity}}} \tag{12}$$

where the third and the fourth terms reflect that the fact that planner internalizes a bank's contribution to its neighbouring banks' utilities. The third term reflects bank i's contribution to its neighbouring banks' indegree centrality. The fourth term reflects bank i' contribution to its neighbouring banks' volatility of their accessible network liquidity.

Rewriting equation (12) in matrix form, we obtain

$$\mathbf{z} = \left(\mathbf{I} + \psi\mathbf{G}'\right)\mu + \mathbf{P}\left(\phi,\mathbf{G}\right)\mathbf{z}$$

where $\mathbf{P}\left(\phi,\mathbf{G}\right) := \phi\left(\mathbf{G} + \mathbf{G}'\right) - \psi^2\mathbf{G}'\mathbf{G}$. This allows us to state the following result.

Proposition 2 *The matrix* $(\mathbf{I}-\mathbf{P}(\phi,\mathbf{G}))^{-1}$ *is well defined and nonnegative if and only if* $|\lambda^{max}(\mathbf{P}(\phi,\mathbf{G}))|<1$. *Then, the planner's optimal solution is uniquely defined and given by*

$$z_i^p(\phi,g)=\left\{[\mathbf{I}-\mathbf{P}(\phi,\mathbf{G})]^{-1}(\mathbf{I}+\psi\mathbf{G}')\right\}_{i.}\mu, \tag{13}$$

where $\{\}_{i.}$ *is the operator that returns the i-th row of its argument.*

Let $\mathbf{M}^P(\phi,\mathbf{G}):=[\mathbf{I}-\mathbf{P}(\phi,\mathbf{G})]^{-1}(\mathbf{I}+\psi\mathbf{G}')$. The planner's optimal solution for the total network liquidity and liquidity risk follows from the decentralized solution:

$$Z^P=\mathbf{1}'\mathbf{M}^P(\phi,\mathbf{G})\bar{\mu}+\mathbf{1}'\mathbf{M}^P(\phi,\mathbf{G})\nu \tag{14}$$

$$Var\left(Z^P(\phi,\mathbf{G})\right)=\mathbf{1}'\mathbf{M}^P(\phi,\mathbf{G})\,diag(\{\sigma_i^2\}_{i=1}^n)\mathbf{M}^P(\phi,\mathbf{G})'\,\mathbf{1}. \tag{15}$$

It is immediate that the planner's solution does not coincide with that of the decentralized equilibrium outcome. To see this clearly, we rewrite (13) as

$$\mathbf{z}^P=\mathbf{z}^*+\mathbf{M}(\phi,\mathbf{G})\left[\psi\mathbf{G}'\mu+\left(\underbrace{\phi\mathbf{G}'}_{\text{outdegree}}-\underbrace{\psi^2\mathbf{G}'\mathbf{G}}_{\text{second-order}}\right)\mathbf{z}^P\right] \tag{16}$$

where the second to the last term reflects network nodes' outdegree centrality, that is, contribution to neighbouring nodes' indegree centrality and hence their liquidity production level; and the last term reflects network nodes' second-order degree centrality, that is, contribution to their neighbouring nodes' indirect volatility. Therefore, the discrepancy between the planner's optimum and the equilibrium outcome rests upon the planner's tradeoff of the liquidity level and liquidity risk in the network. When planner cares more about the level of liquidity production than the liquidity risk in the network, the second term is more pronounced relative to the last term. In this case, banks that have higher outdegree centralities tend to hold less than the socially optimal amount of liquidity. Planner might subsidise or inject liquidity to these banks to increase the liquidity generated by the network. Conversely, when planner cares more about the liquidity risk in the network, banks that have higher second-degree centralities tend to hold more than the socially optimal amount of liquidity. Planner might impose tax on these banks to reduce the risk in the banking network.

III Empirical Methodology

In order to estimate the network model presented in Section II, we need to map the observed total liquidity holding of a bank at time t, $l_{i,t}$, into its two components – the idiosyncratic and the network ones – identified in equations (2) and (5) respectively. This can be done by reformulating the theoretical model as a spatial error model (SEM). That is, we decompose the total bank liquidity holding into a function of observables and an error term that captures the spatial dependence generated by the network:

$$l_{i,t}=\mu+\alpha_t^{time}+\alpha_i^{bank}+\sum_{m=1}^{M}\beta_m^{bank}x_{i,t}^m+\sum_{p=1}^{P}\beta_p^{time}x_t^p+\epsilon_{i,t} \tag{17}$$

$$\epsilon_{i,t}=\phi\sum_{j=1}^{n}g_{i,j,t}\epsilon_{j,t}+\nu_{i,t}\sim iid\left(0,\sigma_i^2\right),\ i=1,...,n,\quad t=1,...,T. \tag{18}$$

The only differences between the theoretical model and the econometric reformulation above are that: *i*) we have made explicit that one of the aggregate factors is a set of common time dummies, α_t^{time}, meant to capture potential trends in the overall interbank market size; *ii*) we allow the network links, $g_{i,j}$, to potentially vary over time (but we construct them, as explained in the data description section below, in a fashion that makes them pre-determined with respect to the time t information set);[13] *iii*) we have replaced z_i with its demeaned counterpart, ϵ_i, since the bank fixed effect cannot be generally decomposed into its network and non-network components. The β_m^{bank} coefficients capture the effect of observable bank characteristics while the β_p^{time} coefficients capture the effect of systematic risk factors on the liquidity choice. Equation (18) describes the process of ϵ_i, which is the residual of individual bank i's level of liquidity in the network that is not due to bank specific characteristics or systematic factors. Here, $\sum_{j=1}^{n} g_{i,j,t}\epsilon_{j,t}$ is the spatial lag term and ϕ is the spatial autoregressive parameter. Consistent with the theoretical model, the spatial dependence is incorporated in the regression disturbance term only. This model is a variation of the Anselin (1988) spatial error model (see also Elhorst (2010a, 2010b)).

The above specification makes clear the nature of the network as a shock propagation mechanism: the shock to any bank liquidity, $\epsilon_{i,t}$, is a function of all the shocks to other banks liquidity; the intensity of the shock spillover is a function of the intensity of the network links across banks captured by the network weights $g_{i,j}$; and wether the network amplifies or dampens the effect of the individual liquidity shocks on aggregate liquidity depends, respectively, on whether the banks in the network act as strategic complements ($\phi > 0$) or strategic substitutes ($\phi < 0$).

To illustrate this last point note that the vector of shocks to all banks at time t can be rewritten as

$$\epsilon_t = (\mathbf{I} - \phi \mathbf{G}_t)^{-1} \nu_t \equiv \mathbf{M}(\phi, \mathbf{G}_t)\, \nu_t \tag{19}$$

where $\epsilon_t = [\epsilon_{1,t}, ..., \epsilon_{n,t}]'$ and $\nu_t = [\nu_{1,t}, ..., \nu_{n,t}]$. This implies that if $\mathbf{G}_t$ is a right stochastic matrix[14] (and this is the case when we model the network weights $g_{i,j}$ as the fraction of borrowing by bank i from bank j) we have that a unit shock to the system equally spread across banks (i.e. $\nu_t = (1/n)\,\mathbf{1}$) would imply a total change in aggregate liquidity equal to $(1-\phi)^{-1}$ – that is, ϕ captures the network multiplier effect of liquidity shocks.

Moreover, Equation (19) implies that any time variation in the network structure, $\mathbf{G}$, or in the network multiplier, $1/(1-\phi)$, would results in time variation in the volatility of total liquidity since the variance of the shocks to total network liquidity ($\mathbf{1}'\epsilon_t$) is

$$Var_t(\mathbf{1}'\epsilon_t) = \mathbf{1}'\mathbf{M}(\phi, \mathbf{G}_t)\, \Sigma_\nu \mathbf{M}(\phi, \mathbf{G}_t)'\, \mathbf{1}.$$

where we used the fact that $\mathbf{G}_t$ is pre-determined with respect to the time t information and $\Sigma_v = E[\nu_t \nu_t']$ is the a diagonal matrix with the variances of the idiosyncratic shocks, $\{\sigma_i^2\}_{i=1}^n$, on the main diagonal.

As outlined in Section A.2 of the Appendix, we can estimate the parameters of the Spatial Error Model jointly using a quasi-maximum likelihood approach. In order to elicit time

[13] To allow for potential time variation in ϕ instead we also perform estimations in subsamples and over a rolling window.

[14] If $\mathbf{G}_t$ is a right stochastic matrix, we have that $\mathbf{G}_t\mathbf{1} = \mathbf{1}$, therefore

$$\mathbf{1} = (\mathbf{I} - \phi\mathbf{G}_t)^{-1}(\mathbf{I} - \phi\mathbf{G}_t)\,\mathbf{1} = (\mathbf{I} - \phi\mathbf{G}_t)^{-1}\,\mathbf{1}\,(1-\phi) \Longrightarrow \mathbf{M}(\phi, \mathbf{G}_t)\,\mathbf{1} = (1-\phi)^{-1}\,\mathbf{1}.$$

variation in the network coefficient ϕ, we perform subsample and rolling window estimates. The estimation frequency is daily, with lagged monthly update of the network matrix $\mathbf{G}_t$.

An estimation issue for network models, is the well-known reflection problem (Manski (1993)): the neighbouring banks' liquidity holding decision affects each others so that we cannot distinguish if a given bank's action is the cause or the effect of neighbouring banks' actions. To address this problem, Bramoullé, Djebbari and Fortin (2009) have shown that the network effect can be identified if there are two nodes in the network with different average connectivities of their direct connected nodes, and this is a condition satisfied in our data.

As a model specification test of our theory driven SEM formulation, we also consider a more general specification that allows for a richer set of network interactions. That is, we model liquidity holding as a Spatial Durbin Model (SDM – see e.g. LeSage and Pace (2009)) where bank specific liquidity is allowed to depend directly on other banks liquidity and characteristics, and pairwise control variables:

$$
\begin{aligned}
l_{i,t} = \mu + \alpha_t^{time} + \alpha_i^{bank} + \sum_{m=1}^{M} \beta_m^{bank} x_{i,t}^m + \sum_{p=1}^{P} \gamma_p^{time} x_t^p + \qquad (20) \\
+ \psi \sum_{j=1}^{n} g_{i,j,t} l_{j,t} + \sum_{j=1}^{n} g_{i,j,t} x_{i,j,t} \theta + \nu_{i,t} \sim iid\left(0, \sigma_i^2\right).
\end{aligned}
$$

where $x_{i,j,t}$ denotes match specific control variables and the characteristics of other banks. The above formulation allows for a specification these of our structural model since restricting $\theta = \mathbf{0}$, and setting $x_{i,j,t} := vec(x_{j \neq i,t}^m)'$, $\theta = -\phi vec(\beta_m^{bank})$, $\gamma_p^{time} = (1-\phi)\beta_p^{time} \, \forall p$, and most importantly $\psi = \phi$, we are back to the SEM specification. These restrictions will be tested formally in Section V.

With the SEM estimated parameter at hand, we can also identify the risk key players of the interbank liquidity market. To do so we define the network impulse-response function as follows.

Definition 3 *Network Impulse-Response Function. Let* $L_t \equiv \mathbf{1}' l_t = [l_{1t}, ..., l_{Nt}]$ *denote total liquidity in the interbank network. The network impulse-response function of total liquidity,* L_t*, to a one standard deviation shock to a given bank* i *is given by*

$$
NIRF_i\left(\phi, \sigma_i\right) \equiv \frac{\partial L_t}{\partial \nu_{i,t}} \sigma_i = \mathbf{1}' \left\{\mathbf{M}\left(\phi, \mathbf{G}_t\right)\right\}_{.i} \sigma_i \qquad (21)
$$

where the operator $\{\}_{.i}$ *returns the* i*-th column of its argument.*

The network impulse-response is identical to the *outdegree* centrality of bank i defined in Equation (6). Note that $NIRF_i\left(\phi, 1\right)$ is smaller or bigger than 1 depending on whether ϕ is positive or negative – that is, if $\phi > 1$ (< 1) individual bank shocks are amplified (reduced) through the system.

The network impulse-response provides a metric to identify which bank's shocks have the largest impact on the overall liquidity. Moreover, it does so accounting for both the size of the bank (via σ_i), the network multiplier, ϕ, and all the direct and indirect links among banks since $\mathbf{1}'\left\{\mathbf{M}\left(\phi,\mathbf{G}_t\right)\right\}_{.i}$ is the solution, for $|\phi|<1$, of

$$\mathbf{1}'\left\{\mathbf{M}\left(\phi,\mathbf{G}_t\right)\right\}_{.i}=\mathbf{1}'\left\{\mathbf{I}+\phi\mathbf{G}_t+\phi^2\mathbf{G}_t^2+...\right\}_{.i}=\mathbf{1}'\left\{\sum_{k=0}^{\infty}\phi^k\mathbf{G}_t^k\right\}_{.i}$$

where the first element in the series captures the direct effect of a unit idiosyncratic shock to bank i, the second element captures the effects through the first order network links, the third elements captures the effect through the second order links and so on. This also implies that $\mathbf{M}\left(\phi,\mathbf{G}_t\right)_{ji}$ measures the total (direct and indirect) effect of a shock to bank i on the liquidity of bank j.

The network impulse-response functions also provide a natural decomposition of the variance of total liquidity in the network system since

$$Var_t\left(\mathbf{1}'\epsilon_t\right)\equiv vec\left(\left\{NIRF_i\left(\phi,\sigma_i\right)\right\}_{i=1}^n\right)'vec\left(\left\{NIRF_i\left(\phi,\sigma_i\right)\right\}_{i=1}^n\right).$$

We can also isolate the purely network part of the impulse-response function, that is, the liquidity effect in excess of the direct effect of a shock to a bank as

$$NIRF_i^e\left(\phi,\sigma_i\right)\equiv NIRF_i\left(\phi,\sigma_i\right)-\sigma_i=\mathbf{1}'\left\{\left(\mathbf{I}-\phi\mathbf{G}_t\right)^{-1}\phi\mathbf{G}_t\right\}_{.i}\sigma_i, \tag{22}$$

and the above, setting $\sigma_i=1$, i.e. considering a unit shock, is exactly the Katz (1953) centrality measure. Note that $NIRF_i^e\left(\phi,\sigma_i\right)$ has by construction the same sign as ϕ.

Moreover, it is straightforward to compute confidence bands for the estimated network impulse-response functions, since they are simply a function of the distribution of $\hat{\phi}$, and $\hat{\phi}-\phi_0$ has a known asymptotic Gaussian distribution (see Section A.3 in the Appendix).

IV Network and Other Data Description

We study the sterling interbank network over the sample period January 2006 to September 2010. The estimation frequency is daily, but we also use higher frequency data to construct several of the control variables defined below. The network of banks we consider is constituted by the banks in the CHAPS system during the sample – a set of 11 banks. These banks play a key role in the Sterling large value payment system since they make payments both on their own behalf and on behalf of banks that are not direct members of CHAPS. The banks in the network are: Halifax Bank of Scotland (owned by Lloyds Banking Group); Barclays; Citibank (the consumer banking arm of Citigroup); Clydesdale (owned by National Australia Bank); Co-operative Bank (owned by The Co-operative Group); Deutsche Bank; HSBC (that incorporated Midland Bank in 1999 – one of the historical "big four" Sterling clearing banks[15]); Lloyds TSB; Royal Bank of Scotland (including Natwest); Santander (formerly Abbey, Alliance & Leicester and Bradford & Bingley, owned by Banco Santander of Spain); Standard Chartered.

[15]During most of the 20th Century the phrase "Big Four" referred to the four largest Sterling banks that acted as clearing houses for bankers cheques. These were: Barclays Bank; Midland Bank (now part of HSBC); Lloyds Bank (now Lloyds TSB Bank and part of Lloyds Banking Group); and National Westminster Bank ("NatWest", now part of The Royal Bank of Scotland Group). Currently, the largest four UK banks are Barclays, HSBC, Lloyds Banking Group and The Royal Bank of Scotland Group (with a combined market capitalization of more than £254bn) closely followed by Standard Chartered (with a market cap of over £37bn) – and all of these banks are part our network.

We split our sample into 3 periods: Pre-crisis period: 1 January 2006 to 9 August 2007; Post Northern Rock/ Hedge Fund Crisis: 10 August 2007 to 19 September 2009; Post Asset Purchase Programme: 20 September 2009 to 30 September 2010. This is explained in more detail below.

Our proxies for the intensity of network links are the interbank overnight borrowing relations. This data is extracted from payment systems data by applying an algorithm developed by Furfine (2000). This is an approach which is common to most papers on the interbank money market. The algorithm identifies pairs of payments between two payment system counterparties where the outgoing payment (the loan) is a multiple of 100,000 and the incoming payment (the repayment) happens the following day and is equivalent to the outgoing payment plus a plausible interest rate. This algorithm has been tested heavily and accurately tracks the LIBOR rate in aggregate. Furfine (2000) showed that the algorithm accurately identifies the Fed Funds rate when applied to Fedwire data.

The data are not perfect. They are inferred data so it is possible that there are some erroneous matches or that some loans are missing. We have no reason to expect this to introduce systematic bias into the data. It is also necessarily incomplete. The data are only for banks which are participants in the payment systems. This creates two problems. First, some loans may be attributed to the settlement bank involved when in fact the payments are made on behalf of one of their customers. Second, where a loan is made between one customer of a settlement bank and another, this transaction will not be settled through the payment system but rather across the books of the settlement bank. This is a process known as internalisation Internalised payments are invisible to the central bank, so they are a part of the overnight money market that will not be captured.

The loan data are compiled to form an interbank lending and borrowing network. In particular, the adjacency matrix $\mathbf{G}_t$ has elements $g_{i,j,t}$ given by the fraction of bank's i overnight loans from bank j. The weights at time t are computed as monthly averages in the *previous* month.

By construction $\mathbf{G}_t$ is a square right stochastic matrix. Its largest eigenvalue is therefore equal to one. This implies that the potential propagation of shocks within the system will be dominated by the second largest eigenvalue of the adjacency matrix.[16] The time series of the second largest eigenvalue of $\mathbf{G}_t$ is reported in Figure 1. The figure shows a substantial increase of the eigenvalue in the aftermath of the Northern Rock/Hedge Fund Crisis period (September 2007), but what is striking is the post Quantitative Easing period that seems to be characterized by a substantial increase in the volatility of the network links.

One way to characterize time variation in the cohesiveness of the network is to look at the behaviour of the Average Clustering Coefficient (ACC – see Watts and Strogatz (1998)) defined as

$$ACC_t = \frac{1}{n}\sum_{i=1}^{n} CL_i(\mathbf{G_t}),$$

$$CL_{i,t} = \frac{\#\{jk \in \mathbf{G}_t \mid k \neq j,\, j \in n_i(\mathbf{G_t}),\, k \in n_i(\mathbf{G}_t)\}}{\#\{jk \mid k \neq j,\, j \in n_i(\mathbf{G}_t),\, k \in n_i(\mathbf{G}_t)\}}$$

where $n_i(\mathbf{G}_t)$ is the set of players between whom and player i there is a direct link. The numerator of $CL_{i,t}$ is the number of pairs of banks linked to i that are also linked to each

[16]Since $\mathbf{G}^k$ can be rewritten in Jordan normal form as $\mathbf{PJ}^k\mathbf{P}^{-1}$ where $\mathbf{J}$ is the (almost) diagonal matrix with eigenvalues (or Jordan blocks in case of repeated eigenvalues) on the main diagonal.

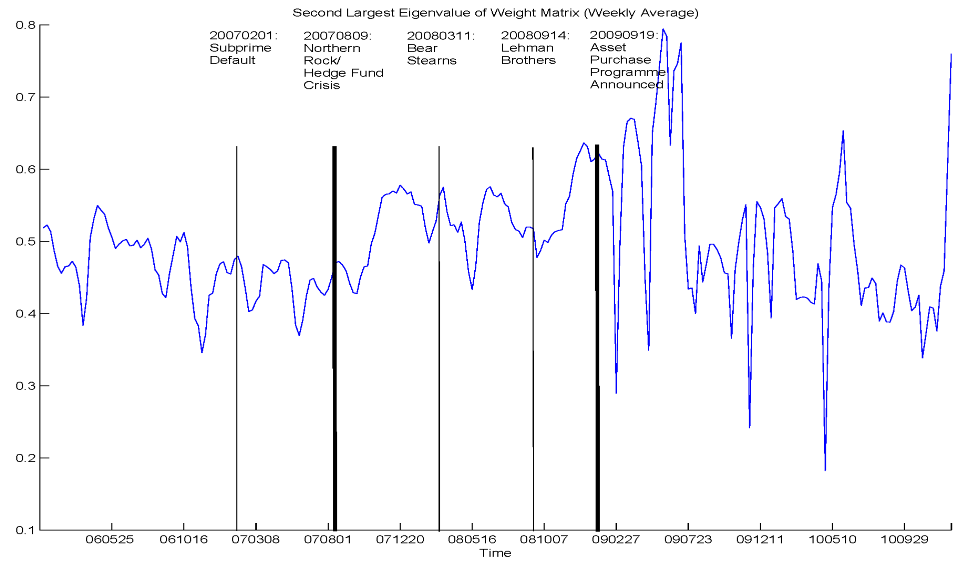

Figure 1: Second largest eigenvalue of $\mathbf{G}_t$.

other, while its denominator is simply the number of pairs of banks linked to i. Therefore, the average clustering coefficient measures the average proportion of banks that are connected to bank i who are also connected to each other. By construction this value ranges from 0 to 1. The time series of the ACC is reported in Figure 2. The figure shows that at the beginning of the sample the network is highly cohesive since, on average, around 80% of the pairs of banks connected to any given bank are also connected to each other.

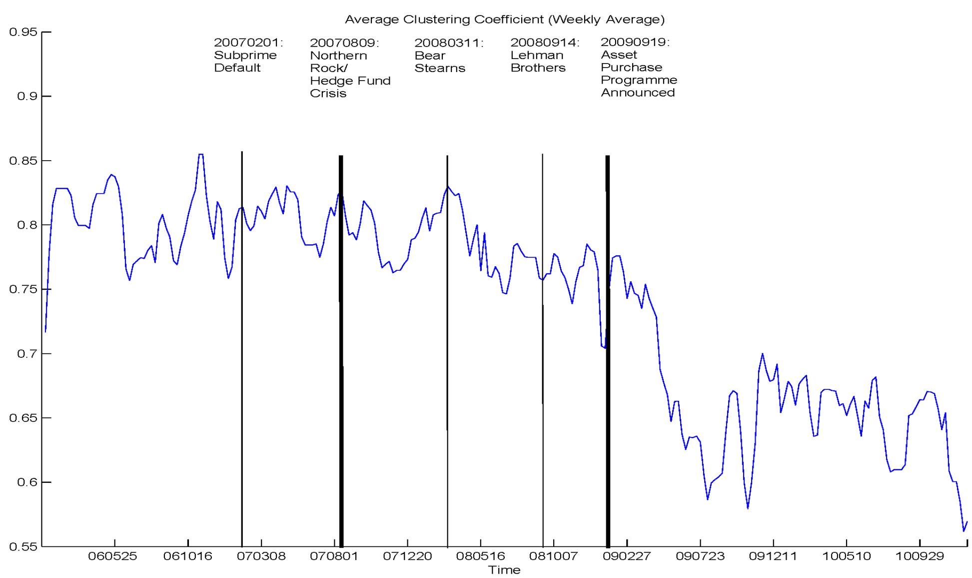

Figure 2: Average clustering coefficient of the interbank network.

The degree of connectedness seems to have a trend reduction during 2007-2008, and a substantial and sudden reduction following the Asset Purchase Programme when the average clustering coefficient gets reduced by about a quarter of its pre-crises average. This might be the outcome of reduced interbank borrowing needs during the Quantitative Easing thanks to the availability of additional reserves from the Bank of England (combined with a move towards increased collateralisation of borrowing and an overall deleveraging of banksbalance sheets).[17] This interpretation is consistent with Figure 3 that depicts the (rolling monthly average of) daily gross overnight borrowing in the interbank network. The data record a substantial increase in overnight borrowing as initial response to the financial market turmoil, possibly caused by a shift toward very short borrowing due to increased difficulties in obtaining long term financing (Wetherilt, Zimmerman, and Soramaki (2010)) , and a substantial decrease in overnight borrowing after the beginning of Quantitative Easing.

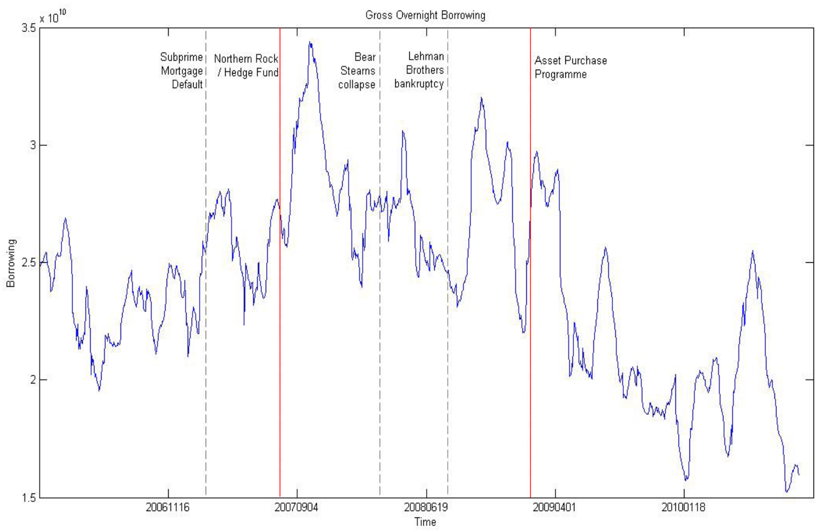

Figure 3: Daily gross overnight borrowing in the interbank network (rolling monthly average).

To measure the dependent variable $l_{i,t}$, that is, the liquidity holding of each bank, we use central bank reserve holdings. We supplement this with collateral that is repo'ed with the Bank intraday in return for intraday liquidity to get a full picture of a bank's most liquid assets. These repos are unwound at the end of each working day. For robustness, we also analyze separately the behaviour of each of these two liquidity components. The weekly average of the total liquidity in the system is reported in Figure 4. The figure depicts a substantial upward trend in the available liquidity in the post Subprime Default subsample and during the various financial shocks registered in the 2008-2009 period, consistently with the evidence of banks hoarding liquidity in response to the financial crisis, but this upward trend is dwarfed by the steep run up registered in response to the Asset Purchase Programme (aka Quantitative Easing) that has almost tripled the average liquidity in the system.

In addition to common monthly time dummies, meant to capture common time effects, and bank fixed effects, meant to capture unobserved heterogeneity, we use a large set of aggregate (x_t^p) and bank specific $(\ x_{i,t}^m)$ control variables. Note that since in the econometric specification in equations (17) and (18) the network effects are elicited through their contribution to the residuals, a potential overfitting of banks' liquidity choice through observable

[17]See Westwood (2011), 'The Money Market Liaison Group Sterling Money Market Survey', Bank of England Quarterly Bulletin: 51(3).

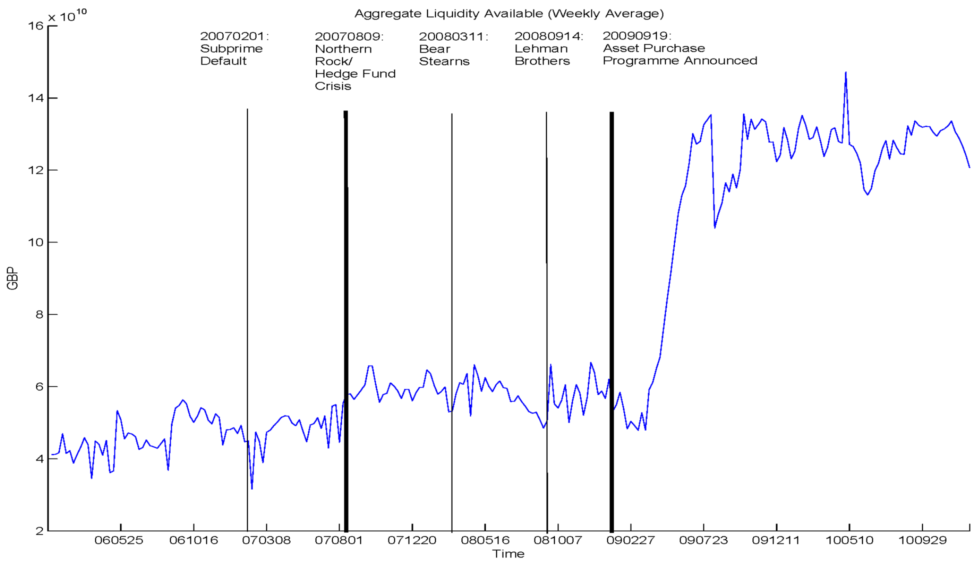

Figure 4: Weekly average of aggregate liquidity available at the beginning of the day.

variables is a conservative approach.

Aggregate Control Variables (x_t^p)**:** All the common control variables, meant to capture aggregate market conditions, are lagged by one day so that they are predetermined with respect to time t innovations. To control for aggregate market liquidity condition we use the total liquidity in the previous day. The proxy for the overall cost of funding liquidity we use the lagged LIBOR rate and the interbank rate premium (computed as the difference between the overnight interest rate and the LIBOR rate).

Since banks' holding decision of liquidity is likely to be influenced by the volatility of their daily payment outflows, we construct a measure of the intraday payments volatility defined as

$$VolPay_t = \sqrt{\frac{1}{88}\sum_{\tau=1}^{88}\left(P_{t,\tau}^{out}\right)^2} \tag{23}$$

where P^{out} denotes payment outflows and 88 is the number of time intervals within a day. The time series of this variable is reported in Figure 5 and it is characterized by a strong upward trend before the subprime default crisis, and a distinctively negative trend during the period of financial turmoil preceding the beginning of QE. During the QE period instead this variable has no clear trend but is characterized instead by a substantial increase in volatility.

We also control for the turnover rate in the payment system (see Benos, Garratt, and Zimmerman (2010)). This variable is constructed as

$$TOR_t = \frac{\sum_{i=1}^{N}\sum_{\tau=1}^{88} P_{i,t,\tau}^{out}}{\sum_{i=1}^{N}\max\left\{\max_{\tau\in[1,88]}\left[CNP(\tau;i,t)\right],0\right\}}$$

where the cumulative net debit position (CNP) is defined as the difference between payment outflows and inflows. The numerator captures the total payments in the system in a day, while the denominator is the sum of the maximum net debt positions of all banks in a given

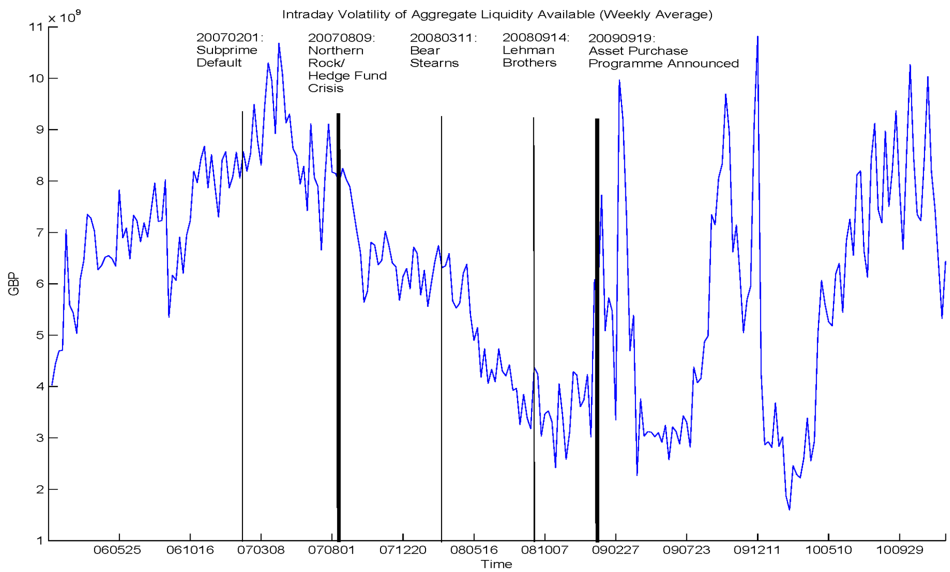

Figure 5: Intraday volatility of aggregate outflows.

day. This variable is meant to capture the velocity of transactions within the interbank system and its time series is reported in Figure 10 of Appendix A.5, and indicates an increased turnover during the financial turmoil, followed by a reduction to levels below the historical average during the QE period.

Since banks have some degrees of freedom in deciding the timing of their intraday outflows, we control for the right kurtosis[18] (rK_t) of intraday payment times. The time series of this variable is reported in Figure 11 of Appendix A.5 and shows a substantial increase during the QE period.

Bank Characteristics $(x_{i,t}^m)$**:** As for the aggregate control variables, all bank characteristic variables are lagged so that they are predetermined with respect to time t innovations. Despite the fact that we control for aggregate interest rates (LIBOR and average overnight borrowing rate), we also control for the bank specific overnight borrowing rate (computed as the average weighted by the number of transactions). We include these variables (reported in Figure 12 of Appendix A.5) because in response to both the Northern Rock and Lehman Brothers collapses there is a substantial increase in the cross-sectional dispersion of the overnight borrowing rates, and this increase in dispersion persists during the QE period (see Figure 13 of Appendix A.5). We also control for: bank specific right kurtosis of the time of

[18]We define as right (rK_t) and left (lK_t) kurtosis the fractions of kurtosis of payments times generated by payments times that are, respectively, above and below the average payment time of the day:

$$rK_t = \frac{\sum_{\tau > m_s} (\frac{\tau - m_t}{\sigma_t})^4}{\sum_{\tau=1}^{88} (\frac{\tau - m_t}{\sigma_t})^4}; \quad lK_t = \frac{\sum_{\tau < m_t} (\frac{\tau - m_t}{\sigma_t})^4}{\sum_{\tau=1}^{88} (\frac{\tau - m_t}{\sigma_t})^4};$$

where m_t and σ_t are defined as

$$m_t = \frac{1}{88} \sum_{\tau=1}^{88} \tau \left(\frac{P_{t,\tau}^{OUT}}{\sum_{t=1}^{T} P_{t,\tau}^{OUT}} \right), \quad \sigma_t^2 = \frac{1}{88-1} \sum_{\tau=1}^{88} \left[\tau \left(\frac{P_{t,\tau}^{OUT}}{\sum_{t=1}^{88} P_{t,\tau}^{OUT}} \right) - m_t \right]^2.$$

intraday payments in ($rK^{in}_{i,t}$, to capture a potential incentive to increase bank liquidity) and out ($rK^{out}_{i,t}$, since banks in need of liquidity might have an incentive to delay their outflows); the intraday volatility of the used liquidity ($VolPay_{i,t}$, defined as in equation (23) but using bank specific flows); the total amount of intraday payments ($LevPay_{i,t} \equiv \sum_{\tau=1}^{88} P^{out}_{i,t,\tau}$); the liquidity used ($LU_{i,t}$) as defined in Benos, Garratt, and Zimmerman (2010);[19] the repo liabilities to total assets ratio; the cumulative change in retail deposits to total asset ratio; the total lending and borrowing in the interbank market; the cumulative change in the 5-year senior unsecured credit default swap (CDS) premia; a dummy variable for the top four banks in terms of payment activity.

V Estimation Results

As a first empirical exercise, we estimate our empirical network model specified in equations (17) and (18) using three subsamples of roughly equal size. These are the pre Northern Rock/Hedge Fund Crisis period (Period 1), the period immediately after the Northern Rock/Hedge Fund Crisis but before the announcement of the Assets Purchase Programme (Period 2), and the period running from the announcement of the Assets Purchase Programme to the end of the sample. We split our sample in these three parts since *a*) they correspond to very different overall market conditions, and *b*), as documented in Section IV, the network structure and behaviour seem to change substantially in these sub-periods. Period 1 corresponds to a relatively tranquil period for the banking sector. Period 2 is characterized by several significant events in world financial markets such as: the run on Northern Rock (the first U.K. bank run for 150 years), the subprime mortgage hedge fund crisis, the Federal Reserve intervention in Bear Stearns and its subsequent sale to JP Morgan Chase and the bankruptcy of Lehman Brothers. Period 3 is characterized by a real regime shift – the beginning of Quantitative Easing – in U.K. monetary policy.[20] Estimation results for these three subsamples are reported in Table 1.

The first row of the Table reports the estimates of the network coefficient ϕ. Recall that a $\phi > 0$ (< 0) implies that banks acts as strategic complements (substitutes) in their liquidity holding decisions and that this tends to amplify (reduce) the effect of bank specific liquidity shocks. In the first period the point estimate of this coefficient is about 0.64 (and highly significant) indicating the presence of a substantial multiplier network multiplier effect for liquidity shocks: a £1 shock idiosyncratic shock equally spread across banks would results in a $1/\left(1-\hat{\phi}\right) = £2.8$ shock to aggregate liquidity.[21] In the second period, the ϕ coefficient

[19] Liquidity used on day t is defined as

$$LU_{i,t} = \max\{\max_{\tau\in[1,88]}[CNP(\tau;i,t)],0\}.$$

[20] See Bank of England, 'Financial Stability Report,' No. 24, October 2008 and 'Financial Stability Report,' No. 25, June 2009.

[21] Note that from Equation (19) we can compute the *average* network multiplier as the total impact on aggregated liquidity resulting from a unit shock equally spread across the n banks. This is given by

$$\mathbf{1}'\mathbf{M}\left(\phi,\mathbf{G}_t\right)\mathbf{1}\frac{1}{n} = \frac{1}{1-\phi}$$

Table 1: Spatial Error Model Estimation

	Period 1	*Period 2*	*Period 3*
$\hat{\phi}$	0.6400^*	0.1660^*	-0.1510^*
	(52.44)	(7.06)	(−6.45)
Aggregate Control Variables (x_t^p)			
$\ln L_{t-1}$	-0.0020	0.3324^*	0.5974^*
	(−0.04)	(4.59)	(14.73)
rK_{t-1}	-0.1654^*	0.0265	0.0031
	(−2.39)	(1.12)	(1.01)
$\ln VolPay_{t-1}$	0.1750	0.1935^*	0.0075
	(1.37)	(7.15)	(0.52)
TOR_{t-1}	0.0097	0.0055^*	0.0049^*
	(1.51)	(2.87)	(2.07)
LIBOR	0.6456^*	0.3216^*	-0.1633
	(2.16)	(6.48)	(−1.12)
Interbank Rate Premium	1.9305^*	-0.0505	0.9514^*
	(2.75)	(−0.61)	(2.86)
Bank Characteristics ($x_{i,t}^m$)			
Interbank Rate	-0.5096	-0.2977^*	0.1414
	(−1.72)	(−6.02)	(1.0428)
$\ln LevPay_{i,t-1}$	-0.1558^*	-0.1595^*	0.0478^*
	(−5.73)	(−8.87)	(2.51)
$rK_{i,t-1}^{in}$	0.0359	-0.0045	-0.0395^*
	(1.90)	(−0.26)	(−3.39)
$rK_{i,t-1}^{out}$	0.1416^*	0.1742^*	0.0426^*
	(8.17)	(15.89)	(4.16)
$\ln VolPay_{i,t-1}$	0.0558^*	0.0524^*	0.0417^*
	(39.72)	(50.23)	(36.73)
$\ln LU_{i,t-1}$	0.0303^*	-0.0023	0.0052
	(3.00)	(−0.34)	(0.68)
Top 4 Bank dummy	1.3374^*	1.6815^*	2.3738^*
	(26.97)	(46.31)	(57.18)
$\frac{\text{Repo Liability}}{\text{Assets}}$	-0.7721	0.7401^*	0.0575
	(−0.92)	(14.46)	(0.64)
$\frac{\Delta\text{Deposit}}{\text{Assets}}$	0.5050	-1.3275^*	-1.2503^*
	(0.68)	(−6.65)	(−3.70)
Total Lending and Borrowing (log)	0.1209^*	0.0249	-0.3231^*
	(3.56)	(0.99)	(−23.70)
CDS (log)	-0.0652	-0.0274^*	0.0514^*
	(−1.49)	(−3.17)	(4.55)
CDS Missing dummy	-2.1893^*	-2.2618^*	-0.8502^*
	(−11.38)	(−32.04)	(−8.37)
R^2	69.11%	89.71%	85.54%

Estimation results of equations (17) and (18). Period 1, 2 and 3, correspond, respectively, to the pre Northern Rock/Hedge Fund Crisis, post Hedge Fund Crisis – pre Asset Purchase Programme, and post the Asset Purchase Programme announcement subsamples. The t-statistics are reported in parenthesis under the estimated coefficients, and * denotes variables significant at confidence levels above 5%.

is still highly significant but it is substantially reduced in magnitude to 0.166 implying weak complementarity with an (average) shock multiplier of about 1.2. This finding suggests that in response to the turbulences in the financial market that have characterize Period 2, the banking system has significantly reduced its own network risk exposure. In period 3 the coefficient ϕ becomes negative, $-.151$, but is still highly significant, implying an average network shock multiplier of about .87. This is particularly interesting since a negative ϕ implies strategic substitution in liquidity holdings, as in Bhattacharya and Gale (1987), that is a situation in which individual banks decide to choose less liquidity when neighbouring banks have more liquidity, having a dampening effect on the aggregate level of liquidity. Since these negative point estimates are obtained in the Quantitative Easing sub-period it suggests that the liquidity multiplier effect is not working at the time of the large inflow of liquidity provided by the Asset Purchase Programme of the Bank of England. The total liquidity injection from the program has been, up to October 2011, of about £275 billion. [22]

Overall, the controls variables, when significant, tend to have the expected signs. Individual liquidity holdings tend to co-move positively and significantly with aggregate liquidity (L_{t-1}), even though in Period 1 the coefficient estimate is not statistically different from zero and has negative sign. Delays in payments in the overall system (captured by rK_{t-1}) significantly reduce the individual bank's available liquidity in Period 1. An increase in the volatility of aggregate ($Volpay_{t-1}$) and individual ($Volpay_{i,t-1}$) daily outflows is associated with higher liquidity holdings from individual banks, but the coefficient on the aggregate variable is statistically significant only in Period 2. This is probably due to multicollinearity between the aggregate and individuals measures, with the individual variable being statistically significant in all periods. The aggregate turnover rate (TOR_{t-1}) is always positively related to banks' liquidity, and statistically significant in Period 1 and 2. This indicates that periods of higher velocity of transactions in the banking system are also periods of higher liquidity holdings. The three interest rates variables are highly collinear, as a consequence their statistical significance and signs tend to alternate across samples.

The lagged total payments of a bank ($LevPay_{i,t-1}$) has a statistically significant effect on its liquidity in all the periods, but its sign is negative in the first two periods while positive in the third one, suggesting that in the third period Quantitative Easing has made it easier for banks to replenish their liquidity in response to an increase in payment outflows. The right kurtosis of the time of payment inflows ($rK^{in}_{i,t-1}$) is statistically significant only in the third period and with the expected negative sign, while the right kurtosis of the time of payment outflows ($rK^{out}_{i,t-1}$) is statistically significant in all subsample and with positive sign, indicating that time in which a bank strategically delays its payment outflows are also the times in which it tends to hoard more liquidity. Liquidity used ($LU_{i,t-1}$) is statistically significant only in the first period with the expected positive sign. The dummy for the top 4 Banks in terms of payment activity is not surprisingly positive and highly significant in all subsamples. The regression coefficient associated with the repo liabilities to assets ratio is significant only in Period 2 and has positive sign, while the change in the deposits to assets ratio is statistically significant only in the last two periods with an associated negative coefficient. Total lending and borrowing, as well as CDS premia, are statistically significant in two periods but the sign of their coefficients change over time. The dummy variable for banks with no outstanding CDS contracts has always a significantly negative coefficient.

[22]See `http://www.bankofengland.co.uk/publications/Pages/news/2011/092.aspx`.

Overall, the fit of the model is quite good in all subsamples with an R^2 in the $69 - 90\%$ range. With the subsample estimates at hand, we can compute the network impulse response functions to identify the risk key players in the interbank market. Results for Period 1 are reported in the upper panel of Figure 6. In particular, in the upper panel we report the excess network impulse response functions to a unit shock ($NIRF^e\left(\hat{\phi}, 1\right)$, defined in Equation (22)), as well as two standard deviations error bands. Also, as a point of reference, we report in the same panel the average network multiplier in excess to the unit shock (i.e. $(1-\phi)^{-1} - 1$). As mentioned earlier, the point estimates in Period 1 implies a large average network multiplier of shocks to individual banks, and the picture shows that in response to a £1 shock idiosyncratic shock equally spread across banks, the final compounded shock to the overall liquidity would be increased by another £1.8. Nevertheless, what the upper panel of Figure 6 stresses is that this large network amplification of shocks is due to a small subset of banks. In particular: a £1 idiosyncratic shock to the liquidity of either Bank 5 or Bank 9 would generate an excess reaction of aggregate liquidity of about £5.8; the same shock to Bank 6 would result in an excess reaction of aggregate liquidity of about about £3.6; instead, a shock to Bank 4 would have an effect that is roughly of the same size as the average network multiplier while a shock to any of the remaining seven banks would be amplified much less by the network system. That is, the network impulse response functions stress that there is a small subset of key players in the interbank liquidity market that generate most of the network risk.

The central panel of Figure 6 shows the average net borrowing during Period 1. Comparing the upper and central panels of the figure, it is interesting to notice that simply looking at the individual lending and borrowing behaviour one cannot identify the riskiest players for the network. In particular, the two riskiest players identified through our structural estimation, are not the largest net borrowers in the network – the largest net borrower, Bank 4, is instead an average bank in network risk terms. Moreover, Bank 5, one of the two risk key players, is not a net borrower – it is instead the second largest net lender. The comparison between the two panels also makes clear that the risk key players are not necessarily the net borrowing banks – net borrowers and net lenders are roughly as likely to be the network risk key players. This result is intuitive: negative liquidity shocks to a bank that lends liquidity to a large share of the network can be, for the aggregate liquidity level, as bad as a negative shock to a bank that is borrowing liquidity from other banks. But the comparison between the two panels makes also clear that, simply looking at the largest players in terms on net borrowing or lending, one would not be able to identify the key risk players for the system. The reasons behind this finding can be understood looking at the lower panel of the figure, where we present the average network structure during Period 1.

In particular, the size of the ellipses identifying individual banks is (log) proportional to their total gross borrowing in the system, incoming arrows to a node indicate borrowing flows to that node while outgoing arrows indicate lending flows from that node, and the thickness of the arrows is (log) proportional to the sterling value of the flows. The lower panel highlights that key risk players are the players that tend to have most borrowing and lending connections in the system, and are directly connected to banks that also have a lot of connections, but are not necessarily the players that borrow or lend more in neither gross nor net terms, but rather the players that borrow from, and lend to, more – well connected – players.

Figure 7 reports excess impulse response functions (upper panel), average net borrowing

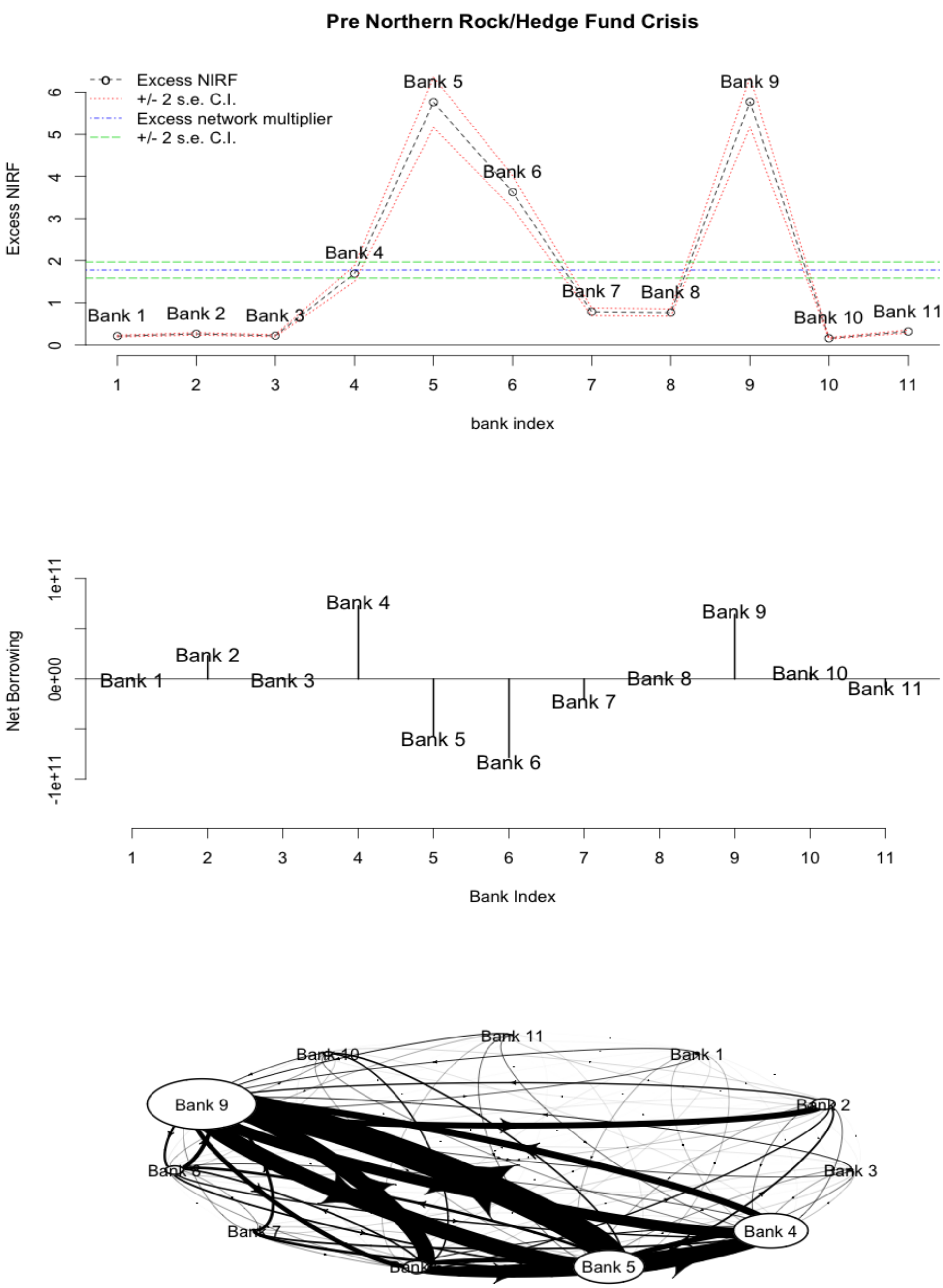

Figure 6: The Pre-Northern Rock/Hedge Fund Crisis Subsample. Network excess impulse-response functions to a unit shock (upper panel); borrowing and lending flows (lower panel).

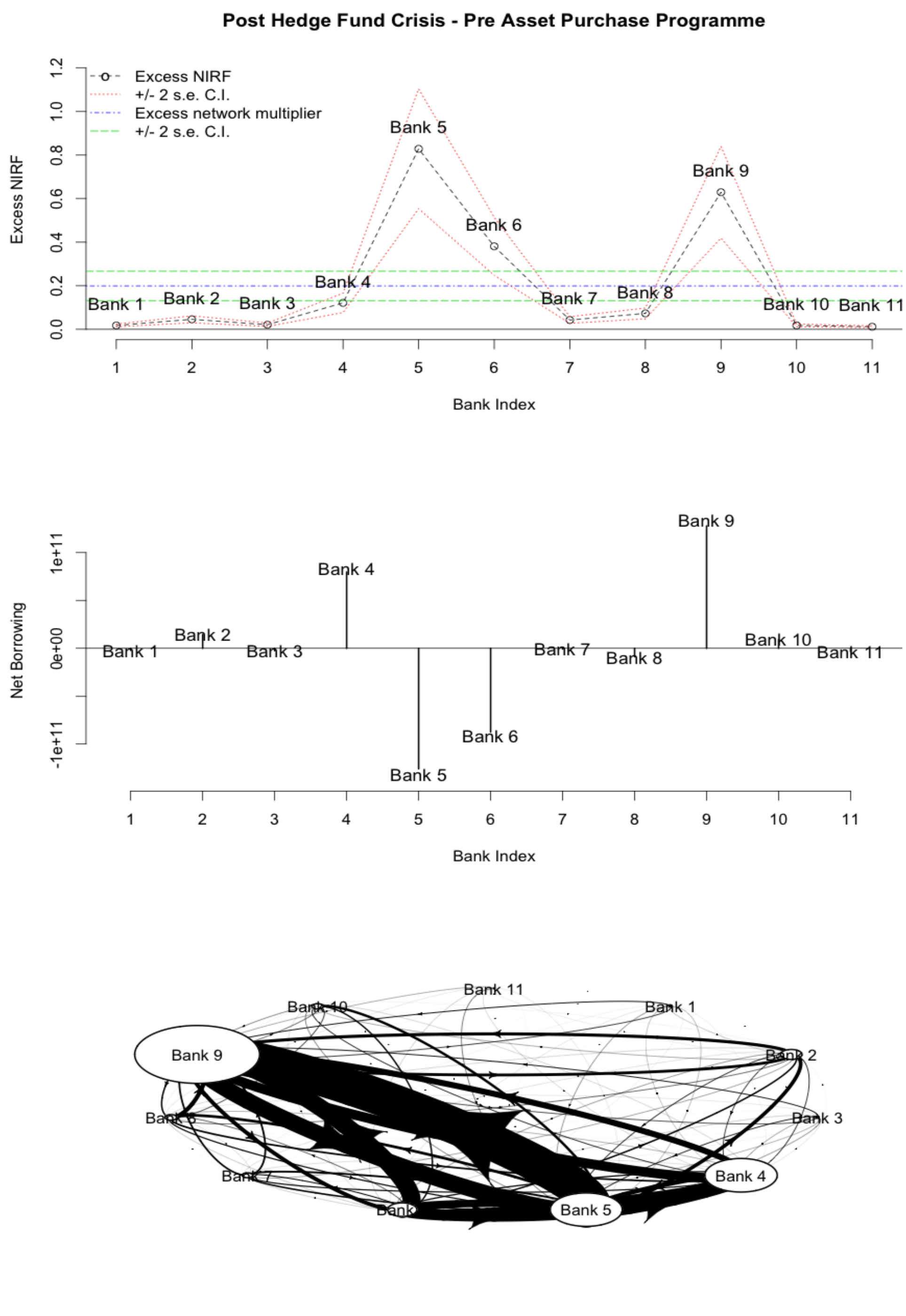

Figure 7: The Post Hedge Fund Crisis and Pre-QE Subsample. Network excess impulse-response functions to a unit shock (upper panel); borrowing and lending flows (lower panel).

positions (central panel), and network flows (lower panel) for Period 2 – the period characterized by a high degree of stress in the financial market. The first thing to notice is that, despite the overall increase in activity in the interbank borrowing and lending market, outlined by both the central and lower panels and by Figure 3, there is a drastic reduction in the average network multiplier reported in the first panel: The average excess network reaction to a unit shock is only about .2. That is, in a period of financial stress, banks seem, on average, to have radically reduced their network risk exposure, and they have done so despite having increased the amount of overnight borrowing and lending needed to fund their liquidity needs. Nevertheless, as stressed by the first panel, the network risk profile, even though substantially reduced overall, is still very high for a small subset of banks. In particular a unit shock to Bank 5, Bank 9 and Bank 6, would result, respectively, in an excess network liquidity change of .83, .63 and .38, while the same shock to Bank 4 would have an effect very similar to the average one, and a shock to the remaining banks would receive minimal amplification from the network system.

The results for Period 3 – the one starting at the onset of Quantitative Easing – are reported in Figure 8, and are radically different from the ones of the previous two periods. First, banks tend to behave as strategic substitutes in their liquidity holdings in this period, therefore the network has a reducing effect on individual bank shocks, implying a negative average excess multiplier of $-.13$, that is, a unit liquidity shock equally spread across banks would result in a $1 - .13 = .87$ shock to aggregate liquidity. But, once again, there is substantial heterogeneity across banks, in the sense that for most banks (Bank 1, 3, 7, 8, 10 and 11) the network has basically no effect on how their own shocks propagate to the system, while for few other banks (4, 5, 6, and 9) the network structure helps reducing the impact of their own idiosyncratic shocks on aggregate liquidity.

This behaviour arises in a period in which the degree of connectedness of the network is substantially reduced (see Figure 2 and the lower panel of Figure 8), the gross borrowing in the system has been substantially reduced (see Figure 3), most banks hold net borrowing positions close to zero (central panel of Figure 8), but at the same time the overall liquidity in the system has been substantially increased thanks to the Asset Purchase Programme (Figure 4). What is also interesting to notice is that the same banks that were the riskiest players in the previous two periods (Banks 5, 6 and 9) are now the less risky ones for the system since an idiosyncratic shock to theses banks, thanks to both their centrality in the network and the overall strategic substitute behaviour of banks, would have a substantially dampened effect on aggregate liquidity.

The results presented so far outlines a substantial change over time of the role played by the network interactions in determining aggregate liquidity. This suggests that the split into just three subsamples might be too coarse to capture the banks' reaction to a changing economic environment. As a consequence we perform estimations of the structural model in Equations (17) and (18) using a 6-month rolling window. These rolling estimates of the network coefficient ϕ are reported (blue line), together with 95% confidence bands (red lines), in Figure 9. The figure also reports the rolling point estimates of the ϕ coefficient implied by the Spatial Durbin Model (green line) in Equation (20) since, if our theory driven Spatial Error specification of the interbank network is correct, the two estimates should not be statistically different.

At the beginning of the sample, the figure shows an extremely large network coefficient, ϕ, implying a substantial network amplification of shocks to banks in the system. The estimated

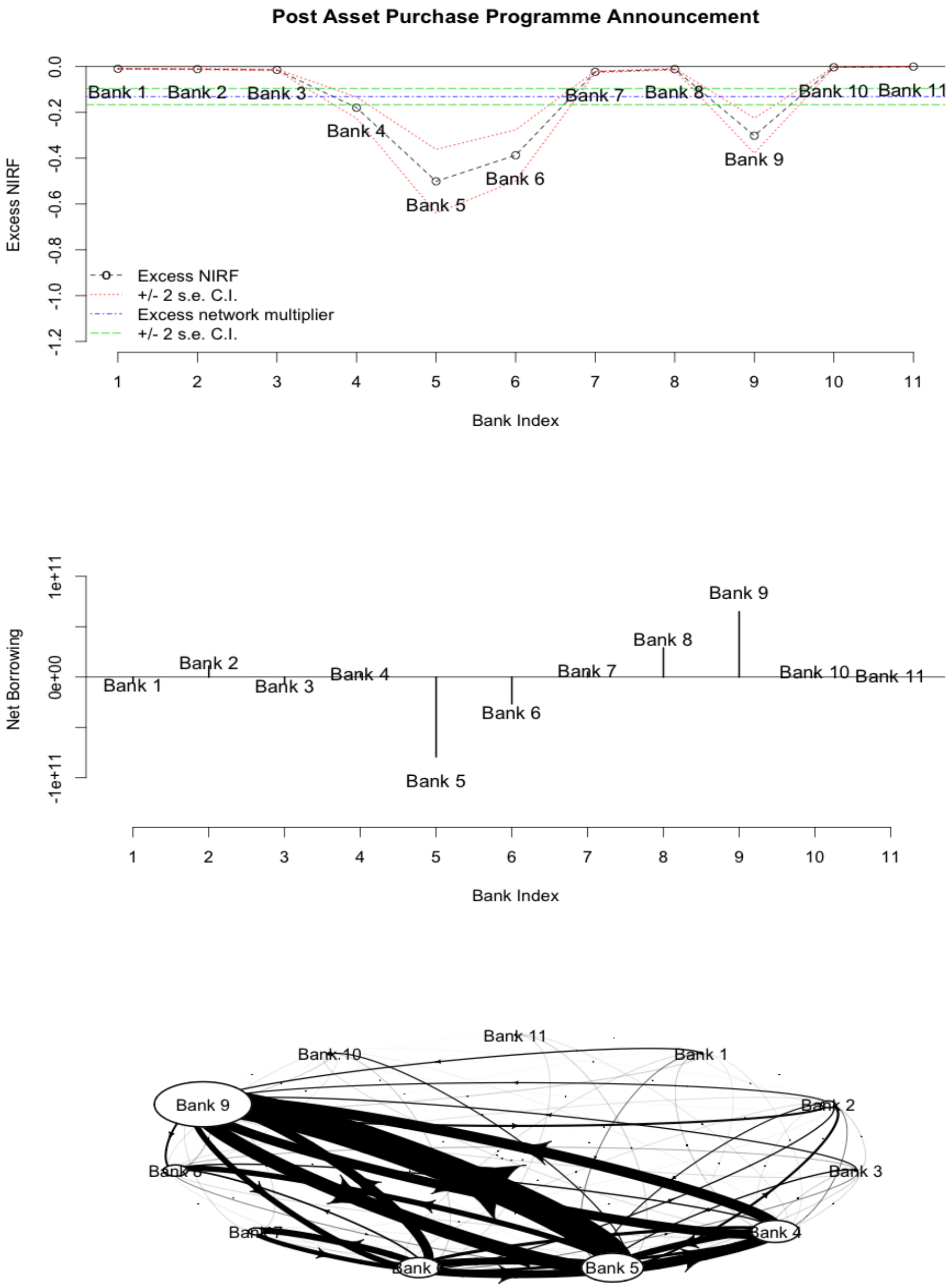

Figure 8: The QE Subsample: Network excess impulse-response functions to a unit shock (upper panel); borrowing and lending flows (lower panel).

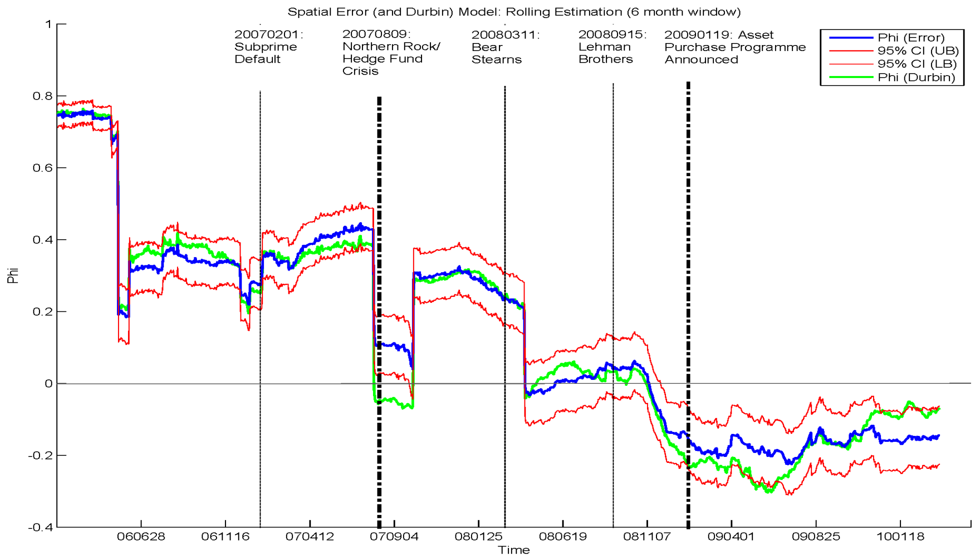

Figure 9: Spatial Error (blue line) and Durbin (green line) rolling estimates of ϕ.

coefficient has a first sharp reduction around the 18th May 2006 when the Bank of England introduced the reserve averaging system described in Section A.1. The network multiplier is relatively stable after May 2006, except for a temporary decrease during the 2007 subprime default, until the Northern Rock bank run when the network multiplier, hence the network risk, is drastically reduced for several months. After this reduction, the coefficient goes back roughly to the previous period average but shows a trend decline that culminates in a sharp drop following the Bear Stearns collapse. Since this period onward, and until long after the Lehman Brothers bankruptcy, the coefficient is statistically indistinguishable from zero, implying a zero excess network multiplier of bank specific shocks. That is, the estimation suggests that in this period there is basically no added risk coming from the network structure of the interbank market, and that individual bank shocks would not be amplified by some sort of domino effect in the U.K. interbank market. The figure suggests that the banks' reaction to the financial market turmoil has been to systematically reduce the overall riskiness of the interbank network. Interestingly, the $\hat{\phi}$ coefficient becomes negative, and statistically significant, right before the announcement of the Asset Purchase Programme, and remains stable during all the Quantitative Easing period. This indicates that, during the liquidity inflow (and also in expectation of it) coming from the Bank of England's QE policy, banks started behaving as strategy substitute in their liquidity holding decision. Note that this a period in which the aggregate supply of central bank reserves is almost completely price inelastic since QE sets a target level for asset purchases (and subsequent reserve creation) and let market forces determine their price. This overall change of the BoE supply of reserves is unlikely to be the driver of our estimates network multiplier coefficient during this period since: a) the change in the ϕ parameter actually occurs before the announcement of QE; b) we estimate the identified optimal response of the banks to market conditions (effectively, the banks equilibrium demand function), and we control for variation in aggregate price and quantities of liquidity, as well as bank deposits held by the private sector.

Finally, the figure outlines that the point estimates of $\hat{\phi}$ coming from our theory driven Spatial Error specification and the ones coming from the more general Spatial Durbin Model are always very similar, both numerically and in terms of their overall evolution during the sample. Moreover, testing formally for discrepancy between the two type of estimates, we find that they are statistically different at the 5% confidence level less than 95% of the times, providing formal support for our formulation of the network model.

VI Conclusions

In this paper, we develop and estimate a network model of interbank liquidity. Based on network topology and the estimated network effects, we construct measures of of systematic risks and identify network players that are most important in contributing to the aggregate liquidity and its risk in the banking system respectively.

We find that the network effect varies significantly through the sample period: January 2006 to September 2010. Prior to the Northern Rock/Hedge Fund crisis, network liquidity is a strategic complement. That is, liquidity shocks propagate through the network and each bank increases its exposure to the network shocks; consequently, the level of liquidity holding is high. That enables a higher velocity of payment transactions. By contrast, during the crisis, the network itself also becomes less cohesive, the network liquidity is significantly reduced and turns into strategic substitute after the commencement of Quantitative Easing. That is, liquidity shocks contract through the network and each bank reduces its exposure to the network shocks. This is associated a lower velocity of payment transactions. Our analysis shows that it is important to analyze not only the network characteristics themselves but also the liquidity decisions that network agents make in order to understand the aggregate impact.

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A Appendix

A.1 Reserves Schemes, Payment Systems, and Interbank Overnight Borrowing

Banks in the UK choose the amount of central bank reserves that they wish to hold to support a range of short term liquidity needs. On a daily basis, reserve balances are used to fund intraday liquidity needs in the large value payment and settlement systems, and to protect against intraday liquidity shocks. Additionally, as central bank reserves are the most liquid asset, they are available to protect banks against a range of unexpected outflows of funds. They are also the ultimate settlement asset for interbank payments. Whenever payments are made between the accounts of customers at different commercial banks, they are ultimately settled by transferring central bank money (reserves) between the reserves accounts of those banks. Therefore, a bank's most liquidity asset is its holding of central bank reserves. Since 2006, the banks in our sample choose their own level of Sterling reserve holdings and reserve holdings are not mandatory. However, their decisions to hold certain stocks of central bank reserves do not happen independently of the policy framework in which they operate.

A.1.1 Monetary Policy Framework

Since the 1998 Banking Act, the Bank has had independent responsibility for setting interest rates to ensure that inflation, as measured by the Consumer Price Index (CPI), meets the inflation target of 2%. Each month the Monetary Policy Committee (MPC) meets to decide the appropriate level of the Bank rate (the policy interest rate) to meet the inflation target in the medium term. The Bank's main mechanism for influencing the inflation rate in the economy is the Sterling Monetary Framework. This framework uses the Bank's balance sheet to influence the level of short-term interest rates, and through this inflation. When banks decide upon the appropriate level of central bank reserves to hold, they do so within the constraints set by this framework. During our sample period (January 2006 to September 2010) the Bank of England had three distinct monetary frameworks: prior to 18 May 2006, the Bank operated an unremunerated reserve scheme; this was then replaced by a reserves average scheme; since March 2009 and the initiation of Quantitative Easing, the reserves average scheme has been suspended. *Pre-2006 Money Market Reform:* The Bank's Sterling Monetary Framework prior to the 2006 reforms was based upon a system of voluntary unremunerated reserves. In this system there were no reserve requirements and no reserve averaging over a maintenance period. The only binding requirement was that banks were obliged to maintain a minimum zero balance at the end of each day. In practice, due to uncertainties about managing end of day cash positions banks opted for small nonzero reserve balances. *Reserve Averaging:* In May 2006, the Bank undertook a major reform of the Sterling Monetary Framework. The new scheme was voluntary remunerated reserves with a period-average maintenance requirement. Each maintenance period – the period between each meeting of the Monetary Policy Committee – banks that participated in the reserve framework were required to decide upon a reserves target. This voluntary choice of reserve balances is a unique feature of the UK system. Over the course of each maintenance period, the banks would manage their balance sheets such that, on average, their reserve balances hit the target. Where banks were unable to meet the target, standing borrowing and deposit facilities were available. If banks balances were more than 1% above the target the Bank would not remunerate the excess reserves; if they were more than 1% below the target the Bank would impose a penalty, reducing the interest paid on the other reserves. At various points during the crisis this ±1% range was increased to give banks more flexibility to manage their liquidity. The amount paid on these reserves was the Bank rate, the interest rate determined by the MPC's monthly meeting. As each bank was free to choose its own reserves target, the level of reserves in the system was almost entirely demand driven.[23] In both schemes before Quantitative Easing (QE), the Bank would use short term Open Market Operations (OMOs), repo and reverse repo transactions backed by high quality liquid assets, to ensure that there were sufficient central bank reserves in the system such that each bank could achieve positive end of day balances or meet their reserves target. The Bank acts as the marginal supplier of funds to the banking system. Banks then use the private interbank money markets to ensure that the reserves are correctly distributed so that all banks meet their targets. *Post Quantitative Easing:* Quantitative Easing started in March 2009 when the MPC decided that, in order to meet the inflation target in the medium term, it would need to supplement using interest rates to influence the price of money (which had hit the practical lower bound of 0.5%) with purchasing assets using central bank reserves. This consisted of the BoE boosting the money supply by creating central bank reserves and using them to purchase assets, predominantly UK gilts. As the quantity of reserves shifted from being demand driven to being influenced by QE, the BoE suspended the average reserve targeting regime, and now remunerates all reserves at the Bank rate.

A.1.2 Payment and Settlement Systems

Banks use central bank reserves to, inter alia, meet their demand for intraday liquidity in the payment and settlement systems. Reserves act as a buffer to cover regular timing mismatches between incoming and outgoing payments, and to cover unexpected intraday liquidity needs, for example, due to exceptionally large payments, operational difficulties, or stresses that impact upon a counterparty's ability, or willingness, to send payments. There are two major interbank payment systems in the UK: CHAPS and CREST. These two systems play a vital role in the UK financial system. On average, in 2011, £700 billion of transactions was settled every day across the two systems. This equates to the UK 2011 nominal GDP settled every two days. CHAPS is the UK's large-value payment system. It is used for real time settlement of payments between its 19 member banks. [24] These 19 banks settle payments on behalf of hundreds of other banks through correspondent banking relationships. Typical payments are business-to-business payments, home purchases, and interbank transfers. Payments relating to unsecured interbank money markets are settled in CHAPS. CHAPS opens for settlement at 8am and closes at 4.20pm. Payments made on behalf of customers cannot be made after 4pm. The system has throughput guidelines which require members to submit 50% of payments by noon and 75% by 14:30. This helps to ensure that payments are settled throughout the day and do not bunch towards the end of the day. In 2011 CHAPS settled an average of 135,550 payments each day valuing 254bn. CHAPS is a Real-Time Gross

[23] There was a ceiling, expressed as the higher of a percentage of eligible liabilities and a fixed value, for the target level of reserves that any bank could choose. However, in practice banks typically chose lower targets, so this was not a binding constraint. The ceiling was raised in May 2008.

[24] This includes Bank of England and CLS. Some banks groups also have multiple memberships due to mergers.

Settlement (RTGS) system. This means that payments are settled finally and irrevocably in real time. To fund these payments banks have to access to liquidity intraday. If a bank has, at any point during the day, cumulatively sent more payments than it has received, then it needs liquidity to cover this difference. This comes either from central bank reserves or intraday borrowing from the central bank. Furthermore, when a bank sends funds to another bank in the system, it exposes itself to liquidity risk. That is, the risk that the bank will not get those funds back during the day, and so will have to use other funds to fulfill its payment obligations. Therefore, it is important to choose an appropriate level of liquidity buffer to manage these intraday liquidity risks. Besides maintaining a liquidity buffer, banks can manage their intraday liquidity exposure settlement banks by borrowing and lending from each other in the unsecured overnight markets. The shortest term for these money markets is overnight. According to Bank of England estimates, payments relating to overnight unsecured money market activity (advances and repayments) account for about 20% of CHAPS values (Wetherilt, Zimmerman, and Soramaki (2010)). CREST, on the other hand, is a securities settlement system. Its Delivery-vs-Payment (DVP) mechanism ensures simultaneous transfer of funds and securities. Liquidity needs in CREST are largely met via Self Collateralising Repos (SCRs), in which the purchase of central bank eligible collateral automatically generates collateralized liquidity from the Bank of England, requiring few intraday resources from the purchasing bank.

A.1.3 The Sterling Unsecured Overnight Money Market

Money markets are the markets where banks and other financial institutions borrow and lend assets, typically with maturities of less than one year. At the shortest maturity, overnight, banks borrow and lend interest bearing central bank reserves. Monetary policy aims to influence the rate at which these markets transact to control inflation in the wider economy. There is very little information available about the size and the structure of the sterling money markets. The Bank estimates suggest that the overnight unsecured market is approximately £20-30 billion per day. Wetherilt, Zimmerman, and Soramaki (2010) describe the network characteristics of the sterling unsecured overnight money market. They find that the network is characterized by a small core of highly connected participants, surrounded by a wider periphery of banks loosely connected with each other, but with connections to the core. It is believed that prior to the recent financial crisis, the unsecured market was much larger than the secured one. But counterparty credit risk concerns, combined with new FSA liquidity regulations, which encourage banks to borrow secured and to increase the maturity of their funding, have increased the importance of the secured markets (Westwood 2011).

A.2 Quasi-Maximum Likelihood Formulation

Writing the variables and coefficients of the spatial error model in equations (17) and (18) in matrix form as[25]

$$B = [\mu, \alpha_1^{time}, ..., \alpha_t^{time}, ..., \alpha_T^{time}, \alpha_1^{bank}, ..., \alpha_i^{bank}, ..., \alpha_N^{bank},$$
$$\beta_1^{bank}, .., \beta_m^{bank}, ..., \beta_M^{bank}, \beta_1^{time}, ..., \beta_p^{time}, ..., \beta_P^{time}]$$

[25]This is similar to the spatial formulation in Lee and Yu (2010).

$$L = [l_{1,1}, ..., l_{1,T}, ..., l_{i,t}, ..., l_{N,1}, ..., l_{N,T}]$$

$$\epsilon = [\epsilon_{1,1}, ..., \epsilon_{1,T}, ..., \epsilon_{i,t}, ..., \epsilon_{N,1}, ..., \epsilon_{N,T}]'$$

$$\nu = [\nu_{1,1}, ..., \nu_{1,T}, ..., \nu_{i,t}, ..., \nu_{N,1}, ..., \nu_{N,T}]'$$

$$\mathbf{G} = diag\left(\mathbf{G}_t\right)_{t=1}^{T} = \begin{bmatrix} \mathbf{G}_1 & \mathbf{0} & ... & \mathbf{0} \\ \mathbf{0} & \mathbf{G}_2 & ... & ... \\ ... & ... & ... & \mathbf{0} \\ \mathbf{0} & ... & \mathbf{0} & \mathbf{G}_T \end{bmatrix}$$

$$\mathbf{X} = \left[\mathbf{1}, \mathbf{D}, \mathbf{F}, \mathbf{X}^{bank}, \mathbf{X}^{time}\right]$$

whit

$$\mathbf{D} = I_T \otimes \mathbf{1}_N, \quad \mathbf{F} = \mathbf{1}_T \otimes I_N$$

$$\mathbf{X}^{time} = \begin{bmatrix} x_1^1 & .. & x_1^p & ... & x_1^P \\ x_2^1 & ... & x_2^p & ... & x_2^P \\ ... & ... & ... & ... & ... \\ x_t^1 & ... & x_t^p & ... & x_t^P \\ ... & ... & ... & ... & ... \\ x_T^1 & ... & x_T^p & ... & x_T^P \end{bmatrix} \otimes \mathbf{1}_N,$$

$$\mathbf{X}^{bank} = \begin{bmatrix} x_{1,1}^1 & ... & x_{1,1}^m & ... & x_{1,1}^M \\ x_{2,1}^1 & ... & x_{2,1}^m & ... & x_{2,1}^M \\ .. & ... & ... & ... & ... \\ x_{N,1}^1 & ... & x_{N,1}^m & ... & x_{N,1}^M \\ ... & ... & ... & ... & ... \\ x_{1,t}^1 & ... & x_{i,t}^m & ... & x_{1,t}^M \\ ... & ... & ... & ... & ... \\ x_{1,T}^1 & ... & x_{1,T}^m & ... & x_{1,T}^M \\ ... & ... & ... & ... & ... \\ x_{N-1,T}^1 & ... & x_{N-1,T}^m & ... & x_{N-1,T}^M \\ x_{N,T}^1 & ... & x_{N,T}^m & ... & x_{N,T}^M \end{bmatrix},$$

we can rewrite the empirical model as

$$\begin{aligned} L &= \mathbf{XB} + \epsilon, \\ \epsilon &= \phi\mathbf{G}\epsilon + \nu, \quad \nu_{i,t} \sim iid\left(0, \sigma_i^2\right). \end{aligned}$$

Implying that

$$\nu\left(\mathbf{B}, \phi\right) = \left(L - \mathbf{XB}\right) - \phi\mathbf{G}\left(L - \mathbf{XB}\right).$$

Finally, using Gaussianity to model the exogenous error terms ν yields the log likelihood

$$\ln \mathcal{L}\left(\mathbf{B}, \phi, \left\{\sigma_i^2\right\}_{i=1}^{N}\right) \equiv -\frac{TN}{2}\ln\left(2\pi\right) - \frac{T}{2}\sum_{i=1}^{N}\ln\sigma_i^2 - \sum_{i=1}^{N}\frac{1}{2\sigma_i^2}\sum_{t=1}^{T}\nu_{i,t}\left(\mathbf{B}, \phi\right)^2, \tag{24}$$

and the above can be estimated using standard optimization methods.

A.3 Confidence Bands for the Network Impulse Response Functions

The ϕ estimator outlined in the previous section has an asymptotic Gaussian distribution with variance s_ϕ^2 (that can be readily estimated as standard from the Hessian and gradient of the log likelihood in Equation (24), or via bootstrap). That is

$$\sqrt{T}\left(\hat{\phi}-\phi_0\right) \xrightarrow{d} N\left(0, s_\phi^2\right).$$

where ϕ_0 denotes the true value of ϕ.

Denoting with

$$a_1(\phi) = \frac{\partial \mathbf{1}'\left\{(\mathbf{I}-\phi\mathbf{G})^{-1}\right\}_{.i}}{\partial\phi};$$
$$a_2(\phi) = \frac{\partial \mathbf{1}'\left\{(\mathbf{I}-\phi\mathbf{G})^{-1}\phi\mathbf{G}\right\}_{.i}}{\partial\phi}$$

we have from Lemma 2.5 of Hayashi (2000) that

$$\sqrt{T}\left[NIRF_i\left(\hat{\phi},1\right) - NIRF_i(\phi_0,1)\right] \xrightarrow{d} N\left(0, a_1(\phi_0)^2 s_\phi^2\right),$$
$$\sqrt{T}\left[NIRF_i^e\left(\hat{\phi},1\right) - NIRF_i^e(\phi_0,1)\right] \xrightarrow{d} N\left(0, a_2(\phi_0)^2 s_\phi^2\right).$$

Therefore, since $a_j\left(\hat{\phi}\right) \xrightarrow{p} a_j(\phi_0)$, $j=1,2$, by continuous mapping theorem, and by Slutsky's theorem $a_j\left(\hat{\phi}\right)\hat{s}_\phi^2 \xrightarrow{p} a_j(\phi_0)^2 s_\phi^2$, where $\hat{s}_\phi^2$ is a consistent variance estimator, we can construct confidence bands for the network impulse response functions using the sample estimates of ϕ and s_ϕ^2.

A.4 Appendix: Variables

A.4.1 Macro control variables

- rK_{t-1}: Right kurtosis of the intraday time of aggregate payment outflow (Yesterday).

$$rK_t = \frac{\sum_{\tau>m_t}\left(\frac{\tau-m_t}{\sigma_t}\right)^4}{\sum_{\tau=1}^{88}\left(\frac{\tau-m_t}{\sigma_t}\right)^4};$$
$$where,\ m_t = \frac{1}{88}\sum_{\tau=1}^{88}\tau\left(\frac{P_{t,\tau}^{OUT}}{\sum_{\tau=1}^{88}P_{t,\tau}^{OUT}}\right)$$
$$\sigma_t^2 = \frac{1}{88-1}\sum_{\tau=1}^{88}(\tau-m_t)^2\left(\frac{P_{t,\tau}^{OUT}}{\sum_{\tau=1}^{88}P_{t,\tau}^{OUT}}\right)$$
$$P_{t,\tau}^{OUT},\ \text{aggregate payment outflow at time interval } \tau.$$

Transactions are recorded for 88 10-minute time intervals within each day (from 5:00 to 19:30). Variable m_t is the average of payment time weighted by the payment outflow.

- $\ln VolPay_{t-1}$: Intraday volatility of aggregate liquidity available (Yesterday, in logarithm). Liquidity available will be defined in the following section on bank specific control variables.

- TOR_{t-1}: Turnover rate in payment system (Yesterday).

  To define turnover rate, we need first to define Cumulative Net (Debit) Position (CNP):

$$CNP(T, i, s) = \sum_{t=1}^{T} (P_{i,s,t}^{OUT} - P_{i,s,t}^{IN}),$$

  where $P_{i,s,t}^{OUT}$ is bank i's the total payment outflow at time t in day s. $P_{i,s,t}^{IN}$ is the payment inflow.

  Turnover rate (in day s) is defined as

$$TOR_s = \frac{\sum_{i=1}^{N} \sum_{t=1}^{88} P_{i,s,t}^{OUT}}{\sum_{i=1}^{N} \max\{\max_T [CNP(T; i, s)], 0\}}$$

  The numerator denotes the total payment made in the system at day s. The denominator sums the maximum cumulative net debt position of each bank at day s. Note that in the denominator, if the cumulative net position of certain bank is always below zero (that is, this bank's cumulative inflow alway exceeds cumulative outflow), this bank actually absorbs liquidity from the system. If there are banks absorbing liquidity from the system, there must be banks injecting liquidity into the system. When we calculate turnover rate (the ratio between total amount circulating and the base), we should only consider only one of the two. That's why we take the first (outside) maximum operator. The reason for the inside operator goes as follows: Any increase in the cumulative net debit position (wherever positive) incurs injection of liquidity into the system, so the maximum of cumulative net position is the total injection from outside to the payment system. And, the sum across different banks gives the total injection through all the membership banks. The higher the turnover rate means the more frequent reuse of the money injected from outside into the payment system.

- *LIBOR*: LIBOR (Yesterday).

- *Interbank Rate Premium:* Average interbank rate in the market minus LIBOR (Yesterday).

A.4.2 Bank-specific variables

- Liquidity Available (LA) is the amount of liquidity to meet payment requirements and is measured as the sum of reserves ($SDAB$, Start of Day Account Balance) plus the value of intraday repos with the BoE (PC, Posting of Collateral). As time goes, the liquidity available is calculated by subtracting the money moved to CREST from the liquidity available in the previous time interval. In this way, we can trace for bank i the liquidity available any time t in day s:

$$LA(t, i, s) = SDAB_{i,s} + PC_{i,s} - \sum\nolimits_{\tau=1}^{t} CREST_{i,s,\tau}$$

- Liquidity holding at the beginning at the day (l): The logarithm of the cash balance plus posting of collateral (the value of intraday repo) at the start of the day.

- *Interbank Rate*: Interbank rate (Yesterday).

- $\ln LevPay_{i,t-1}$: Total intraday payment level (Yesterday, in logarithm).

- $rK^{in}_{i,t-1}$: Right kurtosis of incoming payment time (Yesterday).

- $rK^{out}_{i,t-1}$: Right kurtosis of outgoing payment time (Yesterday).

- $\ln VolPay_{i,t-1}$: Intraday volatility of liquidity available (Yesterday, in logarithm).

- $\ln LU_{i,t-1}$: Liquidity used (Yesterday, in logarithm).

  Liquidity Used:

  $$LU(i,s) = \max\{\max_T [CNP(T;i,s)], 0\}$$

  A positive cumulative net debit position means that at this time interval the bank is consuming liquidity. If a positive cumulative net position never happens for a bank, this bank only absorbs liquidity from the system. That is the reason for the first (outside) maximum operator. The second (inside) maximum operator helps us to trace the highest amount of liquidity a bank uses.

- $\frac{Repo\ Liability}{Assets}$: Repo liability to total asset ratio (monthly).

- *Total Assets (log)*: Total asset (monthly, in logarithm).

- $\frac{\Delta Deposit}{Assets}$: Cumulative change in retail deposit to total asset ratio $\times$ 100 (monthly).

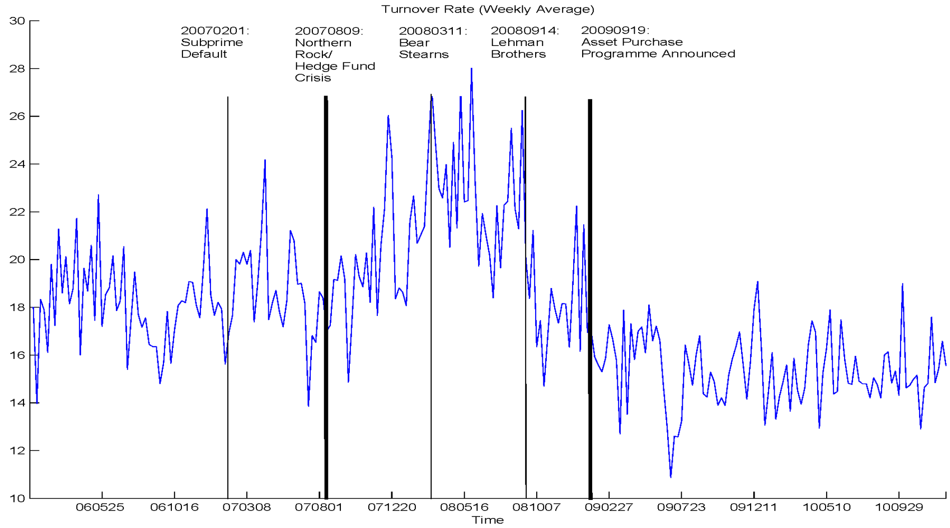
- *Total Lending and Borrowing (log)*: Total lending and borrowing in the interbank market (Yesterday, in logarithm).

- *CDS (log)*: CDS relative price (daily).

- *Stock Return (Inc. Dividend)*: Stock return including dividend (daily).

A.5 Additional Figures

Figure 10: Turnover rate in the payment system.



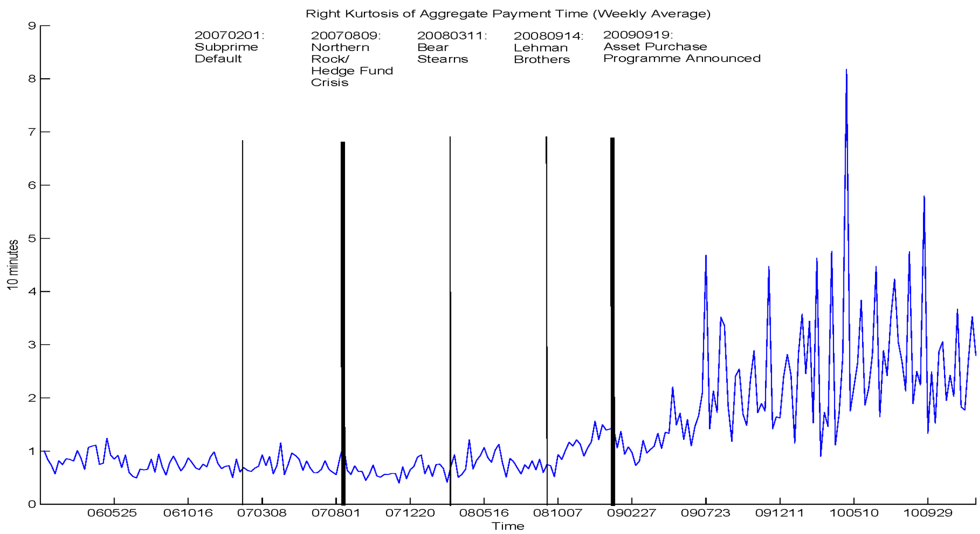


Figure 11: weekly average of the right kurtosis of aggregate payment times.

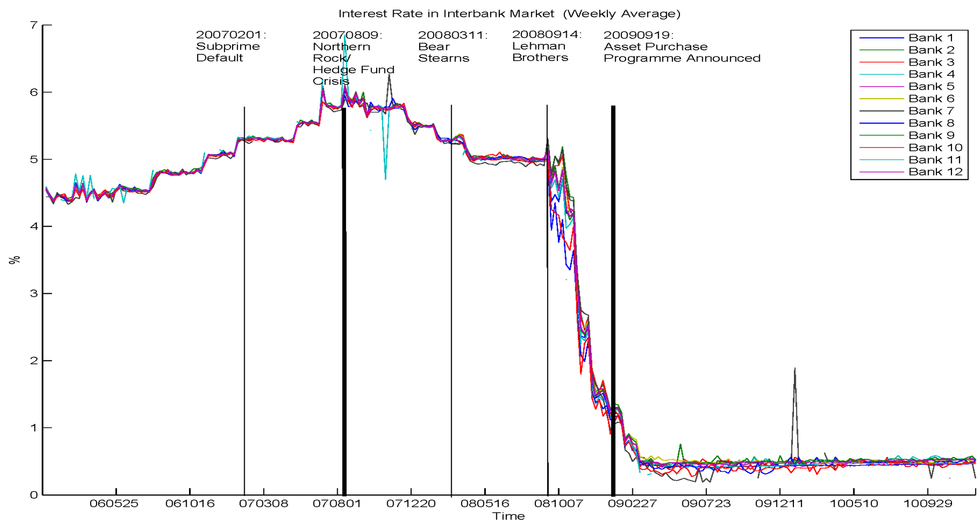


Figure 12: interest rates in the interbank market.

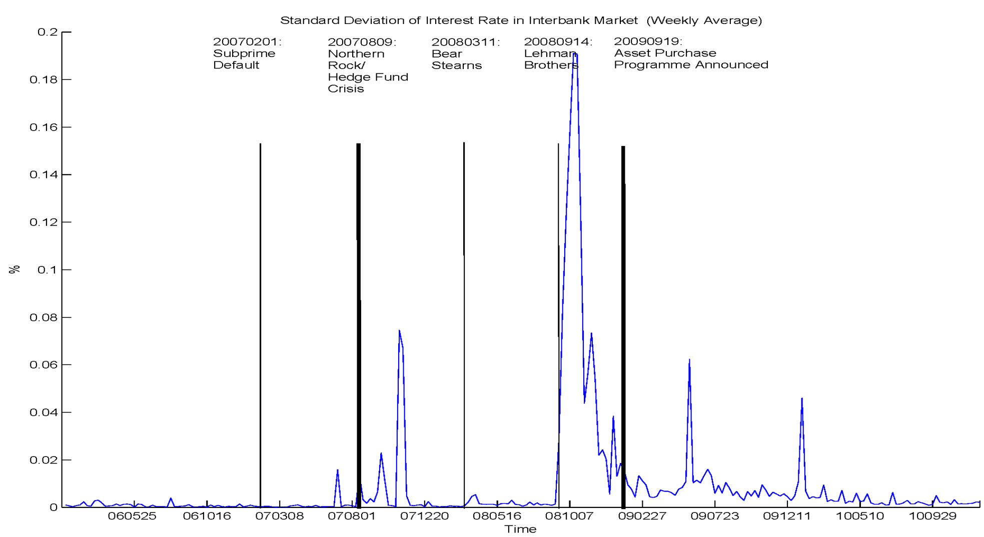


Figure 13: cross-sectional dispersion of interbank rates.