Risk-Sharing and the Creation of Systemic Risk

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The last several decades have seen explosive growth in financial innovation.

New contracts were designed to facilitate risk sharing ((eg.) markets in securitization, credit derivatives).

Simultaneously, there has been a fall in bank liquidity holdings, and increased financial fragility.

Alessandri and Haldane - Bank capital ratios have fallen over the last several decades.
In a world without risk-sharing, agents choose to hold sufficient liquidity to withstand both idiosyncratic and aggregate shocks.

Risk-sharing arrangements such as clearinghouses are most effective in hedging against (uncorrelated) idiosyncratic shocks.

With risk-sharing, agents increase risky investment, while lowering liquidity in the system.

Risk sharing can improve welfare and lead to efficient holdings of liquidity.

However, in the presence of a Lender of Last Resort, risk sharing can also lead to liquidity shortfalls and increased systemic risk.
This paper

- Builds a model of risk-sharing leading to increased systemic risk.
- Intuition expressed in one-bank and many-bank framework.
- 1 bank model - optimal risk taking for a bank in autarky.
- Many bank model.
  - Banks share risks and co-insure each other by forming a mutually owned clearinghouse.
  - Banks are better off ex-ante and hold first-best levels of liquidity.
  - In the presence of a Lender of Last Resort, banks are still better off, but there is a liquidity shortfall and they are more vulnerable to bad aggregate shocks through clearinghouse failure.
There are three periods and two assets - a risky and a riskfree asset.

- Risky asset returns \( R > 1 \).
- Risky project may need refinancing with probability \( \alpha \).
- The riskfree portfolio can fund this refinancing requirement.
- There is only one bank, so there is no pooling of risk.
Model - One Bank Setting

Refinancing required

Prob $\alpha$

Refinancing not required

Prob $1-\alpha$

Refinanced
Payoff = $R$

Not Refinanced
Payoff = 0

Time 0

Time 1

Time 2
Let bank invest amount $\ell$ in riskless asset and $(1 - \ell)$ in risky project.

Bank optimizes over $\ell$.

If $\ell < 1/2$, refinancing of risky project not possible.

$$E\Pi(\ell) = \ell + (1 - \alpha)(1 - \ell)R$$

If $\ell > 1/2$, bank always refinances if shock hits.

$$E\Pi(\ell) = \ell + (1 - \ell)R - \alpha(1 - \ell)$$
Bank chooses $\ell$ to optimize over expected payoff described above.

Investment in riskless asset ($\ell$) is governed by $\alpha$ and $R$ and is intuitive.

When refinancing is unlikely, bank chooses maximal risky investment.

But optimally self-hedges when refinancing is more probable.

\[
\alpha < \frac{R - 1}{2R - 1} \implies \ell = 0
\]

\[
\alpha \in \left[ \frac{R - 1}{2R - 1}, R - 1 \right] \implies \ell = \frac{1}{2}
\]

\[
\alpha > R - 1 \implies \ell = 1
\]
Now, we model several banks sharing risk by owning a clearinghouse.

A clearinghouse allows mutualization of returns and risk, and allows transfers from successful to failed banks.

Banks choose amount of margin they deposit into clearinghouse, and liquidity carried over.

If clearinghouse fails, insolvent banks sell assets in fire sale. Solvent banks can pledge future earnings to purchase these assets.
Continuum of banks (of measure 1) pay premium \( k \) to the clearinghouse.

Bank \( i \) is exposed to an idiosyncratic shock \( (\epsilon_i \sim N(0, 1)) \) and an aggregate shock \( (a \sim N(0, 1)) \).

Total shock to bank \( i \), \( z_i = \sqrt{\rho} a + \sqrt{1 - \rho} \epsilon_i \)

Bank \( i \) needs refinancing if \( z_i < c \); it is bailed out if clearinghouse survives; \( \alpha = N(c) \) is the autarkic probability of failure.
The clearinghouse collects up-front margin and can make capital calls on solvent banks.

Clearinghouse can call on liquidity held by banks, and pledge fraction $\tau$ of banks’ future revenues to make transfers from solvent to insolvent banks.

Size of the transfer is contingent on the number of failures.

The clearinghouse becomes insolvent when the required bailout exceeds available revenue, and a fire sale takes place.
Clearinghouse: A Co-Insurance Model

- Banks contribute margin \( k \) to the clearinghouse and carry over liquidity \( \ell \).
- Let \( f \) be the number of banks requiring refinancing \( \implies \) total refinancing need \( = f(1 - k - \ell) \).
- Revenue of banks not requiring refinancing
  \[ = R(1 - f)(1 - k - \ell). \]
- Define \( \eta(f) \) as the portion of revenue transferred by successful banks to refinance failed firms.
  \[ \eta(f) = \frac{f(1 - k - \ell) - k - \ell}{\tau R (1 - k - \ell)(1 - f)} \]
- Clearing house fails if
  \[ \eta(f) > 1 \iff f > \frac{\tau R (1 - k - \ell) + k + \ell}{(\tau R + 1)(1 - k - \ell)} \iff a < a_0(k, \ell) \]
  \[ a_0(k, \ell) = \frac{c - \sqrt{1 - \rho N^{-1}[\tau R/(1 + \tau R) + (k + \ell)/(1 + \tau R)(1 - k - \ell)]}}{\sqrt{\rho}} \]
Clearinghouse failure and fire sale

- If the clearinghouse fails, margin in the clearinghouse is rebated (randomly) to insolvent banks to bail them out.
- Those banks which do not get bailed out sell assets in fire sale, which is then purchased by solvent banks.
- Solvent banks take prices as given, and submit demand functions to purchase assets.
- Solvent banks can use liquidity carried over and pledge fraction $\tau$ of future payoffs. These banks only generate a return of $(R - \Delta)$ from acquired assets.
As before, denote the number of banks that have failed by $f(k, \ell)$.

Clearinghouse uses margins to bail out $g(k, \ell)$ banks before declaring insolvency. $g(k, \ell) = k/(1 - k - \ell)$.

$y(p, k, \ell)$ is demand function submitted by each bank in the fire sale.

Market clearing:

$$y(p, k, \ell)[1 - f(a)] = (1 - k - \ell)[f(a) - g(k, \ell)]$$

Also,

$$y(p, k, \ell) = \frac{(\ell + \tau R(1 - k - \ell) - y(p, k, \ell))^+}{p}$$

where $x^+ = \max(x, 0)$
Fire sale demand functions and prices

\[ p(k, \ell) = \max(0, -1 + [\ell + \tau R(1 - k - \ell)] \frac{(1 - f(a))}{(1 - k - \ell)(f(a) - g(k, \ell))} ) \]

- Fire sale price \( p(k, \ell) \) decreases with number of failures \( f \).
- If number of failures is low enough \( (f < f_1) \), price is \( (R - \Delta - 1) \), and acquiring banks do not make a profit on purchased assets.
- If number of failures is high \( (f > f) \), price is zero

<table>
<thead>
<tr>
<th>Region</th>
<th>Fire sale price</th>
<th>Fire sale demand</th>
<th>Profits (for acquiring firms)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f \in [f_0, f_1] )</td>
<td>( R - \Delta - 1 )</td>
<td>( y(R - \Delta - 1, k, \ell) )</td>
<td>0</td>
</tr>
<tr>
<td>( f \in [f_1, f] )</td>
<td>( p(k, \ell) )</td>
<td>( y(p, k, \ell) )</td>
<td>( (R - \Delta - 1 - p)y(p, k, \ell) )</td>
</tr>
<tr>
<td>( f \in [f, 1] )</td>
<td>0</td>
<td>( y(0, k, \ell) )</td>
<td>( (R - \Delta - 1)y(0, k, \ell) )</td>
</tr>
</tbody>
</table>
Equilibrium:

- Clearinghouse sets margin level, \( k \), paid by each bank. Banks choose liquidity \( \ell \) taking as given liquidity \( \bar{\ell} \) carried over by other banks.
- We focus on symmetric equilibria where all banks carry the same liquidity.
- The equilibrium quantities \( k^* \) and \( \ell^* \) solve the following system:

\[
\ell^*(k) = \arg \max_{\ell} \mathbb{E}\Pi(k, \ell, \bar{\ell}) \quad \text{and} \quad \ell^* = \bar{\ell}
\]

\[
k^* = \arg \max_{k} \mathbb{E}\Pi(k, \ell^*(k), \ell^*(k))
\]
Properties of Equilibrium:

- Expected profits $\Pi(k, \ell, \bar{\ell})$ is linear in $\ell$.

$$\Pi(k, \ell, \bar{\ell}) = \alpha_0(k, \bar{\ell}) + \alpha_1(k, \bar{\ell})\ell$$

There is a bang-bang solution to the bank’s choice of $\ell$.

**Case 1:** $\alpha_1(k, \bar{\ell}) < 0$. Then, $\ell^*(k, \bar{\ell}) = 0$. For a symmetric equilibrium to exist, $\bar{\ell} = 0$, and for consistency, $\alpha_1(k, 0) < 0$. This situation corresponds to the case where the bank carries over no liquidity from time 0.

**Case 2:** $\alpha_1(k, \bar{\ell}) = 0$ Bank is indifferent to the choice of $\ell$. For a symmetric equilibrium, the bank chooses $\ell^*(k, \bar{\ell}) = \bar{\ell}$.

**Case 3:** $\alpha_1(k, \bar{\ell}) > 0$ In this case, the bank chooses $\ell^*(k, \bar{\ell}) = 1$ and in equilibrium, $\ell^* = \bar{\ell} = 1$. There is no systemic risk or investment in the risky asset and the clearing house never fails.
Properties of Equilibrium:

- For every \( \rho \) and for every \( k \), there exists a unique \( \ell^*(k) \) such that \( (k, \ell^*(k)) \) is an equilibrium.
- For every \( \rho \), \( \ell^*(k^*) = 0 \), where \( k^* = \arg \max_k \mathbb{E} \Pi(k, \ell^*(k)) \).
- In the absence of the clearinghouse, banks choose to carry over enough liquidity to always be able to refinance themselves if required, i.e. \( \tilde{\ell} = 1/2 \).
- In the absence of the clearinghouse, profit is the same as under autarky, and equals \( \Pi^{\text{aut}} = (1 + R - \alpha)/2 \).
- \( \mathbb{E} \Pi(k^*, 0) > \Pi^{\text{aut}} \), so expected payoffs under the clearinghouse always dominates autarky.
- In the presence of the clearinghouse, there is always systemic risk.
So far, we have focused on the outcomes of symmetric equilibria where all banks carry same liquidity $\ell^*$.  

We can generalize framework to allow for asymmetric equilibria, where there are $n$ “types” of banks. 

In particular, let $w_i$ banks carry liquidity $\bar{\ell}_i$, where $\sum_{i=1}^n w_i = 1$. 

Bank chooses liquidity $\ell$ taking as given weights ($w_i$) and liquidity holdings ($\bar{\ell}_i$). 

Claim: For any asymmetric equilibrium $(\mathbf{w}, \bar{\ell})$, $\exists$ a unique symmetric equilibrium $\ell^*(\mathbf{w}, \bar{\ell})$ which delivers the same profits and systemic risk for all the banks.
Equilibrium under coinsurance and fire sale
- Margins rise with correlation, converging to autarkic levels.
- Systemic risk first rises with correlation, and then decreases.
- An increase in $\tau$, the amount of future income that can be pledged in a fire sale increases systemic risk for all values of $\rho$.
- Under autarky, banks continue to self-hedge and there is no aggregate risk.
Dependence of profits and systemic risk on aggregate shock
Regulation and First-best outcomes

- How efficient is the clearinghouse in raising profits for banks? Is it possible for a regulator to do better?

- Regulator sets margin \((k^{FB})\) and liquidity \((\ell^{FB})\) levels for all banks to maximize expected profits.

\[
(k^{FB}, \ell^{FB}) = \arg \max_{k,\ell} \mathbb{E}\Pi^{FB}(k, \ell)
\]

- For every value of correlation \(\rho\), \(k^{FB}(\rho) = k^*(\rho)\) and \(\ell^{FB}(\rho) = \ell^*(k^*, \rho) = 0\)

- For every value of \(\rho\), \(\mathbb{E}\Pi^{FB}(\rho) = \mathbb{E}\Pi(\rho)\), and systemic risk is as large under the first-best outcome as under equilibrium.
Without external intervention, the clearinghouse is able to deliver first-best welfare and liquidity outcomes.

In practice, however, there is a Lender of Last Resort that injects liquidity into a clearinghouse in the case of an emergency.

The Federal Reserve extended credit to the CME following the 1987 crash.

We extend the model allowing for the presence of a Lender of Last Resort.

This can lead to liquidity shortfalls and lower welfare.
Assume that the Lender of Last Resort (LoLR) injects funds $g(a)$ into the economy at cost $c(g) = a_{gov}g^2$.

The LoLR refineses $g(a)/(1 - k - \bar{\ell})$ banks, and the total benefit to the economy through liquidity injections is $\Delta g(a)$.

The maximal LoLR injection $g^*$ satisfies $c'(g^*) = \Delta$.

Clearinghouse and banks take LoLR injection as given, and choose margins $k^*$ and liquidity $\bar{\ell}$.

There is a fire sale if not all banks can be refinanced even if $g = g^*$. 
Assume that the LoLR injects $g = g^*$ if $a < a_g$.

If $a \in (a_g, a_0)$, the clearinghouse fails, but the LoLR injects $g < g^*$ and there is no fire sale.

If $(a > a_0)$, then the clearinghouse survives and $g = 0$.

Welfare is given by

$$W(k, \ell, \bar{\ell}) = \mathbb{E} \Pi(k, \ell, \bar{\ell}) - c(g^*)P(a < a_g(k, \bar{\ell})) - \int_{a_g}^{a_0} c(g(a))\phi(a)da$$

Let us define

$$k_{pub}^* = \arg \max_k W(k, \ell, \bar{\ell}(k)) ; \quad k_{pvt}^* = \arg \max_k \mathbb{E} \Pi(k, \ell, \bar{\ell}(k))|g^*$$

$$k_{eqm}^* = \arg \max_k \mathbb{E} \Pi(k, \ell, \bar{\ell}(k))|g^* = 0$$
Outcomes with Lender of last resort

- **Margin**
  - k*(pub)
  - k*(pvt)
  - k*(eqm)
  - k*(aut)

- **Systemic Risk**
  - S.risk(pub)
  - S.risk(pvt)
  - S.risk(eqm)
  - S.risk(aut)

- **Welfare**
  - W(pub)
  - W(pvt)
  - W(eqm)
  - W(aut)
Conclusions

- This paper builds a model showing how risk sharing can increase systemic risk in a framework where there are several banks mutually owning a clearinghouse.
- However, the presence of risk sharing while increasing systemic risk can also generate first-best outcomes.
- In the presence of Lender of Last Resort provisions, however, a clearinghouse can lead to inefficiently high systemic risk and lower welfare.
- This provides a rationale for regulation in the form of margin requirements for clearinghouses.