Trading Networks and Equilibrium Intermediation

Maciej H. Kotowski[1] C. Matthew Leister[2]

[1]John F. Kennedy School of Government
Harvard University

[2]Department of Economics
Monash University

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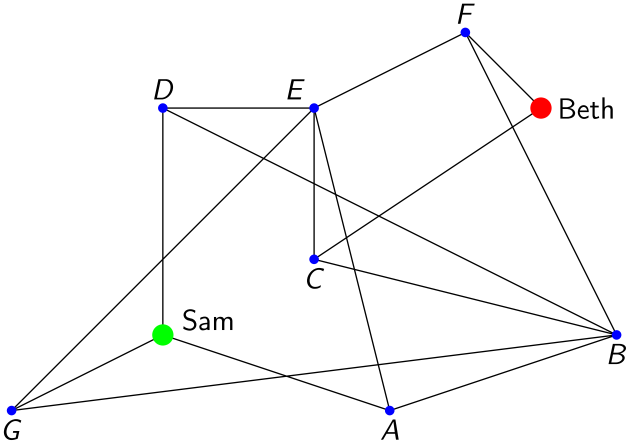
Intermediation

"Intermediation" is 25% of the U.S. Economy (Spulber 1996, JEP)

- Retail & Wholesale Trade
- Finance
- Other (Real Estate Brokers, Transport, . . .)

Trading Networks



Trading Networks

$$\text{Seller} \underbrace{\leftrightarrow \textit{Intermediary} \leftrightarrow \cdots \leftrightarrow \textit{Intermediary} \leftrightarrow}_{\text{We Study This Part of the Market}} \text{Buyer}$$

- Intermediaries have a network of relationships
- Intermediaries have different (private) costs of trade
- Intermediaries bid competitively to provide "intermediation services" that move goods from the seller to the buyer

Some Related Work

- Networks and exchange
  - Kranton & Minehart (2001)
  - Manea (2015)
  - Condorelli, Galeotti, Renou (2015)
- Middlemen
  - Rubinstein & Wolinsky (1987)
- Experiments
  - Gale & Kariv (2009)
- Many others cited in the paper.

Outline

1. Model $\begin{cases} \text{A tractable network structure} & \text{``Multipartite Networks''} \\ \\ \text{A tractable trading protocol} & \text{Second Price Auctions} \\ \\ \text{A tractable cost structure} & \text{Binary} \end{cases}$

Outline

1. Model
   - A tractable network structure — "Multipartite Networks"
   - A tractable trading protocol — Second Price Auctions
   - A tractable cost structure — Binary

2. Analysis
   - Stability — Network Persistence / "No Mergers"
   - Equilibrium — Network Formation / "Free Entry"

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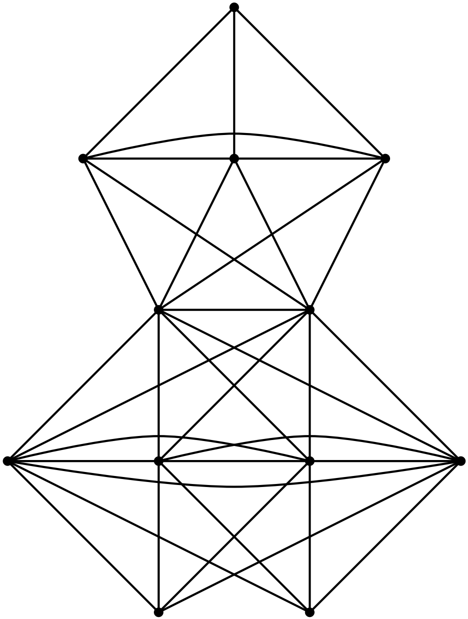
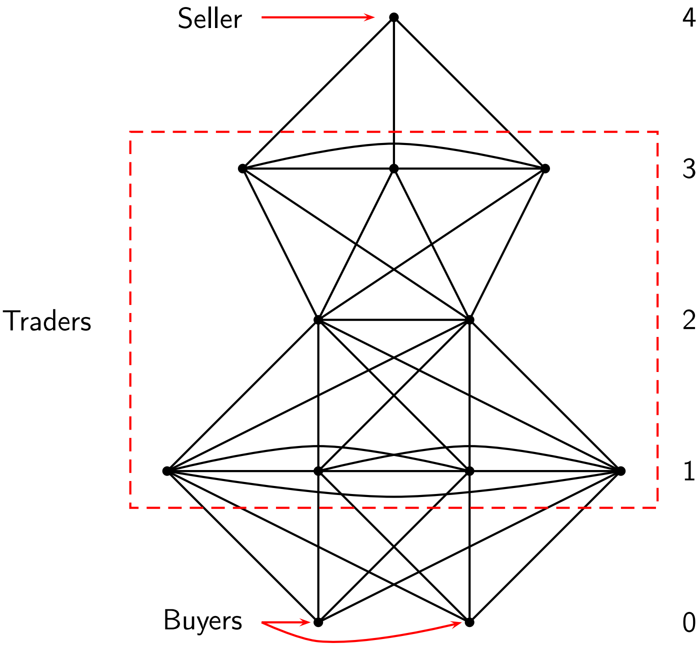
3. Conclusion
   - Stability + Equilibrium — Just an Example
   - Final Remarks




Seller
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Traders
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Buyers
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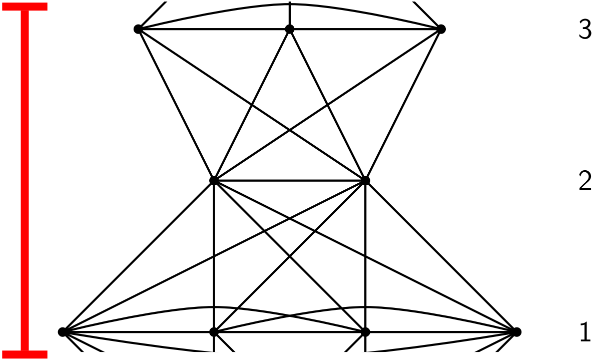
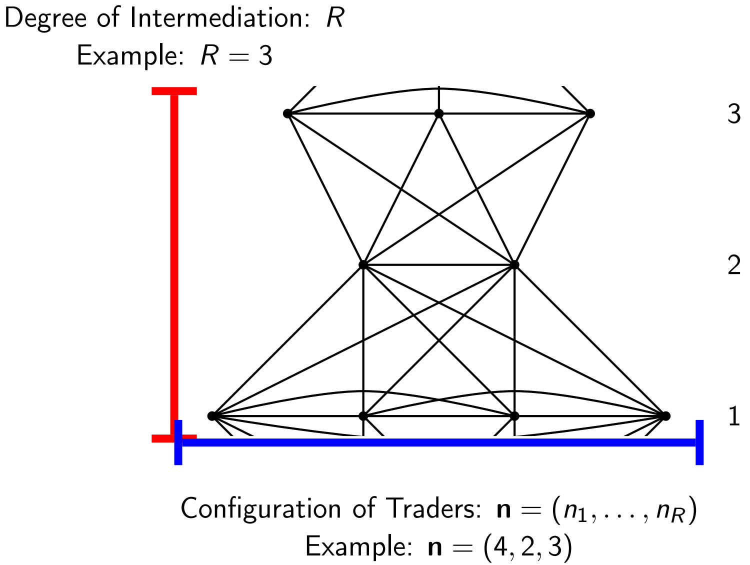
Degree of Intermediation: R

Example: $R = 3$




Degree of Intermediation: R
Example: $R = 3$
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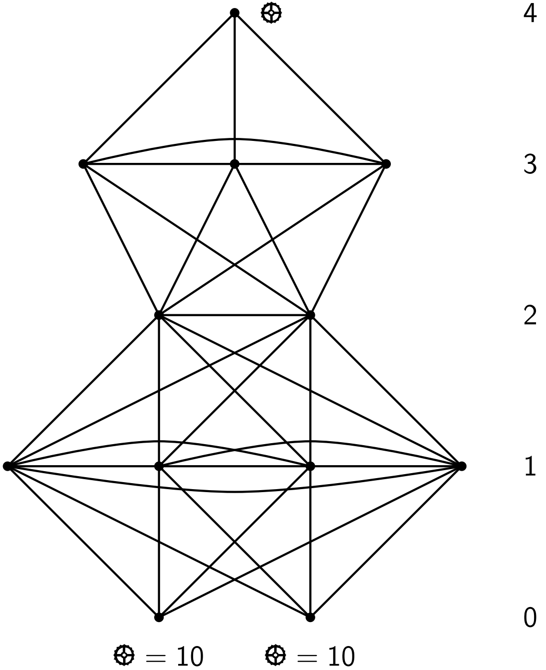
Configuration of Traders: $\mathbf{n} = (n_1, \ldots, n_R)$
Example: $\mathbf{n} = (4, 2, 3)$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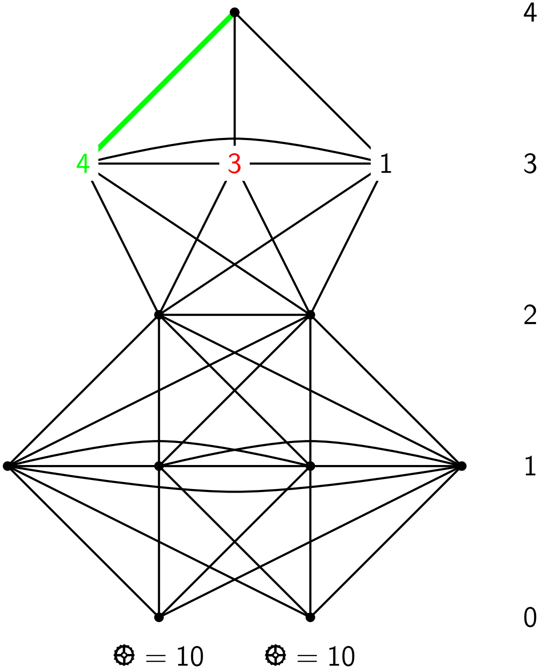
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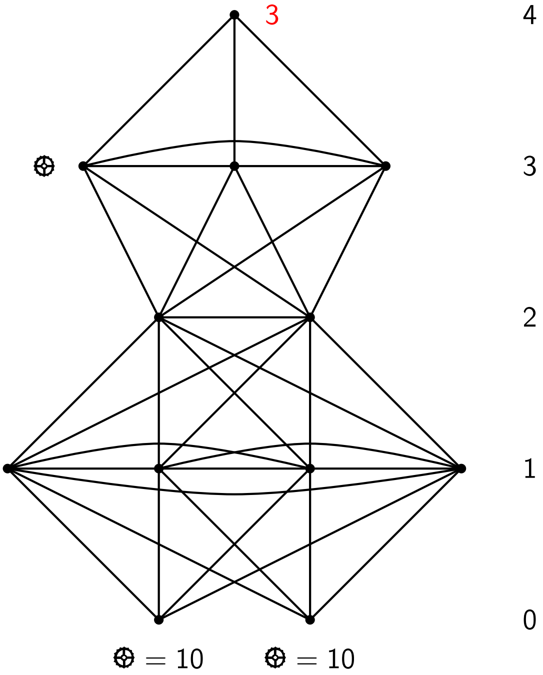
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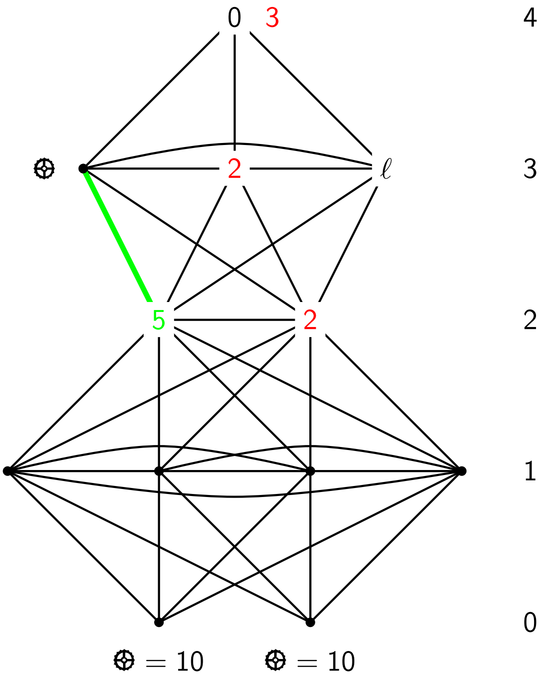

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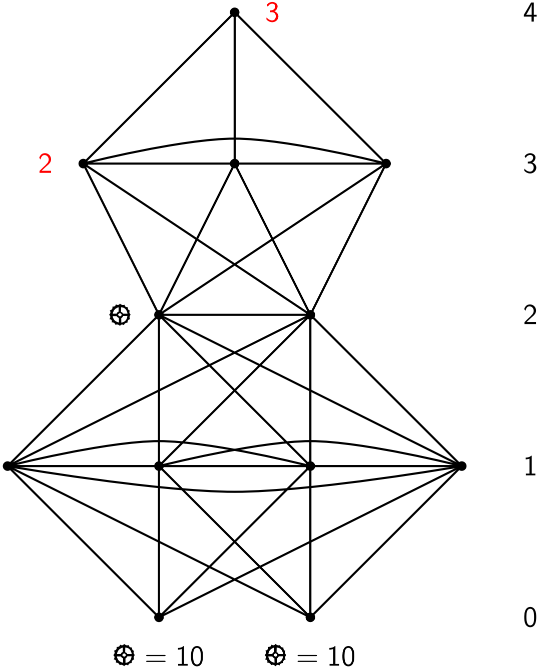

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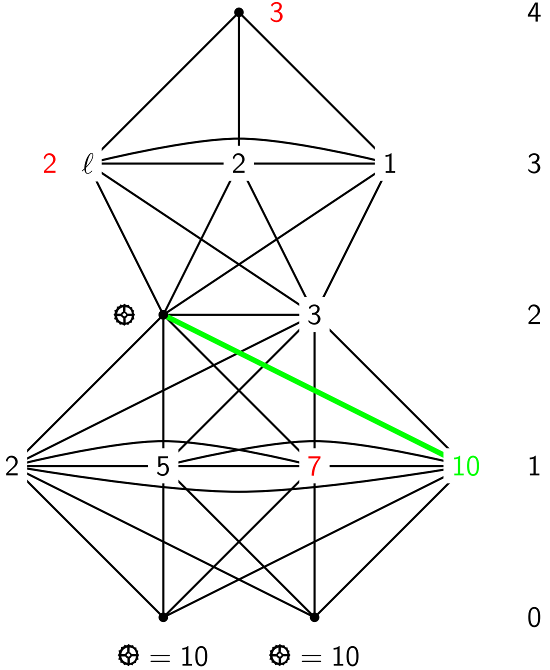

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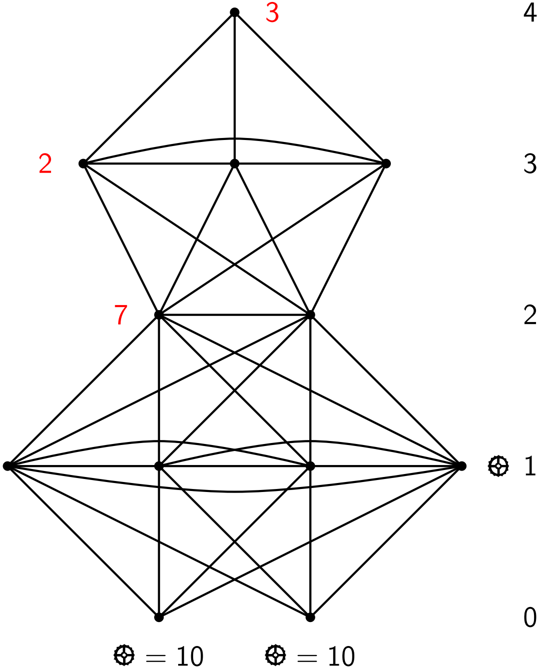

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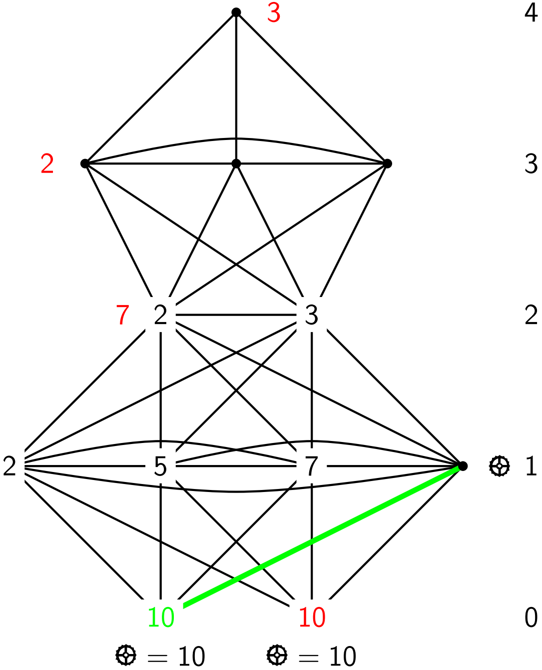

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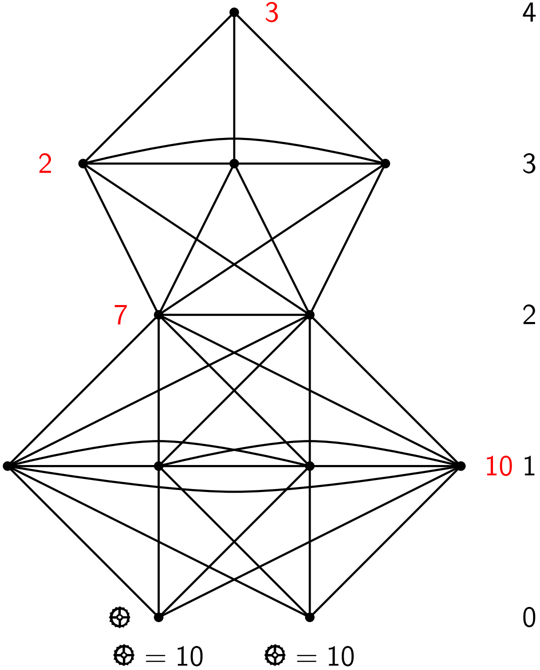

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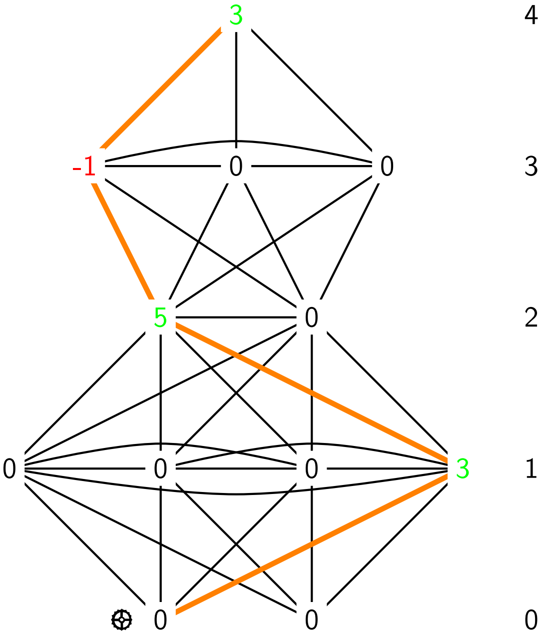

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Model: Odds and Ends

- Network structure common knowledge.
- Buyers' valuations are henceforth normalized to 1 and are common knowledge.
- Ties are broken at random.
- Trade "breaks down" if all bidders/traders bid "ℓ."

Model: Trading Costs

Each trader has a private trading (inventory cost) that he must incur when he receives the item.

- p — probability trading cost is 0.
- $1 - p$ — probability trading cost is $\bar{c} > 1$.

Distribution of trading costs is common knowledge. Realized trading costs are private information.

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Trader's Payoffs

(Re)sale Revenue - Purchase Costs - Trading Cost

Exchange in a Fixed Network

Theorem

There exists a perfect Bayesian equilibrium of the trading game where each agent i (in row r) adopts the following strategy:

1. *If the agent's costs are low and the asset is being sold by an agent in row $r+1$, the agent places a bid equal to the asset's expected resale value conditional on all available information and on others' strategies.*

2. *Otherwise, the agent bids ℓ.*

Buyers bid their value for the asset.

NB. Multiple second price auctions $\Longrightarrow$ Many other equilibria.

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- Asset does not backtrack or stall.
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$$\nu_1 = 1$$

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$$\nu_2 = \delta(n_1)$$

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$$\delta(n) := 1 - (1-p)^n - np(1-p)^{n-1}$$

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\nu_3 &= \delta(n_2)\delta(n_1) \\
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\nu_r &= \prod_{k=1}^{r-1} \delta(n_k) = \delta(n_{r-1})\nu_{r-1} \\
&\vdots \\
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Expected Payoffs

Ex ante expected trading profit of a row r trader given $\mathbf{n} = (n_1, \ldots, n_R)$:

$$\pi_r(\mathbf{n}) = \underbrace{\prod_{k=1}^{r-1} \delta(n_k)}_{[1]} \times \underbrace{p}_{[2]} \times \underbrace{(1-p)^{n_r - 1}}_{[3]} \times \underbrace{\prod_{k=r+1}^{R} \mu(n_k)}_{[4]}$$

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Fact: $\pi_r(n_r, \mathbf{n}_{-r})$ is decreasing in n_r and increasing in $\mathbf{n}_{-r}$.

- Traders in the same row are substitutes.
- Traders in others rows are complements.

Stability

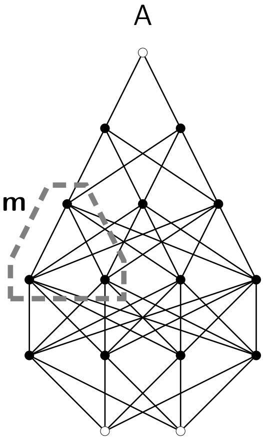
Persistence of a trading network is a puzzel.

Why? Adjacent traders have an incentive to merge or collude.

We call such deviations "partnerships."

In a stable market, traders should not deviate in this manner, i.e. the network is valuable.

A partnership is any group of adjacent traders that function as a single entity.



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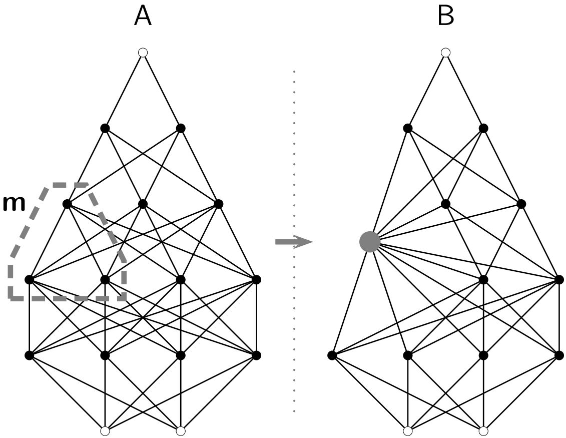

Partnerships

- Timing: A partnership forms conditional on $\mathbf{n}$ but before trading costs are realized.
- Once present, a partnership can trade just like any trader.
- Denote partnership membership by $\mathbf{m} = (m_1, \dots, m_R)$.

  Example: $\mathbf{m} = (0, 2, 1, 0)$

  - $\bar{m}$ — highest row with a partnership member.
  - $\underline{m}$ — lowest row with a partnership member.

Partnerships: Benefits and Costs

- Probability that partnership **m** has low trading cost:

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- Costs of partnership formation

$$\zeta(\mathbf{m}) = \underbrace{c_h \sum_{r=\underline{m}}^{\bar{m}} (m_r - 1)}_{[1]} + \underbrace{c_v \cdot (\bar{m} - \underline{m})}_{[2]}$$

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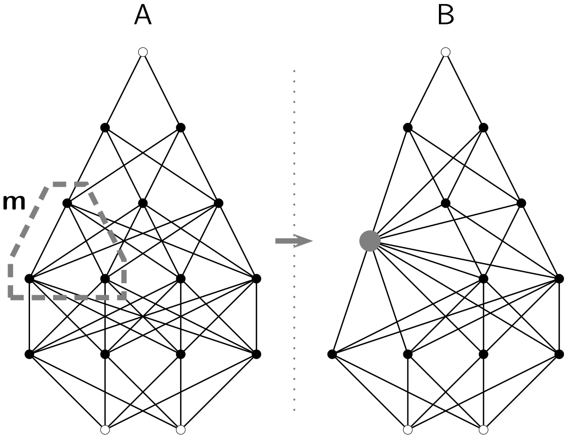
Exchange

The trading game can be analyzed as before, but a partnership enjoy direct and indirect advantages.



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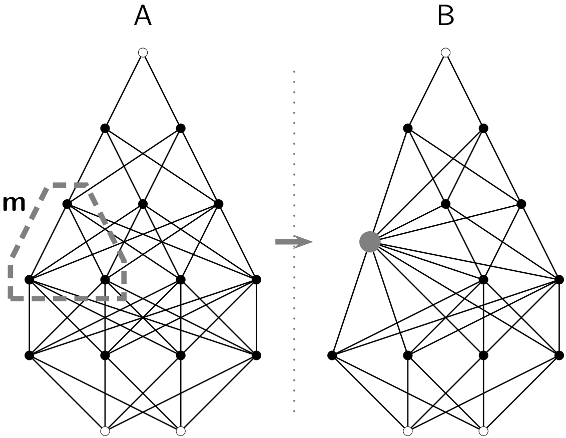

Stability

A trading network $\mathbf{n}$ is *stable* if for all feasible partnerships $\mathbf{m} \leq \mathbf{n}$,

$$\sum_r m_r \pi_r(\mathbf{n}) \geq \pi_{\mathbf{m}}(\mathbf{n}) - \zeta(\mathbf{m}).$$

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Theorem

If $c_h > 0$ and $c_v \geq 0$, then there exists a $\hat{p} > 0$ such that for all $p < \hat{p}$, the trading network is stable.

Equilibrium Networks

Our model of network formation.

1. R is fixed.
2. There is a large pool of potential traders.
3. A trader can enter any row at an entry cost of $\kappa > 0$.
4. Traders make entry decision before learning their cost-type.
5. Traders enter until expected profits are zero.*

Equilibrium

The network configuration $\mathbf{n}^* = (n_1^*, \ldots, n_R^*)$ is an *equilibrium configuration* if for all r,

$$\pi_r(\mathbf{n}^*) - \kappa \geq 0$$

and

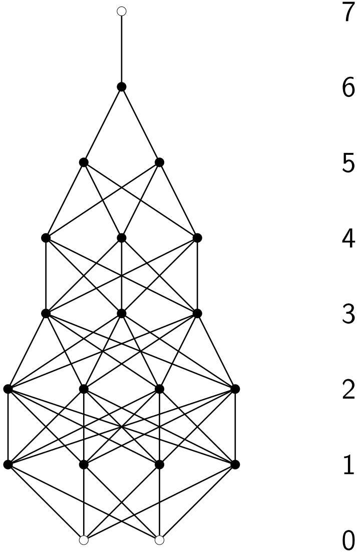
$$\pi_r(n_1^*, \ldots, n_{r-1}^*, n_r^* + 1, n_{r+1}^*, \ldots, n_R^*) - \kappa < 0.$$

See also Gary-Bobo (1990).

Existence and Example

- There exists a nontrivial equilibrium $\mathbf{n}^*$ iff there exists $\mathbf{n}$ such that for all r, $\pi_r(\mathbf{n}) - \kappa \geq 0$.
- If $\mathbf{n}^*$ is an equilibrium, $n_r^* \geq n_{r+1}^*$.
- Multiple equilibria may exist.
- Equilibria form a directed set.
  ($\mathbf{n}^* \geq \mathbf{n}^{**} \iff n_r^* \geq n_r^{**}$ for all r.)
- There exists a unique "maximal" equilibrium.

An example: $R = 6$, $p = 0.5$, $\kappa = 0.01$



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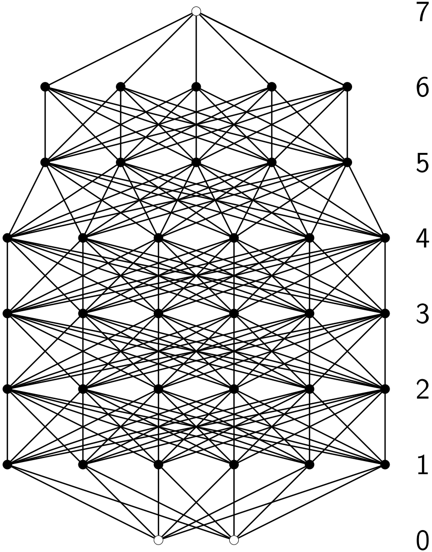

Welfare

Aggregate Welfare

$$\Omega(\mathbf{n}) = \underbrace{n_0\pi_0(\mathbf{n})}_{\text{Buyers' Payoffs}} + \underbrace{\sum_{r=1}^{R} n_r(\pi_r(\mathbf{n}) - \kappa)}_{\text{Traders' Payoffs}} + \underbrace{\pi_{R+1}(\mathbf{n})}_{\text{Seller's Payoff}} .$$

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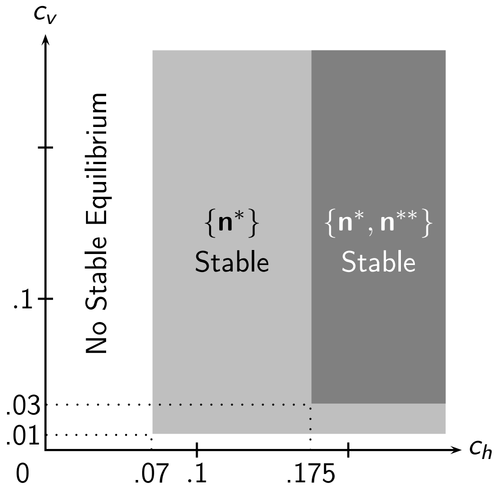
Theorem

If $\hat{\mathbf{n}}$ maximizes $\Omega(\mathbf{n})$, then $\hat{n}_r = \hat{n}_{r'}$ for all r and r'.
Moreover, if $\mathbf{n}^*$ is an equilibrium configuration, then $\hat{\mathbf{n}} \geq \mathbf{n}^*$.

(cf. Mankiw & Whinston 1986)

Stability and Equilibrium: An Example

If $R = 5$, $p = 1/2$, and $\kappa = 0.015$, there are two equilibrium configurations: $\mathbf{n}^* = (4, 3, 3, 2, 1)$ and $\mathbf{n}^{**} = (5, 5, 5, 5, 4)$.



Concluding Remarks

- Developed a tractable model of exchange in a network.
- Proposed definition of stability (no mergers) and equilibrium configurations (free entry).
  - (Network) Externalities $\Longrightarrow$ Multiple Equilibria.
  - A stability-efficiency tradeoff: A trading network may be stable, but improving efficiency may lead to instabilities.