Financial Linkages, Portfolio Choice and Systemic Risk

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Motivation

- Financial linkages reflect cross-ownership and borrowing between banks and corporations.
- Linkages can smoothen the shocks and uncertainties faced by individual components of the system. But they also create a wedge between ownership and control on the other hand.
- We wish to understand how the empirically observed core-periphery networks mediate this agency problem:
  1. does deeper financial integration reduce volatility and raise welfare?
  2. what are the properties of an ideal financial network?
The model

- **Two ingredients:**

- **Contribution:**
  1. Relationship: Network topology, risk taking and welfare
  2. Optimal design of networks
• **Existing work:** More extensive ties are beneficial for individuals.

• However, the empirical evidence is mixed. Greater international integration sometimes increases volatility at the individual country level Kose et al (2009), Obstfeld and Rogoff (1996).

• **Our theory:** greater integration leads to greater volatility in returns as well as greater expected returns.

• Welfare consequences depend on network: goes up in homogenous networks but may fall in asymmetric and heterogenous networks (core-periphery network).

• **Our work:** origin of the shocks – the investments in risky assets – is endogenous. Complementary to complementary to the existing body of work.
The Model

- $N = \{1, 2, \ldots n\}$ agents (firms, financial institutions, households)
- Agent $i$ with endowment $w_i$, invests in a project with sure return $r$ and in a risky project $i$ with return $z_i \sim \mathcal{N}(\mu_i, \sigma_i^2)$, $\mu_i > r$.
- Returns of projects are independent.
- Let $\beta_i \in [0, w]$ be agent $i$’s risky investment.
- $\beta = \{\beta_1, \ldots, \beta_n\}$ is the investment profile.
The Financial Network: Ownership

- A network of cross-holdings; $n \times n$ matrix $S$, with $s_{ii} = 0$, $s_{ij} \geq 0$ and $\sum_{j \in \mathcal{N}} s_{ji} < 1$ for all $i \in \mathcal{N}$.

- Let $D$ be a $n \times n$ diagonal matrix, in which the $i^{th}$ diagonal element is $1 - \sum_{j \in \mathcal{N}} s_{ji}$.

- Define $\Gamma = D[I - S]^{-1}$.

- Interpret $\gamma_{ij}$ as $i$’s ownership of $j$. 

$$
\gamma_{ij} = \left[ 1 - \sum_{j \in \mathcal{N}} s_{ji} \right] \left[ 0 + s_{ij} + \sum_{k} s_{ik}s_{kj} + \ldots \right].
$$
Example: sectors

Figure: ownership $\gamma = 0.20$, $\gamma_{\text{across}} = 0.10$, $\gamma_{\text{within}} = 0.133$
Value, Utility and Choice

- The expected returns to individual $i$
  \[ W_i = \beta_i z_i + (w_i - \beta_i) r \]  
  (1)

- The economic value of individual $i$ is
  \[ V_i = \sum_j \gamma_{ij} W_j. \]  
  (2)

- Individuals seek to maximize a mean-variance utility function.
  \[ U_i(\beta_i, \beta_{-i}) = E[V_i(\beta)] - \frac{\alpha}{2} \text{Var}[V_i(\beta)]. \]
We begin by characterizing optimal agent investments. Observe that cross partial derivatives are zero. So:

\[
\beta^*_i = \arg \max_{\beta_i \in [0, w_i]} \gamma_{ii} [w_i r + \beta_i (\mu_i - r)] - \frac{\alpha}{2} \gamma_{ii}^2 \beta_i^2 \sigma_i^2.
\]

If agent \( i \) has no cross-holdings then \( \gamma_{ii} = 1 \) and:

\[
\hat{\beta}_i = \frac{\mu_i - r}{\alpha \sigma_i^2}.
\]

\( \hat{\beta}_i \) is agent \( i \)'s autarchy investment.
Proposition: Optimal Portfolio Choice

Optimal investment of individual $i$ is:

$$
\beta_i^* = \min \left\{ w_i, \frac{\hat{\beta}_i}{\gamma_{ii}} \right\}.
$$

(3)

- Remark: Investment in risky asset is inversely related to self ownership.
- Agency problem: individual $i$ optimizes the mean-variance utility of $\gamma_{ii} W_i$, not of $W_i$. 
Mean, variance and correlations

- Expected value and variance for individual are:

\[
E[V_i] = r \sum_{j \in \mathcal{N}} \gamma_{ij} w_j + \sum_{j \in \mathcal{N}} \hat{\beta}_j (\mu_j - r) \frac{\gamma_{ij}}{\gamma_{jj}} \\
\text{Var}[V_i] = \sum_{j \in \mathcal{N}} \hat{\beta}_j^2 \sigma_j^2 \left( \frac{\gamma_{ij}}{\gamma_{jj}} \right)^2,
\]

- More ownership of individuals with low self-ownership: greater expected value and variance.

- The covariance between \( V_i \) and \( V_j \) is:

\[
\text{Cov}(V_i, V_j) = \sum_{l \in \mathcal{N}} \hat{\beta}_l^2 \sigma_i^2 \gamma_{il} \gamma_{jl} \frac{\gamma_{il}}{\gamma_{ll}}.
\]

- **Systemic risk:** covariance between \( V_i \) and \( V_j \) is higher with common ownership of low self-ownership individuals.
Correlations across Sectors

Figure: $\beta_i = 0.32$; correlation within 0.48; correlation across 0.41
Integration and Diversification: General Observations

• Financial interconnections have deepened over last 3 decades. Kose et al (2006), Lane and Milesi-Ferretti (2003).

• Traditional argument
  • Individuals invest in risky assets that have independent returns: deeper or more extensive linkages should lower variance of earnings. Since individuals are risk averse this raises overall utility.

• Our result:
  • Greater linkages encourage more risk taking. Improve welfare in symmetric networks but lower welfare in asymmetric networks such as a core-periphery network.
Integration and Diversification

• For a vector \( s_i = \{s_{i1}, \ldots, s_{in}\} \) define the variance of \( s_i \) as
  \[ \sigma_{s_i}^2 = \sum_j (s_{ij} - \eta_{i}^{out}/(n - 1))^2. \]

• Integration All links are stronger, some strictly so.

• Diversification Variance of out-going links is smaller for every node.
Integration

Agent A
Diversification
Core-periphery Network: motivation

- Ownership of transnational corporations: Vitali et al. (2011): a giant bow-tie structure, a large portion of control flows to a small tightly-knit core of financial institutions.
- International financial flows: McKinsey Global Institute (2014). Core constituted of United States and Western Europe, rest of the world comprising the periphery (links with the core countries).
Core-periphery Network
Core-periphery network: description

- There are $n_p$ peripheral agents and $n_c$ central agents, $n_p + n_c = n$; $i_c$ and $i_p$ refer to the (generic) central and peripheral agent.
- A link between two central agents has strength $s_{i_ci_c} = s$, and a link between a central and a peripheral agent $s_{i_ci_p} = s_{i_pi_c} = \hat{s}$, and there are no other links.
Special Case I: Complete Network

- Every (ordered) pair of agents has a directed link of strength $s$.
- The ownership matrix $\Gamma$ in a complete network is
  \[ \gamma_{ij} = \frac{s}{s+1} \quad \text{and} \quad \gamma_{ii} = 1 - (n-1)\gamma_{ij}. \]
- Greater $s$ lowers self-ownership: all agents raise their risky investments.
- Expected value $E[V_i]$ and variance $Var[V_i]$ increase in $s$.
- Expected utility of each agent is increasing in $s$. 
Special Case II: Star Network

- The self-ownerships of central and peripheral agents are, respectively:

\[
\gamma_{i_c i_c} = \frac{1 - n_p \hat{s}}{1 - n_p \hat{s}^2}
\]

and

\[
\gamma_{i_p i_p} = \frac{[1 - \hat{s}][1 - \hat{s}^2(n_p - 1)]}{1 - n_p \hat{s}^2}
\]
Proposition: integration and volatility

- Suppose $\sigma_i^2 = \sigma^2$, $\mu_i = \mu$ and $w_i = w$ is large for all $i$. All links equal, $\hat{s} \in [0, 1/(n-1)]$.

1. The central agent makes larger investments in the risky asset relative to the other agents. An increase in $\hat{s}$ increases the investment in the risky asset of each agent.

2. There exists $0 < \underline{s} < \bar{s} < 1/n_p$ so that an increase in $\hat{s}$ increases aggregate utilities if $\hat{s} < \underline{s}$ and it decreases aggregate utilities if $\hat{s} > \bar{s}$. 
Thought experiment: changes in core-periphery network

- We change strength of ties in our network and study effects
- Example: $n_c = 4$, $n_p = 10$, $\sigma = 0.4$, $\alpha = 0.5$, $\mu = 2$, $r = 1$, $w = 700$, $s = 0.1$,
- Vary strength of core-periphery tie: $\hat{s} = \{0...0.065\}$. 
Core-periphery Network
Integration in Core-periphery Network

**Investments**

- Investment core
- Investment periphery

**Utilities**

- Utility core
- Utility periphery

**Aggregate utility**

- Welfare

**Systemic Risk**

- Systemic risk
Normative analysis

- What is the welfare maximizing investment for a given network?
- How does it differ from what individuals do: what are the externalities?
- What is the optimal design of financial networks?
Welfare Maximizing Investments

• The ‘planner’ seeks to maximize aggregate utilities:

\[ W^P(\beta, S) = \sum_{i \in N} E[V_i] - \frac{\alpha}{2} \sum_{i \in N} Var[V_i]. \]  \hspace{1cm} (4)

• For a given \( S \), the planner chooses investments in risky assets, \( \beta^P = \{\beta_1^P, \beta_2^P, \ldots, \beta_n^P\} \), to maximize (4).
The optimal investment of the social planner in risky project $i = 1, \ldots, n$ is given by

$$\beta_i^P = \min \left[ w_i, \frac{1}{\sum_{j \in N} \gamma_{ji}^2 \hat{\beta}_i} \right]. \quad (5)$$
Externalities

• Compare marginal utility of increasing $\beta_i$ for agent $i$, with the marginal utility of planner. We have:

$$\frac{\partial U_i}{\partial \beta_i} = (\mu_i - r)\gamma_{ii} - \alpha\sigma_i^2 \beta_i \gamma_{ii}^2,$$

$$\frac{\partial W(S)}{\partial \beta_i} = (\mu_i - r) - \alpha\sigma_i^2 \beta_i \sum_{j \in N} \gamma_{ji}^2.$$

• Agent underestimates the impact of his investment on the aggregated expected value and on sum of variances.
Proposition: Individual vs collective optimum

Assume that $w_i$ is large for all $i \in \mathcal{N}$. Agent $i$ over-invests as compared to the planner, $\beta_i > \beta_i^P$ if, and only if,

$$\gamma_{ii} < \sum_{j \in \mathcal{N}} \gamma_{ji}^2.$$
Consider interior solutions.

- The first best network design is the complete network with maximum link strength $s_{ij} = \frac{1}{n - 1}$ for all $i \neq j$.
- The second best network design is the complete network with link strength

$$s_{ij} = \frac{1}{n - 1} \frac{\alpha - \phi}{\alpha}, \text{ for all } i \neq j.$$
First best: intuitions

- We first derive the optimal $\Gamma$, and then we derive the network $S$ that induces the optimal $\Gamma$.
- Homogeneous networks dominate heterogeneous networks: this is because agents are risk-averse, and concentrated and unequal ownership exacerbates the costs of variance.
- This leads to a preference for homogeneous networks: networks where, for every $i$, $\gamma_{ji} = \gamma_{j'i}$ for all $j,j' \neq i$.
- In the first-best, within homogeneous networks, stronger links are better, as they allow for greater smoothing of shocks, and this is welfare-improving due to agents’ risk aversion.
Second best: Intuitions

• Within homogeneous network, the designer has to choose between networks in which agents have high self-ownership (and, therefore, make large investments in the risky asset) versus low self-ownership (when they take little risk).

• When the social planner is utilitarian, $\phi = 0$, the optimal network is invariant: $s_{ij} = 1/n - 1$ for all $i, j$ both in the first-best and the second-best case.

• If social planner cares about correlation across agents, then the larger the weight placed on systemic risk, the greater the aversion to correlations in agents’ values. second best network is less integrated than the optimal network in the first-best scenario.
Discussion: Ownership and control

• Suppose that $\gamma_{ij}$ signifies that agent $i$ has control over $\gamma_{ij}$ fraction of agent $j$’s initial endowment $w_j$. So $\gamma_{ij}w_j$ is a transfer from $j$ to $i$ that occurs before shocks are realized. Therefore, $\Gamma$ redefines the agents’ initial endowments. No network effects, due to absence of income effects.

• Control is ‘local’: agent $i$ can invest $w\gamma_{ij}$ in the risk-free asset and in the risky project of agent $j$. Individually optimal investment levels are independent of network, and choices mimic those of a central planner with mean-variance preferences over aggregate returns $V = \sum_i V_i$. 
Discussion: Correlated returns

- In basic model, any form of correlation across agents’ economic value is driven by the architecture of the cross-holdings network. The assumption that projects are uncorrelated allows us to isolate the effects of cross-holdings on risk-taking behavior and aggregate outcomes.
- We extend the model to allow for correlations across assets.
- Existence and sufficient conditions for uniqueness of interior equilibrium.
- We then show, via examples, that asymmetric networks may lead to over-investment in risky assets, as in the case of uncorrelated projects.
Summary

- Financial networks reflect cross-ownership across corporations, short term borrowing and lending among banks.
- Financial linkages smoothen the shocks and uncertainties faced by individual components, but they also give rise to an agency problem: there is a wedge between ownership and control.
- We develop a framework of endogenous risk taking by decision makers connected via financial obligations. It formalizes a basic agency problem: decision makers do not internalize entirely the consequence of risk taking.
The standard argument on benefits of pooling risk is valid when the network is homogenous. When the ownership of some agents is concentrated, the agency problem becomes salient. Greater integration and diversification may lead to excessive risk taking and volatility; result in lower welfare.

Optimal networks are homogenous and dense.