Systemic Risk and Central Clearing Counterparty Design

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What this paper is about

- Examine effects of central clearing counterparty (CCP) on a financial network from ex post and ex ante (systemic risk measure) perspective
- Propose CCP design with “hybrid” guarantee fund that is netted against liabilities
- Simple enough for exact analysis of trade off between systemic risk reduction and banks’ incentive to join CCP
- Sophisticated enough to capture real world orders of magnitude of capital, guarantee funds, and fees (stylised CDS OTC market data BIS 2010)
Main findings

- **Ex post**: CCP reduces banks’ liquidation and shortfall losses, improves aggregate surplus
- **Ex ante**: find explicit threshold on CCP capital and guarantee fund for systemic risk reduction
- Design of “hybrid” guarantee fund netted against liabilities is superior to (“pure” guarantee) default fund plus margin fund
  - hybrid implies similar systemic risk
  - hybrid gives much larger banks’ incentive compatibility
Outline

1. Financial network
2. Central counterparty clearing
3. Ex post effects of central counterparty clearing
4. Systemic risk and incentive compatibility
5. Simulation study
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Setup

- Two periods $t = 0, 1, 2$
- Values at $t = 1, 2$ are random variables on $(\Omega, \mathcal{F})$
- $m$ interlinked banks $i = 1 \ldots m$
Instruments

Bank $i$ holds

- Cash $\gamma_i$: zero return
- External asset (e.g. long-term investment maturing at $t = 2$):
  - fundamental value $Q_i$ at $t = 1, 2$
  - liquidation value $P_i < Q_i$ at $t = 1$
- Interbank liabilities:
  - formation at $t = 0$
  - realization/expiration at $t = 1$: $L_{ij}$
- No external debt

Example of interbank liabilities: CDS (premiums paid before $t = 0$. At $t = 1$ change in credit spreads or defaults)
Interbank liabilities realize at $t = 1$

- $L_{ij}(\omega)$ cash-amount bank $i$ owes bank $j$
- $L_i = \sum_{j=1}^{m} L_{ij}$ total nominal liabilities of bank $i$
- $\sum_{j=1}^{m} L_{ji}$ total nominal receivables from other banks (assets)
Financial network

Bank $i$’s nominal balance sheet at $t = 1$

- **Assets**
  \[ \gamma_i + \sum_{j=1}^{m} L_{ji} + Q_i \]

- **Liabilities**
  \[ L_i + \text{nominal net worth} \]

- **Nominal cash balance**
  \[ \gamma_i + \sum_{j=1}^{m} L_{ji} - L_i \]
Liquidation of external asset at $t = 1$

- If bank $i$’s cash balance is negative,
  \[ \gamma_i + \sum_{j=1}^{m} L_{ji} < L_i, \]
  it sells external assets at liquidation price $P_i < Q_i$.
- Bank $i$ is bankrupt if
  \[ \gamma_i + \sum_{j=1}^{m} L_{ji} + P_i < L_i, \]
  liquidation value of assets
  and then bank $j$ receives a part of liquidation value of bank $i$’s assets.
Interbank liability clearing equilibrium defined as \( (L_{ij}^*) \) satisfying

1. **Fair allocation:**
   \[ 0 \leq L_{ij}^* \leq L_{ij} \]

2. **Clearing:**
   \[ L_i^* = \sum_{j=1}^{m} L_{ij}^* \]
   satisfies
   \[ L_i^* = L_i \wedge \left( \gamma_i + \sum_{j=1}^{m} L_{ji}^* + P_i \right), \quad i = 1 \ldots m \]

**Assumption:** Let \( (L_{ij}^*) \) be an interbank liability clearing equilibrium
Example of interbank clearing equilibrium

Eisenberg and Noe (2001): proportionality rule $\Pi_{ij} = L_{ij}/L_i$ and

$$L_{ij}^* = \Pi_{ij} L_i^*$$

with clearing vector $L^* = (L_1^*, \ldots, L_m^*)$ determined as fixed point

$$\Phi(L^*) = L^*$$

where $\Phi : [0, L] \rightarrow [0, L]$ is given by

$$\Phi_i(\ell) = L_i \wedge \left( \gamma_i + \sum_{j=1}^m \ell_j \Pi_{ji} + P_i \right), \ i = 1 \ldots m$$

Eisenberg and Noe (2001): If $\gamma_i + P_i > 0$ for all $i$ then there exists a unique interbank clearing equilibrium.
Bank $i$’s terminal net worth at $t = 2$

- Fraction of liquidated external asset
  \[ Z_i = \left( \frac{L_i - \gamma_i - \sum_{j=1}^{m} L_{ji}^*}{P_i} \right)^+ \wedge 1 \]

- Assets
  \[ A_i = \gamma_i + \sum_{j=1}^{m} L_{ji}^* + Z_i P_i + (1 - Z_i) Q_i \]

- Net worth
  \[ C_i = A_i - L_i \]
Bankruptcy characterization

- Shortfall of bank $i$ equals
  \[ C_i^- = L_i - L_i^* \]
- Bank $i$ is bankrupt if and only if
  \[ C_i < 0 \quad \text{(or} \quad L_i^* < L_i) \]
- If bank $i$ is bankrupt then all its external assets are liquidated
  \[ Z_i = 1 \]
Lemma: The aggregate surplus depends on interbank liabilities only through implied liquidation losses:

$$\sum_{i=1}^{m} C_i^+ = \sum_{i=1}^{m} \gamma_i + \sum_{i=1}^{m} Q_i - \sum_{i=1}^{m} Z_i(Q_i - P_i).$$

→ Forced liquidation of external assets lowers aggregate surplus.
→ Absent external asset, cash gets only redistributed in network. No dead weight losses.
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Central Clearing Counterparty (CCP)

- We label the CCP as \( i = 0 \)
- All liabilities are cleared through the CCP
  - star shaped network
- Proportionality rule: CCP liabilities have equal seniority
  - interbank clearing equilibrium is trivial (no fixed point problem)
The CCP is endowed with
- external equity capital $\gamma_0$
- guarantee fund
  \[ \sum_{i=1}^{m} g_i \]

where $g_i \leq \gamma_i$ is received from bank $i$ at time $t = 0$

- Guarantee fund is hybrid of margin fund and default fund:
  - GF payment $g_i$ netted against bank liability (margin fund)
  - GF absorbs shortfall losses of defaulting banks (default fund)

- Banks’ shares in the guarantee fund have equal seniority
Liabilities

- Bank $i$’s net exposure to CCP

$$\Lambda_i = \sum_{j=1}^{m} L_{ji} - \sum_{j=1}^{m} L_{ij}$$

- Bank $i$’s nominal liability to the CCP (netting)

$$\hat{L}_{i0} = (\Lambda_i^- - g_i)^+$$

- CCP’s nominal liability to bank $i$

$$\hat{L}_{0i} = (1 - f)\Lambda_i^+$$

→ CCP charges a volume based fee $f$ on bank $i$’s receivables

$$f \times \Lambda_i^+$$
Bank $i$’s nominal share in the guarantee fund:

$$G_i = (\Lambda_i + g_i)^+ - \Lambda_i^+$$

**Figure:** $G_i$ and $\hat{L}_{i0}$ as functions of $\Lambda_i$
Central counterparty clearing

CCP’s nominal balance sheet at \( t = 1 \)

Denote \( G_{\text{tot}} = \sum_{i=1}^{m} G_i \) total nominal value of guarantee fund

- **Assets:** \( \gamma_0 + \sum_{i=1}^{m} g_i + \sum_{i=1}^{m} \L_i^0 \),
- **Liabilities:** \( \L_0^0 + G_{\text{tot}} + \text{nominal net worth} \left( \gamma_0 + \sum_{i=1}^{m} f \Lambda_i^+ \right) \).
Central counterparty clearing

Liability clearing equilibrium

- Fraction of external assets liquidated \((\hat{L}_{i0} \times \hat{L}_{0i} = 0)\)

\[
\hat{Z}_i = \frac{(\gamma_i - g_i - \hat{L}_{i0})^-}{P_i} \land 1
\]

- Clearing payment of bank \(i\) to CCP

\[
\hat{L}^*_i = \hat{L}_{i0} \land (\gamma_i - g_i + P_i)
\]

- Value of CCP’s total assets become

\[
\hat{A}_0 = \gamma_0 + \sum_{i=1}^{m} g_i + \sum_{i=1}^{m} \hat{L}^*_i
\]

- Clearing payment of CCP

\[
\hat{L}^*_0 = \hat{L}_0 \land \hat{A}_0
\]

- Bank \(i\) receives (proportionality rule)

\[
\hat{L}^*_{0i} = \frac{\hat{L}_{0i}}{\hat{L}_0} \times \hat{L}^*_0
\]
Liquidation of the guarantee fund at $t = 2$

- Guarantee fund = first layer, prior to nominal net worth

$$G_{tot}^* = G_{tot} \wedge \left( \hat{A}_0 - \hat{L}_0^* - \gamma_0 - \sum_{i=1}^{m} f \wedge_i^+ \right)^+$$

- Bank $i$ receives (proportionality rule)

$$G_i^* = \frac{G_i}{G_{tot}} \times G_{tot}^*$$
Central counterparty clearing

Terminal net worth

- **CCP**
  \[
  \hat{C}_0 = \hat{A}_0 - \hat{L}_0 - G^*_{\text{tot}}
  \]

- **Bank } i\text{’s assets**
  \[
  \hat{A}_i = \gamma_i + \hat{Z}_i P_i + (1 - \hat{Z}_i) Q_i + \frac{\hat{L}_{0i}}{\hat{L}_0} \times \hat{L}^*_0 + G^*_i - g_i
  \]

- **Bank } i\text{’s net worth**
  \[
  \hat{C}_i = \hat{A}_i - \hat{L}_{i0}
  \]

- **Shortfall of CCP and banks becomes**
  \[
  \hat{C}^-_i = \hat{L}_i - \hat{L}^*_i
  \]
Lemma: The aggregate surplus with CCP depends on clearing mechanism only through implied liquidation losses:

\[
\sum_{i=0}^{m} \hat{C}_i^+ = \sum_{i=0}^{m} \gamma_i + \sum_{i=1}^{m} Q_i - \sum_{i=1}^{m} \hat{Z}_i(Q_i - P_i).
\]
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Independence from fee and guarantee fund policy

Write \( g = (g_1, \ldots, g_m) \).

Lemma:
- Number of liquidated assets \( \hat{Z}_i \) does not depend on \( (f, g) \)
- Shortfall of bank \( i \) does not depend on \( (f, g) \)

\[
\hat{C}_i^- = (\Lambda_i + P_i + \gamma_i)^-
\]
- Aggregate surplus does not depend on \( (f, g) \)
Ex post effects of central counterparty clearing

Scope

- Compare financial network with and without CCP
- **Convention:** For comparison we set

\[
C_0 = \gamma_0
\]
Ex post effects of central counterparty clearing

**CCP ex post effects**

**Theorem:**

The CCP reduces

- liquidation losses $\hat{Z}_i \leq Z_i$
- bank shortfalls (bankruptcy cost) $\hat{C}_i^- \leq C_i^-$

The CCP improves

- aggregate terminal bank net worth $\sum_{i=1}^{m} \hat{C}_i \geq \sum_{i=1}^{m} C_i$
- aggregate surplus

$$\sum_{i=0}^{m} \hat{C}_i^+ = \sum_{i=0}^{m} C_i^+ + (Q_i - P_i) \sum_{i=1}^{m} (Z_i - \hat{Z}_i) \geq 0$$

The CCP imposes shortfall risk $\hat{C}_0^- \geq 0$
CCP impact on banks’ net worth decomposition

**Theorem:** Difference in net worth of bank $i$ is decomposed in

$$\hat{C}_i - C_i = T_1 + T_2 + T_3$$

corresponding to

- **counterparty default:**
  $$T_1 = -\frac{\Lambda_i^+}{\sum_{i=1}^{m} \Lambda_i} \hat{C}_0^+ + \sum_{j=1}^{m} (L_{ji} - L_{ji}^*)$$

- **liquidation loss:**
  $$T_2 = (Z_i - \hat{Z}_i)(Q_i - P_i) \geq 0$$

- **fees and losses in guarantee fund:**
  $$T_3 = -f \Lambda_i^+ - \frac{G_i}{G_{tot}} (G_{tot} - G_{tot}^*) \leq 0$$
Figure: Expected differences in stand-alone risk components with and without CCP as functions of guarantee fund contribution $g$. Number of banks is $m = 14$. CCP equity is $\gamma_0 = 5 \times 10^9$. Fee is $f = 2\%$. 
Systemic risk and incentive compatibility

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Systemic risk measure

- Write $\mathbf{C} = (C_0, \ldots, C_m)$ and $\hat{\mathbf{C}} = (\hat{C}_0, \ldots, \hat{C}_m)$
- Generic coherent risk measure $\rho(X)$
- Aggregation function, $\alpha \in [1/2, 1]$,
  \[
  A_\alpha(\mathbf{C}) = \alpha \sum_{i=0}^{m} C_i^- - (1 - \alpha) \sum_{i=0}^{m} C_i^+
  \]
  
  bankruptcy cost \hspace{5cm} tax benefits

- Systemic risk measure (Chen, Iyengar, and Moallemi 2013)
  \[
  \mathcal{R}(\mathbf{C}) = \rho(A_\alpha(\mathbf{C}))
  \]
Impact on aggregation function

**Lemma:**

\[ A_\alpha(\hat{C}) - A_\alpha(C) = \alpha\hat{C}_0 - \Delta_\alpha \]

where

\[ \Delta_\alpha = \alpha \sum_{i=1}^{m} (C_i - \hat{C}_i) + (1 - \alpha)(Q - P) \sum_{i=1}^{m} (Z_i - \hat{Z}_i) \]

is nonnegative, \( \Delta_\alpha \geq 0 \), and does not depend on \((f, g)\). Hence

\[
R(\hat{C}) - R(C) = \rho \left( A_\alpha(\hat{C}) \right) - \rho \left( A_\alpha(C) \right) \leq \rho \left( A_\alpha(\hat{C}) - A_\alpha(C) \right) \\
\leq \alpha \rho \left( \hat{C}_0 \right) + \rho(-\Delta_\alpha)
\]

with equality if \( \rho(X) = \mathbb{E}[X] \).
Theorem: The CCP reduces systemic risk, $\mathcal{R}(\hat{C}) < \mathcal{R}(C)$, if

$$\alpha \rho \left( \hat{C}_0^- \right) < -\rho \left( -\Delta \alpha \right)$$

where

$$\Delta \alpha = \alpha \sum_{i=1}^{m} \left( C_i^- - \hat{C}_i^- \right) + (1 - \alpha) \sum_{i=1}^{m} \left( Z_i - \hat{Z}_i \right) \left( Q_i - P_i \right) \geq 0$$

does not depend on $(f, g)$.

\[^1\] If and only if for $\rho(X) = \mathbb{E}[X]$
Acceptable equity, fee, and guarantee fund policies

- CCP and banks are risk neutral
- Utility function = expected surplus $\mathbb{E}[C_i^+]$
- Policy $(\gamma_0, f, g)$ is incentive compatible if
  \[
  \mathbb{E} [\hat{C}_i^+] \geq \mathbb{E} [C_i^+] \quad \forall i = 0 \ldots m. 
  \]
- Policy $(\gamma_0, f, g)$ is acceptable if incentive compatible and
  \[
  \mathcal{R}(\hat{C}) \leq \mathcal{R}(C)
  \]
Symmetric case

**Assumption:** \( \gamma_i \equiv \gamma, \ g_i \equiv g, \) and

\[
(Q_i, P_i, \{L_{ij}\}_{j=1}^m, \{L_{ji}\}_{j=1}^m), \quad i = 1 \ldots m
\]

is exchangeable.

**Theorem:**

- Policy \((\gamma_0, f, g)\) incentive compatible if and only if

\[
\gamma_0 \leq \mathbb{E} \left[ \hat{C}_0^+ \right] \leq \gamma_0 + \sum_{i=1}^m \mathbb{E} \left[ (Z_i - \hat{Z}_i) (Q_i - P_i) \right]
\]

- Existence theorem for acceptable policies
- Every acceptable policy is Pareto optimal
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Simulation study

Parameters

- Symmetric CDS inter dealer network based on BIS 2010 data
- gross market value $W = 1\text{tn}$
- $m = 14$ banks
- $\gamma_i = \gamma = 10\text{bn}$
- $Q_i = Q = 11\text{bn}$, $P_i = Q_i/2$
- CCP: $\gamma_0 = 5\text{bn}$, fee $f = 2\% \approx 1\text{bp}$ of notional
- Systemic risk measure $\mathcal{R}(C) = \mathbb{E}[A_{0.9}(C)]$
- Model:

$$W = \sum_{i \neq j} \mathbb{E}[|X_{ij}|], \quad X_{ij} \text{ i.i.d. } N(0, \sigma)$$

$$L_{ij} = (|X_{ij}| - |X_{ji}|)^+$$
∃ acceptable and incentive compatible policies: $g_{\text{reg}}, g_{\text{comp}} < g_{\text{mon}}$

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Systemic Risk and CCP Design

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Incentive compatible utility indifference curves and systemic risk zero line in \((f, g)\)
Simulation study

Systemic risk as functions of $g$ for $m = 14$ vs. 10 banks

$g_{\text{reg}}$ doubles: concentration risk matters!
Systemic risk, banks’ and CCP utility as functions of $g$, $\gamma_0$
Hybrid vs. pure (default) guarantee fund

Pure guarantee fund: not netted against liabilities, $L_{i0} = \Lambda_i^−$.

Assets remaining with bank $i$, $\gamma_i - g_i + P_i$, form margin fund.

Systemic risk improvement is limited, while banks have no incentive compatibility: $g_{\text{mon}} < g_{\text{reg}}$. 
Conclusion

- General financial network setup with and without CCP
- CCP improves aggregate surplus due to lower liquidation losses
- CCP reduces banks’ bankruptcy cost
- CCP introduces tail risk, and may increase systemic risk
- Find exact condition for systemic risk reduction
- Simulation study illustrates range of acceptable CCP equity, fee, and guarantee fund policies
- Hybrid guarantee fund design greatly improves banks incentives to join CCP