Quantifying Contagion Risk in Funding Markets: A Model-Based Stress-Testing Approach

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“Bad news”

- The subprime crisis was put in motion on Aug 9th, 2007
  - BNP Paribas announced it had suspended withdrawals from three investment funds exposed to U.S. subprime mortgages

- News triggered general market anxiety about the extent of other banks’ exposures to sub-prime mortgages and solvency
  - Exacerbated by the opacity of banks’ balance sheets

- Funding conditions deteriorated for all banks
“Good news”

- Flip side – good news can have a positive market impact

- The Supervisory Capital Assessment Program (SCAP)
  - Stress-tests conducted by the Federal Reserve on U.S. banks
  - First conducted in 2009 – midst of the crisis
  - Yielded credible results for prospective losses for banks
  - Helped restore confidence in the banking system
Information contagion and stress testing

- **Information contagion** – key driver in financial crises

- Modeling / quantifying contagion is crucial for **stress testing**
  - Identify vulnerabilities within financial systems
  - Support crisis management and resolution

- We present a new model-based stress-testing framework
  - Banks’ solvency risks, funding liquidity risks and market risks are intertwined due to information contagion
  - Frictions – coordination failure and asymmetric information
Outline of Presentation

Motivation

Overview

Model

Equilibrium

Stress testing

Conclusion
Related literature

- Chen (1999) – Heterogenous information amongst depositors are responsible for runs

- Acharya and Yorulmazer (2008) – Ex-post information contagion leads to ex-ante herding, with banks undertaking correlated investments

- Li and Ma (2013) – Most similar to our paper; coordination failure and adverse selection mutually reinforce each other, leading to bank runs and fire-sales

- Many models of stress-testing, e.g., Elsinger et al. (2006), Alessandri et al. (2009), and Gauthier et al. (2012)
Overview
Our model

- Solvency risk – exogenous macroeconomic shock

- Funding liquidity risks
  - Endogenous runs – global games (Morris and Shin, 2009)
  - Coordination failures between a bank’s creditors
Our model

- **Market risks**
  - Pro-cyclical collateral haircuts
  
  - Macro-economy = \[
    \begin{cases}
      \text{“Good”} & \rightarrow \text{low haircuts} \\
      \text{“Bad”} & \rightarrow \text{large haircuts}
    \end{cases}
  \]

- Investors entertain prior beliefs on the macro-economy

- Bank failure $\rightarrow$ Beliefs updated $\rightarrow$ “Bad" state more probable
Our results

- **Vicious illiquidity:** Investors’ pessimism over the macro-economy hampers the bank’s recourse to liquidity
  - Influences the incidence of bank runs
  - Investors turn more pessimistic
  - Driving down other banks’ recourse to liquidity

- **Virtuous liquidity:** Investors’ are optimistic to start with
  - Banks are more likely to survive solvency shocks
  - Investors turn more optimistic over asset quality
  - Other banks’ recourse to liquidity improves
Our results

- **Price and Spread:** An increase in the haircut-spread heightens the illiquidity channel
  - Larger spread $\rightarrow$ greater uncertainty over asset quality
  - Investors are more inclined to believe that banks fail because their assets are low quality than high quality

- **Convergence:** For a system of $N \geq 2$ banks, a unique equilibrium is always reached after, at most, $N$ iterations
  - Simple induction argument
MODEL
Agents and environment

- Three dates $t = 0, 1, 2$, and no time discounting
  - Map to an annual time-horizon

- $N = 2$ leveraged financial institutions or banks, $b \in \{1, 2\}$

- Two groups of risk-neutral agents
  - Creditors – unit endowments; can consume in $t = 1$ or $t = 2$
  - Investors – deep-pocketed; consume at $t = 2$

- Interim date $t = 1$ is divided into two rounds
Balance sheet in period 2

Risky Investments
\[ Y^b - S_1^b - S_2^b \]

Short-term Debt
\[ ST^b \]

Long-term Debt
\[ LT^b \]

Liquid Assets
\[ M^b \]

Capital
\[ E^b - S_1^b - S_2^b \]
Asset side

- $Y^b$ – value of risky investments in period 2
- $S^b_1$ – semi-annual loss in period 1
  - Support – $[S^b_1, \bar{S}^b_1]$; pdf – $f^b_1(S)$; cdf – $F^b_1(S)$
- $S^b_2$ – semi-annual loss in period 2
  - Support – $[S^b_2, \bar{S}^b_2]$; pdf – $f^b_2(S)$; cdf – $F^b_2(S)$
- $M^b$ – amount of liquid assets from period 0
Liability side

- $ST^b$ – rolled-over short-term debt
- $LT^b$ – long-term debt to be repaid
- $E^b$ – CET1 capital + income earned - dividends paid
Balance sheet in period 2

- Bank $b$ is insolvent in period 2 whenever $E^b - S_1^b - S_2^b < 0$

- Insolvency can also be triggered in period 1 due to illiquidity
Recourse to liquidity in period 1 (round 1)

- Banks repo risky assets with investors for liquidity
  - Reversed in period 2

- **Pro-cyclical haircuts**: depend on the macro-economy
  - “Good” \( (m = 1) \) – small haircut; \( \psi_H < 1 \) of liquidity
  - “Bad” \( (m = 0) \) – large haircut; only \( \psi_L < \psi_H \) of liquidity
Recourse to liquidity in period 1 (round 1)

- State $m$ realized in period 1
  - Investors do not know $m$, and cannot observe credit shocks
  - Prior belief for round 1: $w_1 = \text{Prob}(m = 1)$

- Bank $b$’s recourse to liquidity is

$$M^b + \left\{ w_1 \psi_H + (1 - w_1)\psi_L \right\} (Y - S^b_1)$$

$$= \psi^1$$
Rollover risk in period 1 (round 1)

- The decisions of bank $b$’s creditors to demand payment at round 1 modeled as a binary-action simultaneous move game

<table>
<thead>
<tr>
<th></th>
<th>Solvent</th>
<th>Insolvent</th>
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<tbody>
<tr>
<td>Not to withdraw</td>
<td>$1 + r^b$</td>
<td>0</td>
</tr>
<tr>
<td>Withdraw</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

- If a fraction $\ell^b_1 \in [0, 1]$ creditors withdraw, bank $b$ is illiquid if

$$\ell^b_1 > \lambda^b \left( S^b_1; \bar{\psi}^1 \right) \equiv \frac{M^b + \bar{\psi}^1 [Y^b - S^b_1]}{ST^b}$$

- We refer to $\lambda^b$ as the balance sheet liquidity for bank $b$
Rollover risk in period 1 (round 2)

- Indicator $\eta_b^1 \in \{0, 1\}$ for the outcome of bank $b$ after round 1

- End of round 1, bank $b$ is either
  \[
  \begin{cases}
    \text{liquid} & \Rightarrow \eta_b^1 = 0 \\
    \text{illiquid} & \Rightarrow \eta_b^1 = 1
  \end{cases}
  \]

- Investors update their belief $w_2 = \text{Prob}(m = 1|\eta_1^1, \eta_2^1)$
Rollover risk in period 1 (round 2)

- Change to liquid bank(s) recourse to liquidity ("margin call")
  \[ \overline{\psi}^2 = w_2 \psi_H + (1 - w_2) \psi_L \]

- Creditors of liquid bank(s) decide to withdraw in round 2
  - Payoffs same as in round 1

- If a fraction \( \ell_2^b \in [0, 1] \) of creditors from (liquid) bank \( b \) withdraw, then bank \( b \) is illiquid if
  \[ \ell_2^b > \lambda^b \left( S_1^b; \overline{\psi}^2 \right) \]
## Model timeline

<table>
<thead>
<tr>
<th>$t = 0$</th>
<th>$t = 1$ (round 1)</th>
<th>$t = 1$ (round 2)</th>
<th>$t = 2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2. Interim shock</td>
<td>2. “Margin calls”</td>
<td>2. Incomes accrued</td>
<td></td>
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</tbody>
</table>
EQUILIBRIUM
Global games framework

- Solve for the Bayes-Nash equilibrium in each round
  - Creditors of bank $b$ receive a noisy signal on $S^b$
  - The noise is i.i.d across creditors and rounds

- **Unique equilibrium in threshold strategies** for each bank $b$ in round $d$, in the limit of vanishing private noise:
  - If $S^b > S^b_d$, all creditors withdraw and bank $b$ is illiquid
  - If $S^b \leq S^b_d$, no creditor withdraws and bank $b$ remains liquid

- Closed-form analytical expressions for investors’ beliefs
Virtuous liquidity

If both banks are liquid at the end of round 1, then \( w_2 > w_1 \). Consequently, both banks remain liquid at the end of round 2.
Suppose bank $i$ is liquid and bank $j$ is illiquid after round 1. The investors become more pessimistic, $w_2 < w_1$, whenever:

$$\frac{\text{Prob}(\eta_i^1 = 0 | m = 1)}{\text{Prob}(\eta_i^1 = 0 | m = 0)} < \frac{\text{Prob}(\eta_j^1 = 1 | m = 0)}{\text{Prob}(\eta_j^1 = 1 | m = 1)}.$$ 

If the downward revision of the belief is large enough, then bank $i$ will also become illiquid at the end of round 2.
Price and spread effects

For a given initial belief, $w^1$, and “bad” state haircut, $\psi_L$, an increase in the “good” state haircut, $\psi_H$, increases the spread, $\Delta = \psi_H - \psi_L$. This, in turn, strengthens the pessimism condition and increases the range of parameters where the investor’s belief is revised downwards.

On the other hand, for a given “good” state haircut, $\psi_H$, an increase in the “bad”, $\psi_L$, leads to a decrease in the spread. This weakens the pessimism condition and reduces the range of parameters where the investor’s belief is revised downwards.
In a game involving \( N \geq 2 \) banks, the cycles of Bayesian updating by investors and withdrawal by creditors terminates after, at most, \( N \) rounds.
STRESS TESTING
Macro Stress Tests in Canada

- Annual exercise conducted jointly by the BoC and OSFI involving Canadian D-SIBS

- **Objective**: Assess the resilience of the financial system to extreme but plausible shocks

- MST scenario development

- Bottom-up exercise
  - Banks apply MST scenario to their balance sheets
  - Focus on solvency risk only

- Top-down exercise
  - MFRAF
The MFRAF: Structure

Solvency risk module
Macroeconomic and financial shocks materialize.
Banks suffer losses due to credit risk and market risk.

Liquidity risk module
Creditors have concerns over banks' funding strategies and solvency.
Creditors withdraw their claims on banks.

Systemic risk module
Contagion between investors' beliefs and creditors' withdrawals and interbank spillovers.
System-wide losses distribution.
The MFRAF: Structure

- Stress scenario
  - Corporate and household defaults
  - Credit losses for banks
  - Banks' capital changes
  - Income Generated

- Banks default
  - Banks' creditors review losses
    - Creditors rollover
    - Creditors withdraw
      - Creditors of solvent banks update their beliefs on fire-sales
      - Bank defaults due to illiquidity

- Interbank network
- Aggregate loss distribution
The MFRAF: Calibration

- Macroeconomic scenario draws on Canada’s 2013 FSAP

- 6 Canadian D-SIBs’ balance sheet – 2013Q1
  - Average CET1 ratio – 8.9%
  - Liabilities maturity within 6 months – 35% of all liabilities

- Front-load income onto bank’s capital

- “Insolvency” if capital falls below 7% CAR

- Losses = credit shock + bankruptcy cost (10% RWA) + \((\psi_H - \bar{\psi}) \times \text{illiquid assets} \) (for illiquid banks)

- Baseline – assume identical balance sheets for all banks
The MFRAF: Results

- Average balance sheet liquidity – 1.08

<table>
<thead>
<tr>
<th>Bank</th>
<th>Solvency</th>
<th>Liquidity</th>
<th>Contagion</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>47.0</td>
<td>22.9</td>
<td>0.0</td>
<td>69.9</td>
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<tr>
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<td>0.0</td>
<td>0.0</td>
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</tr>
<tr>
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<tr>
<td>4</td>
<td>47.0</td>
<td>0.0</td>
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<td>66.2</td>
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<tr>
<td>5</td>
<td>47.0</td>
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<td>0.0</td>
<td>47.0</td>
</tr>
<tr>
<td>6</td>
<td>47.0</td>
<td>22.2</td>
<td>0.8</td>
<td>70.0</td>
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The MFRAF: Results

![Diagram showing the relationship between losses/total assets and liquidity/contagion/solvency risks.](image)

- **Contagion**
- **Liquidity**
- **Solvency**

The graph illustrates the impact of losses on total assets, highlighting the risks associated with contagion, liquidity, and solvency.
The MFRAF: Results

- Lower BSLs for banks 2 and 5

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Conclusion

- MFRAF is a top-down stress testing tool that investigates the interactions between solvency and liquidity risk

- Results depend starting capital ratios and balance sheets

- Uses in policy
  - Consistency check for bottom-up results
  - Considers impact of second-round effects over and above the (solvency only) bottom-up stress-test
  - Quantifies liquidity assistance required to avoid runs

- Next steps – macro-feedbacks, and endogenous haircuts, would be nice to have!

Thank you!


