Asset Management Contracts and Equilibrium Prices

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Motivation

- Most wealth invested in financial markets is managed professionally.
  - Individual investors held only 21.5% of US stocks in 2007 (French 2008).
- Agency problem between investors and asset managers.
- Questions:
  - What are optimal contracts between investors and asset managers?
  - What are the implications for managers' portfolio choices?
  - What are the implications for equilibrium asset prices?

This Paper – Optimal Contracts

- Static model of optimal contracting between investors and managers.
  - Moral hazard arising from managers' effort to acquire information.
  - Adverse selection arising from:
    - Information that managers acquire.
    - Managers' preferences.

- Result 1: Optimal contract involves **risk limits**.
  - Risk of managers' portfolio is kept within bounds.
  - Optimal portfolio given managers' private information may exceed the bounds.

- Intuition:
  - Investors pay managers a high fee for high return to induce information acquisition by skilled manager types.
  - This induces unskilled manager types to gamble for the high fee.

This Paper – Equilibrium Asset Prices

- Embed contracting model into dynamic asset-pricing model.
  - One riskless and multiple risky assets.
  - Random demand by noise traders $\rightarrow$ Mispricing.
  - Skilled manager types can observe noise-trader demand.

- Result 2: Risk limits generate **risk-return inversion**.
  - Overvalued assets have high volatility.

- Result 3: Risk limits generate **overvaluation bias**.
  - Overvalued assets become more overvalued and undervalued assets become more undervalued.
  - Effect on overvalued assets dominates, biasing aggregate market upward.

Risk-Return Inversion

- Overvalued assets have high volatility because of an amplification effect.
  - Positive news to asset fundamentals
  - $\rightarrow$ Managers' positions become larger and risk limits become more binding
  - $\rightarrow$ Managers cut down on their positions
  - $\rightarrow$ Managers *buy* overvalued assets (since they short/underweigh them to begin with).

- Amplification effect concerns distortions during bubbles rather than crises.
  - BIS (2003): "... Overvalued assets/stocks tend to find their way into major indices, which are generally capitalization-weighted and therefore will more likely include overvalued securities than under-valued securities. Asset managers may therefore need to buy these assets even if they regard them as overvalued; otherwise they risk violating agreed tracking errors..."
  - IMF (2015): "... Another source of friction capable of amplifying bubbles stems from the captive buying of securities in momentum-biased market capitalization-weighted benchmarks. Underlying constituents that rise most in price will see their benchmark weights increase irrespective of fundamentals, inducing additional purchases from fund managers seeking to minimize benchmark tracking error..."

Overvaluation Bias

- Risk limits exacerbate mispricings in both directions because they prevent managers from absorbing noise-trader demand.
- Effect on overvalued assets dominates because risk limits are more likely to bind for those assets.
  - Overvalued assets have higher share price and volatility than undervalued assets.
  - $\rightarrow$ Risk limits are more likely to bind for a short position in an overvalued asset than for a long position of an equal number of shares in an undervalued asset.
- <u>Common theme:</u> Corrective forces in asset markets are weaker during bubbles than during crises.

Static Contracting Model

Model

- Two periods, 0 and 1.
- Riskless rate is zero.
- One risky asset.
  - Price S in period 0.
  - Payoff $S + d$ or $S - d$ in period 1. Prior probability of $S + d$ is $\frac{1}{2}$.
- Investor:
  - Risk-averse with utility $-\exp^{-\rho W}$.
  - Can invest in the risky asset by employing manager.
- Manager can be of two types:
  - Risk-averse with utility $-\exp^{-\bar{\rho}\bar{W}}$. Can observe signal about asset payoff.
    - Signal costs K to observe and yields posterior probability $\pi \in [1 - \bar{\pi}, \bar{\pi}]$ for $S + d$. π is distributed symmetrically around $\frac{1}{2}$ with density $h(\pi)$.
  - Risk-neutral and cannot observe signal. Probability λ.
- Extend contracting model to asymmetric distributions.
  - Required for equilibrium asset pricing model.

Contracting

- Investor pays fee $f(W)$ to manager.
- $f(W)$ general function of W, except for:
  - Limited liability: $f(W) \geq 0$.
  - Monotonicity:
    - $f(W_1) - f(W_2) \geq \epsilon(W_1 - W_2)$ for all (W_1, W_2) that can arise in equilibrium, where $\epsilon > 0$. Take limit $\epsilon \to 0$.
    - Manager cannot gain by reducing W.
- Investor chooses $f(W)$ to maximize utility.
- Incentive compatibility constraints for manager:
  - Whether or not to observe signal.
  - Which position z in the risky asset to choose.

Incentive Compatibility: Position Choice

- Risk-averse type:
  - Position $z(\pi)$ when posterior probability for $S+d$ is π.
  - Symmetry $\rightarrow z(\pi) = z(1-\pi)$.
  - Incentive compatibility $\rightarrow$ Fee difference $\Delta(\pi) \equiv f(z(\pi)d) - f(-z(\pi)d)$ is non-decreasing.
  - Monotonicity $\rightarrow z(\pi)$ is non-decreasing.
- Risk-neutral type:
  - Symmetry $\rightarrow$ Indifferent between position $\hat{z} \geq 0$ and $-\hat{z}$.
  - Investor ensures $\hat{z} \leq z(\bar{\pi})$. (Can set $f(W) = 0$ for $W > z(\bar{\pi})d$.)
  - Monotonicity $\rightarrow \hat{z} < z(\bar{\pi})$ only if $\hat{\Delta} \equiv f(\hat{z}d) - f(-\hat{z}d) < \Delta(\bar{\pi})$.
    - Uninformed risk-neutral type chooses a less variable fee profile than most informed risk-averse types.

Incentive Compatibility: Information Acquisition

- Manager observes signal only if $\Delta(\bar{\pi})$ exceeds a bound, which increases in K.
  - Investor pays manager a high fee for high return to induce information acquisition.
- For K sufficiently large, $\hat{z} = z(\bar{\pi})$.
  - High fee for high return attracts risk-neutral type.
  - Uninformed risk-neutral type is pooled with most informed risk-averse types.

Optimal Contract

- Risk-neutral type is pooled with interval of risk-averse types.
  - Maximum long position is chosen by risk-neutral type and interval $[\pi^*, \bar{\pi}]$ of most optimistic risk-averse types.
  - Maximum short position is chosen by risk-neutral type and interval $[1-\bar{\pi}, 1-\pi^*]$ of most pessimistic risk-averse types.

- Pooling threshold π^* is unique solution in $[\pi^*, \bar{\pi}]$ of

$$\underbrace{2(1-\lambda)\int_{\pi^*}^{\bar{\pi}}(\pi-\pi^*)h(\pi)d\pi}_{\substack{\text{Cost of tighter}\\ \text{risk limit}}} = \underbrace{\lambda\left(\pi^*-\frac{1}{2}\right)}_{\substack{\text{Benefit of tighter}\\ \text{risk limit}}}.$$

- Optimal positions:
  - Separation for $\pi \in [\frac{1}{2}, \pi^*)$:

$$z(\pi) = \frac{1}{2\rho d}\log\left(\frac{\pi}{1-\pi}\right) + \frac{\Delta(\pi)}{2d}.$$

  - Pooling for $\pi \in [\pi^*, \bar{\pi}]$:

$$z(\bar{\pi}) = \frac{1}{2\rho d}\log\left(\frac{\pi^*}{1-\pi^*}\right) + \frac{\Delta(\bar{\pi})}{2d}.$$

Special Case

- Manager's risk-aversion coefficient $\bar{\rho}$ is much larger than investor's.
  - Fee $f(W)$ is small relative to investor's wealth W.
- Uncertainty d is small. Probability π is $\frac{1}{2} + \mu d$.
  - Embed model in continuous time.
- Equivalence to reduced-form model without manager.
  - With probability $1 - \lambda$: Investor optimizes knowing μ, but with risk limit $|z| \leq \frac{\mu^*}{\rho}$. When risk limit does not bind, optimal position is $z = \frac{\mu}{\rho}$.
  - With probability λ: Investor randomizes between L and $-L$.

Dynamic Asset-Pricing Model

Model

- Continuous time $t \in [0, \infty)$.
- Riskless asset, exogenous return r.
- Risky asset.
  - Dividend flow D_t follows squared-root process

$$dD_t = \kappa \left(\bar{D} - D_t \right) dt + \sigma \sqrt{D_t} dB_t$$

    where $(\kappa, \bar{D}, \sigma)$ are positive and dB_t is Brownian motion.
    - Dividends are always positive.
    - Volatility of dividend per share increases with dividend level. (Important)
  - Endogenous price S_t.
  - Supply of θ shares. Can result from the asset issuer or from noise traders.
    - $\theta < 0$: Demand from noise traders exceeds supply from asset issuer.

Investors

- Overlapping generations living over infinitesimal periods.
- Continuum with measure 1.
- Utility $-\exp(-\rho dW_t)$. Equivalent to mean-variance objective

$$\mathbb{E}_t(dW_t) - \frac{\rho}{2}\mathbb{V}\mathrm{ar}_t(dW_t)$$

- Expert investors:
  - Observe (θ, D_t) and invest on their own.
  - Measure $1 - x \in (0, 1]$.
- Non-expert investors:
  - Do not observe (θ, D_t) and can only invest through managers. Previous model and reduced form.
    - Adapt previous model to asymmetric distributions for π.
    - Modify beliefs of risk-neutral types so that their positions cancel out in equilibrium.

Equilibrium – Non-Binding Risk Limit

- All investors choose same position in risky asset. $z_{1t} = z_{2t} = z_t$ shares.

- First-order condition:

$$\mathbb{E}_t(dR_t^{sh}) = \rho z_t \mathbb{V}\mathrm{ar}_t(dR_t^{sh}),$$

  where $dR_t^{sh} \equiv D_t dt + dS_t - rS_t dt$ is asset's return per share.

- Using market-clearing $(1-x)z_{1t} + x(1-\lambda)z_{2t} = \theta$ and Ito's lemma on $S_t = S(D_t)$, write FOC as

$$D_t + \kappa(\bar{D} - D_t)S'(D_t) + \frac{1}{2}\sigma^2 D_t S''(D_t) - rS(D_t) = \frac{\rho\theta}{1-\lambda x}\sigma^2 D_t S'(D_t)^2.$$

Equilibrium – Binding Risk Limit

- Position z_{1t} of experts satisfies FOC.
- Position z_{2t} of non-experts meets risk limit

$$\sqrt{\frac{\mathbb{V}\mathrm{ar}(z_{2t} dR_t^{sh})}{dt}} = |z_{2t}| \sigma \sqrt{D_t} S'(D_t) \leq \frac{\mu^*}{\rho}$$

  with equality.

- Previous calculation yields

$$D_t + \kappa(\bar{D} - D_t) S'(D_t) + \frac{1}{2}\sigma^2 D_t S''(D_t) - rS(D_t) = \rho z_{1t} \sigma^2 D_t S'(D_t)^2.$$

- Using market-clearing $(1-x) z_{1t} + x(1-\lambda) z_{2t} = \theta$, rewrite as

$$\begin{aligned} D_t + \kappa(\bar{D} - D_t) S'(D_t) + \frac{1}{2}\sigma^2 D_t S''(D_t) - rS(D_t) &= \frac{\rho\theta}{1-x}\sigma^2 D_t S'(D_t)^2 \\ &\quad - \frac{\mathrm{sign}(\theta)(1-\lambda) x \mu^*}{1-x} \sigma \sqrt{D_t} S'(D_t). \end{aligned}$$

- Two non-linear second-order ODEs, with free boundary. Smooth-pasting at boundary.

ODE Solution – No Risk Limit

- Proposition: If $\mu^* = \infty$ (suboptimal contract), then $S(D_t) = a_0 + a_1 D_t$ (affine price), with

$$a_0 = \frac{\kappa}{r} a_1 \bar{D},$$
$$a_1 = \frac{2}{r + \kappa + \sqrt{(r+\kappa)^2 + 4\frac{\rho\theta}{1-\lambda x}\sigma^2}}.$$

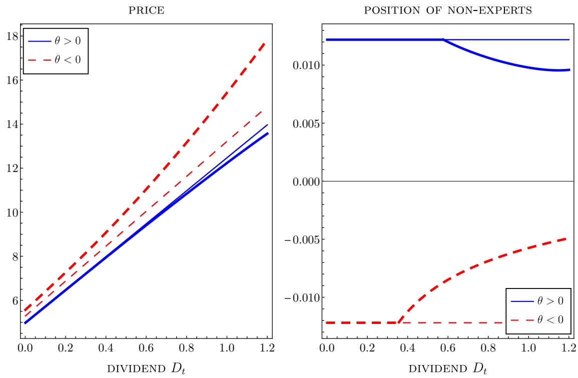
- Price S_t decreases in supply θ.
  - Low-supply assets trade at high price (overvalued).
  - High-supply assets trade at low price (undervalued).

- Volatility $\sqrt{\mathbb{V}\mathrm{ar}_t(dR_t)} = \sqrt{\mathbb{V}\mathrm{ar}_t(\frac{dS_t}{S_t})}$ is independent of θ.

ODE Solution – Risk Limit

- Theorem: If $\mu^* < \infty$ (optimal contract), then
  - $S(D_t)$ is convex and lies above affine solution for $\theta < 0$.
  - $S(D_t)$ is concave and lies below affine solution for $\theta > 0$.
- Comparison of solutions $\rightarrow$ Risk limits exacerbate mispricings in both directions.
- Convexity $\rightarrow$ Amplification.
- Proposition: Volatility $\sqrt{\mathbb{V}\mathrm{ar}_t(dR_t)}$ is larger for $\theta < 0$ than for $\theta > 0$.
  - Overvaluation and high volatility go together.

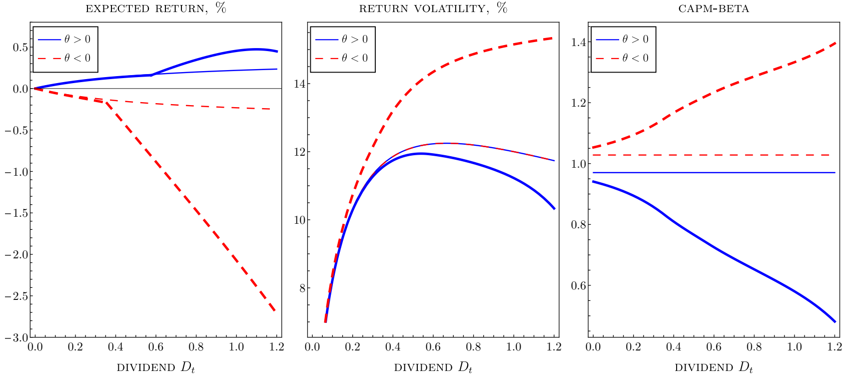
Numerical Example: Prices and Positions



$r = 0.03$, $\kappa = 0.1$, $\bar{D} = 0.2$, $\sigma = 0.2$, $\theta \in \{0.01, -0.01\}$, $\rho = 1$, $x = 0.9$, $\lambda = 0.2$.

- Risk limits exacerbate mispricing.
- Risk limits have a larger effect on prices and positions when $\theta < 0$ than when $\theta > 0$.

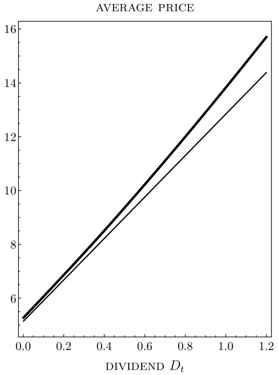
Numerical Example: Return Moments



$r = 0.03$, $\kappa = 0.1$, $\bar{D} = 0.2$, $\sigma = 0.2$, $\theta \in \{0.01, -0.01\}$, $\rho = 1$, $x = 0.9$, $\lambda = 0.2$.

- Overvaluation and high volatility go together.
- Same result when replace volatility by CAPM beta.
  - Compute CAPM beta in multi-asset extension of the model.
  - Assets have independent payoffs and managers specialize in different assets.

Numerical Example: Aggregate Market



$r = 0.03$, $\kappa = 0.1$, $\bar{D} = 0.2$, $\sigma = 0.2$, $\theta \in \{0.01, -0.01\}$, $\rho = 1$, $x = 0.9$, $\lambda = 0.2$.

- Risk limits raise average price.

Extensions and Conclusion

Benchmarks

- Replace risk limit

$$|z_{2t}|\sigma\sqrt{D_t}S'(D_t) \leq \frac{\mu^*}{\rho}$$

by

$$|z_{2t} - \eta|\sigma\sqrt{D_t}S'(D_t) \leq \frac{\mu^*}{\rho}.$$

- Bound concerns volatility of position relative to *benchmark* position.
- Results carry through identical provided that all comparisons between θ and zero are replaced by ones between θ and η.
- Can contracting model be extended to derive risk limit relative to benchmark?

Concluding Remarks

- Joint determination of asset management contracts and equilibrium prices.
- Contracting results: Optimal contract involves risk limits.
- Asset-pricing results:
  - Risk limits generate risk-return inversion.
  - Risk limits generate overvaluation bias.
- Future research: Normative and policy implications.
  - How do privately optimal and socially optimal risk limits compare?
  - Should asset management contracts be designed differently?