Networks in Production: Asset Pricing Implications

Bernard Herskovic

UCLA Anderson

Third Economic Networks and Finance Conference
London School of Economics

December 2015
Introduction

- Input-output network and technology

- How are changes in the input-output network priced?

- Theory – general equilibrium model
  Network factors: priced sources of risk

- Data – new asset pricing factors
Introduction: input-output network
Introduction: concentration and sparsity

- **Concentration** (nodes/circles)
  - Large sectors – concentrated network
  - Output concentration
  - Decreases output

- **Sparsity** (edges/arrows)
  - Few thick arrows – sparse network
  - Input specialization
  - Increases output

(a) Low Concentration  
Low Sparsity

(b) High Concentration  
High Sparsity

(c) Low Concentration  
High Sparsity
Introduction: how are the network factors priced?

- **Concentration** innovations
  Decrease consumption growth and increase marginal utility
  *Negative* price of risk
  \[ \therefore \text{more exposure to concentration} \implies \text{lower returns} \]
  Return spread of $-4\%$ with similar FF/CAPM alpha

- **Sparsity** innovations
  Increase consumption growth and decrease marginal utility
  *Positive* price of risk
  \[ \therefore \text{more exposure to sparsity} \implies \text{higher returns} \]
  Return spread of $6\%$ with similar FF/CAPM alpha
Related Papers

- Multisector models, input-output and aggregation:
  - Long and Plosser (1983)
  - Acemoglu, Carvalho, Ozdaglar, and Tahbaz-Salehi (2012)

- Networks and asset pricing:
  - Ahern (2012)
  - Kelly, Lustig, and Van Nieuwerburgh (2012)

- Production-based asset pricing:
  - Papanikolaou (2011)
  - Loualiche (2012)
  - Kung and Schmid (2013)

- Sectoral composition risk:
  - Martin (2013)
  - Cochrane, Longstaff, and Santa-Clara (2008)
Multisector Model
Representative Household

- $n$ goods
- Epstein-Zin recursive preferences

\[ U_t = \left[ (1 - \beta) C_t^{1-\rho} + \beta \left( \mathbb{E}_t \left( U_{t+1}^{1-\gamma} \right) \right)^{\frac{1-\rho}{1-\gamma}} \right]^{\frac{1}{1-\rho}} \]

w/ Cobb-Douglas consumption aggregator: $C_t = \prod_{i=1}^n c_{i,t}^{\alpha_i}$

- Budget constraint

\[
\sum_{i=1}^n P_{i,t} c_{i,t} + \sum_{i=1}^n \varphi_{i,t+1} (V_{i,t} - D_{i,t}) = \sum_{i=1}^n \varphi_{i,t} V_{i,t}
\]

$V_{i,t}$ cum-dividend price of firm $i$

$\varphi_{i,t}$ share holding of firm $i$

$D_{i,t}$ dividend of firm $i$

$c_{i,t}$ consumption of good $i$
Firms

- $n$ firms and $n$ goods: firm $i$ produces good $i$
- $i$ buys inputs $\{y_{i1,t}, \ldots, y_{in,t}\}$ from other firms
- Final output $Y_{i,t}$: combination of inputs
- Maximization problem

$$
D_t = \max \{ y_{ij,t} \} \quad P_i,tY_{i,t} - \sum_{j=1}^{n} P_{j,t}y_{ij,t} \\
\text{s.t.} \quad Y_{i,t} = \varepsilon_{i,t}I_{i,t}^{\eta} \\
I_{i,t} = \prod_{j=1}^{n} w_{ij,t}y_{ij,t}
$$

$\eta < 1$ diminishing returns
$\varepsilon_{i,t}$ sector specific productivity
$w_{ij,t}$ network weight of firm $i$ on firm $j$
Network

\[ I_{i,t} = \prod_{j=1}^{n} y_{ij,t}^{w_{ij,t}} \]

\[ W_t = \begin{bmatrix} w_{11,t} & \cdots & w_{1n,t} \\ \vdots & \ddots & \vdots \\ w_{n1,t} & \cdots & w_{nn,t} \end{bmatrix}_{n \times n} \]

- **Network Weights**
  - \( w_{ij,t} \): fraction \( i \) spends on inputs from \( j \)
  - \( w_{ij,t} \): elasticity of \( I_{i,t} \) with respect to input \( j \)

- **Network Properties**

\[ \sum_{j=1}^{n} w_{ij,t} = 1 \quad \text{and} \quad w_{ij,t} \geq 0 \]

- **\( W_t \)**: exogenous, stochastic, arbitrary dynamics
Competitive Equilibrium

Definition
A competitive equilibrium consists of spot market prices \((P_1,t, \cdots, P_{n,t})\), value of the firms \((V_1,t, \cdots, V_{n,t})\), consumption bundle \((c_1,t, \cdots, c_{n,t})\), shares holdings \((\varphi_1,t, \cdots, \varphi_{n,t})\) and inputs bundles \((y_{ij,t})_{ij}\) such that

1. Given prices, household and firms maximize

2. Markets clear

\[
c_{i,t} + \sum_{j=1}^{n} y_{ji,t} = Y_{i,t} \quad \forall i, t \quad \text{(goods)}
\]

\[
\varphi_{i,t} = 1 \quad \forall i, t \quad \text{(assets)}
\]
Output Shares

- Output share of firm $i$

$$\delta_{i,t} = \frac{P_{i,t} Y_{i,t}}{\sum_{j=1}^{n} P_{j,t} Y_{j,t}}$$

- In equilibrium

$$\delta_{j,t} = (1 - \eta) \alpha_j + \eta \sum_{i=1}^{n} w_{ij,t} \delta_{i,t}$$

$$= (1 - \eta) \alpha_j + \eta \sum_{i=1}^{n} \alpha_i w_{ij,t} + \eta^2 \sum_{i=1}^{n} \sum_{k=1}^{n} \alpha_i w_{ik,t} w_{kj,t} + \ldots$$

- Feedback effects: decaying rate $\eta$
Theorem

- In equilibrium, consumption growth is given by

\[
\frac{1}{1 - \eta} \left[ (e_{t+1} - e_t) - (1 - \eta) (N_{t+1}^C - N_t^C) + \eta (N_{t+1}^S - N_t^S) \right]
\]

where

\[
e_{t} = \sum_{i=1}^{n} \delta_{i,t} \log \varepsilon_{i,t} \quad \text{(residual TFP)}
\]

\[
N_{t}^{C} = \sum_{i=1}^{n} \delta_{i,t} \log \delta_{i,t} \quad \text{(concentration)}
\]

\[
N_{t}^{S} = \sum_{i=1}^{n} \delta_{i,t} \sum_{j=1}^{n} w_{ij,t} \log w_{ij,t} \quad \text{(sparsity)}
\]

and \( \delta_{j,t} \) is the equilibrium output share of firm \( j \)

\[
\delta_{j,t} = (1 - \eta) \alpha_j + \eta \sum_{i=1}^{n} \alpha_i w_{ij,t} + \eta^2 \sum_{i=1}^{n} \sum_{k=1}^{n} \alpha_i w_{ik,t} w_{kj,t} + \ldots
\]
Network Concentration

\[ \mathcal{N}_t^C = \sum_{i=1}^{n} \delta_{i,t} \log \delta_{i,t} \]

- **Sectoral Output Concentration**
  - Min if \( \delta_{j,t} = \frac{1}{n} \) (equal shares)
  - Max if \( \delta_{s,t} = 1 \) and \( \delta_{j,t} = 0 \ \forall j \neq s \) (concentrated shares)

- Good news for consumption? No
  - Decreases consumption
  - Production relies on fewer sectors: diminishing returns
Network Sparsity

\[ N_{t}^{S} = \sum_{i} \delta_{i,t} \sum_{j} w_{ij,t} \log w_{ij,t} \equiv N_{i,t}^{S} \]

- High \( N_{i,t}^{S} \) \( \implies \) row \( i \) with few high entries (thick arrows)
- High \( N_{t}^{S} \) \( \implies \) sparse network

\[ W_{t} = \begin{bmatrix} w_{11,t} & \cdots & 0 & \cdots & w_{1n,t} \\ \vdots & \ddots & \vdots & & \vdots \\ w_{n1,t} & \cdots & 0 & \cdots & w_{nn,t} \end{bmatrix}_{n \times n} \]

- Dispersion of marginal product and output elasticities
- Gains from input specialization
Example: why does sparsity increase consumption?

- Firm $i$ has $k$ to buy inputs, what is the optimal output?
- $\varepsilon_j = 1$, $P_j = 1$ for every $j = 1, \ldots, n$

Scenario 1: high sparsity
  - $w_{ij} = 1$ for some $j$ and $w_{is} = 0$ for every $s \neq j$
  - $y_{ij} = k$ for some $j$ and $y_{is} = 0$ for every $s \neq j$

$$Y_i = k^n$$

Scenario 2: low sparsity
  - $w_{ij} = \frac{1}{n}$
  - $y_{ij} = \frac{k}{n}$

$$Y_i = \frac{k^n}{n^n}$$
Why Does Sparsity Increase Consumption?

- (Partial eq.) If $i$ spends $k$, then

$$y_{ij,t} = w_{ij,t} \frac{k}{P_{j,t}} \implies Y_{i,t} = \frac{\varepsilon_{i,t} \left( \prod_j w_{ij,t}^{w_{ij,t}} \right)^{\eta}}{\left( \prod_j P_{j,t}^{w_{ij,t}} \right)^{\eta}} k^{\eta}$$

- substitution of inputs: input specialization
- changes in marginal cost: different input bundle

- (General eq.) Sparsity increases output

$$\Delta \log \sum_i P_{i,t+1} Y_{i,t+1} = \frac{\eta}{1 - \eta} \Delta \sum_i \delta_{i,t+1} \log \prod_j w_{ij,t+1}^{w_{ij,t+1}}$$

- keeping network concentration constant
Data
Network Factors

Level, −0.34 correlation

Innovations, 0.06 correlation

Networks in Production: Asset Pricing Implications
Bernard Herskovic
Dec. 2015
Constructing Beta-Sorted Portfolios

1. CRSP monthly data: form annual returns for each stock
2. For each stock, regress excess returns on the factors’ innovations over a 15 year window:

\[ r_{t+1}^i - r_t^f = \alpha^i + \beta_{NS}^i \Delta N_{t+1}^S + \beta_{NC}^i \Delta N_{t+1}^C + \text{Controls} + \xi_t^i \]

- \( \beta_{NS}^i \) and \( \beta_{NC}^i \): exposure of stock \( i \) to factors’ innovations
- Sample: stocks with network data
- Controls: factors in level and orthogonalized TFP

3. Form portfolios sorted by \( \beta_{NS}^i \) and \( \beta_{NC}^i \) terciles
4. Compute subsequent year’s return for the sorted portfolio
5. Verify return spread
### Table: One Way Sorted Portfolios

#### Panel A: Sparsity

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(3)-(1)</th>
<th>t-stat</th>
</tr>
</thead>
<tbody>
<tr>
<td>Avg. Exc. Returns (%)</td>
<td>5.24</td>
<td>8.61</td>
<td>11.25</td>
<td>6.01</td>
<td>2.26</td>
</tr>
<tr>
<td>$\alpha_{CAPM}$</td>
<td>-3.15</td>
<td>2.29</td>
<td>4.78</td>
<td>7.92</td>
<td>3.11</td>
</tr>
<tr>
<td>$\alpha_{FF}$</td>
<td>-3.21</td>
<td>1.47</td>
<td>3.84</td>
<td>7.04</td>
<td>2.91</td>
</tr>
<tr>
<td>Volatility (%)</td>
<td>17.60</td>
<td>13.78</td>
<td>15.13</td>
<td>11.60</td>
<td>-</td>
</tr>
<tr>
<td>Book/Market</td>
<td>0.76</td>
<td>0.67</td>
<td>0.70</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Avg. Market Value ($bn)</td>
<td>1.53</td>
<td>2.18</td>
<td>1.23</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

#### Panel B: Concentration

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(3)-(1)</th>
<th>t-stat</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_{CAPM}$</td>
<td>2.62</td>
<td>2.43</td>
<td>-1.60</td>
<td>-4.21</td>
<td>-2.26</td>
</tr>
<tr>
<td>$\alpha_{FF}$</td>
<td>2.00</td>
<td>1.64</td>
<td>-2.00</td>
<td>-4.01</td>
<td>-2.12</td>
</tr>
<tr>
<td>Volatility (%)</td>
<td>16.18</td>
<td>13.60</td>
<td>16.27</td>
<td>8.05</td>
<td>-</td>
</tr>
<tr>
<td>Book/Market</td>
<td>0.74</td>
<td>0.69</td>
<td>0.70</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Avg. Market Value ($bn)</td>
<td>0.91</td>
<td>2.03</td>
<td>2.00</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>
Why do sectors have different network betas?

- Dividend growth:

\[ D_{i,t} = (1 - \eta)\delta_{i,t}z_t \implies \Delta d_{i,t+1} = \Delta \log \delta_{i,t+1} + \Delta \log z_{t+1}. \]

- Cross-sectional heterogeneity: changes in output shares

- Concentration beta
  
  Network centrality / size

- Sparsity beta

\[ N_t^S \equiv \sum_{i=1}^{n} \delta_{i,t} \sum_{j=1}^{n} w_{ij,t} \log w_{ij,t} = \sum_{j=1}^{n} \sum_{i=1}^{n} \delta_{i,t} w_{ij,t} \log w_{ij,t} \]

\[ \text{out-sparsity of sector } j \]
Concluding Remarks

- New production-based asset pricing factors
  - Network sparsity and concentration
- Sources of aggregate risk
- Sparsity-beta sorted portfolios
  - 6% return spread per year on avg
- Concentration-beta sorted portfolios
  - -4% return spread per year on avg
- Spreads not explained by CAPM or Fama French factors
- Calibrated model replicates return spreads
Annex
Firms

Maximization problem

\[ D_t = \max \{ y_{ij,t} \} \cdot I_{i,t} \quad P_{i,t} Y_{i,t} - \sum_{j=1}^{n} P_{j,t} y_{ij,t} \]

s.t. \quad Y_{i,t} = \varepsilon_{i,t} I_{i,t}^{\eta} L_{i,t}^{1-\eta}

\[ I_{i,t} = \prod_{j=1}^{n} w_{ij,t} \]

- \( \eta < 1 \) diminishing returns
- \( \varepsilon_{i,t} \) sector specific productivity
- \( w_{ij,t} \) network weight of firm \( i \) on firm \( j \)
- \( L_{i,t} = 1 \)
Robustness: sorted portfolios

Table: Return Spreads

<table>
<thead>
<tr>
<th></th>
<th>Sparsity-beta sort</th>
<th>Concentration-beta sort</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(3)-(1) t-stat</td>
<td>(3)-(1) t-stat</td>
</tr>
<tr>
<td>Benchmark</td>
<td>6.01 2.26</td>
<td>-4.04 -2.19</td>
</tr>
<tr>
<td>No level control</td>
<td>4.47 1.90</td>
<td>-3.50 -1.55</td>
</tr>
<tr>
<td>All CRSP stocks</td>
<td>5.78 2.17</td>
<td>-3.83 -2.13</td>
</tr>
<tr>
<td>Out of Sample</td>
<td>0.31 0.14</td>
<td>-3.25 -1.61</td>
</tr>
<tr>
<td>R. TFP Cons.</td>
<td>6.03 2.09</td>
<td>-3.42 -1.64</td>
</tr>
<tr>
<td>No TFP</td>
<td>5.49 1.92</td>
<td>-4.89 -2.51</td>
</tr>
<tr>
<td>16-year window</td>
<td>5.51 1.92</td>
<td>-5.35 -2.73</td>
</tr>
<tr>
<td>17-year window</td>
<td>4.91 1.46</td>
<td>-6.00 -2.52</td>
</tr>
<tr>
<td>18-year window</td>
<td>4.57 1.22</td>
<td>-5.15 -2.19</td>
</tr>
<tr>
<td>19-year window</td>
<td>8.54 2.02</td>
<td>-5.93 -2.45</td>
</tr>
<tr>
<td>20-year window</td>
<td>6.48 1.73</td>
<td>-3.60 -1.63</td>
</tr>
</tbody>
</table>