Quality of tick values

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Counting moves

- Robert and Rosenbaum (2009)

\[ \hat{\eta}_1 = \frac{1}{2} \frac{N_{C,k}}{N_{A,k}} \]

- Count continuations and alternations
- \( 2 \cdot \eta \cdot \alpha \) is a natural spread
Two steps forward, one step back

- Estimate time to reach frontiers of UZ
- UZ had size $2 \cdot \eta \cdot \alpha$ and is centered at mid-ticks $(P_i + \frac{\alpha}{2})$
As time goes by

- Durations ($\Delta t$ to next price change) are different
- But average durations can be estimated: $Dur \approx 2 \cdot \eta \cdot \left(\frac{\alpha}{\sigma S}\right)^2$
- Then number of price changes is inversely proportional to $2 \cdot \eta \cdot \left(\frac{\alpha}{\sigma S}\right)^2$
Informed trading

- Two FX contracts in Brazil, same tick size, underlying and settlement, but different size
- Open contracts / traded volume very different $\Rightarrow$ informed traders
- Trades / Price changes: 2.71 DOL, 2.97 WDO
Imbalance

- Predictive power of imbalance (trade as expected - trade as not expected)
- Smaller $\eta$ means imbalance is more predictive
- Equivalent to microprice leaving earlier a smaller UZ
Fight or flight

- Depletions by cancel or trade
- Smaller $\eta$ means more depletions by trade, not by cancel
Once depletions by trade happened, smaller $\eta$ means more fills by the original side.

Once a fill happens, smaller $\eta$ means more depletions on the opposite side.
What is being measured?

- Market makers hope to earn the spread but fear the informed trader
- Top of the book valuable but total size of best level important (buffer against informed trading)
- Summarize $VTB = 1 - 2 \cdot \eta$
Futures

- Availability of spot for price formation
- Leverage and liquidity might bring diverse ecology of traders
- Global futures exchanges - liquidity over a large period of the day
- But how to choose size of contract and tick size?
Shakespeare in 160 milliseconds

- Why choose large ticks?

- Hamlet => Macbeth

- Avoid excessive quotes with low amount of information
Factors to consider

1. Spread of underlying
2. Time-weighted average spread
3. Average price change (related to $\lambda_i$)
4. $\eta$ (assuming the factors above validate the assumption of a large tick asset)
5. Average cost curve
6. Duration (incorporates volatility and relative tick size)
7. Direct costs of trading (exchange fees)
## Averages

<table>
<thead>
<tr>
<th>Product</th>
<th>Tick</th>
<th>$\bar{\delta P}$</th>
<th>$\zeta$</th>
<th>Volume</th>
<th>M</th>
<th># $\delta P$</th>
<th>Calc</th>
<th>$\eta$</th>
<th># S=</th>
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I’ve seen the future

- Predict next $\eta$
I’ve seen the future

- Predict durations given tick value and spot, volatility, $\eta$

**Chart:**

- Realized Duration ($y$) vs Predicted Duration ($x$): EUR
  - Status: prior, after

- Realized Duration ($y$) vs Predicted Duration ($x$): CAD
  - Status: prior, after
I’ve seen the future

- Predict number of price changes given durations (tick value and spot, volatility, $\eta$)
I've seen the future

- Predict number of trades given number of price changes (ratio $k$)
- After: EUR 2.5, CAD 3.2
Looking behind the curtain

- Predictive power of imbalance
Looking behind the curtain

- Depletions

![Depletion Graphs](image-url)
Looking behind the curtain

- Depletions and fills

![Graphs showing depletions and fills](image)
Posted liquidity

<table>
<thead>
<tr>
<th>Currency</th>
<th>Tick</th>
<th>Bid</th>
<th>Ask</th>
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<tbody>
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All together now

- \( \frac{V_1}{V_0} = \frac{\beta_{V,M,1}}{\beta_{V,M,0}} \cdot \frac{M_1}{M_0} = \frac{\beta_{V,M,1}}{\beta_{V,M,0}} \cdot \frac{k_1}{k_0} \cdot \frac{\#\delta P_1}{\#\delta P_0} \)

- \( \frac{V_1}{V_0} = \frac{\beta_{V,M,1}}{\beta_{V,M,0}} \cdot \frac{k_1}{k_0} \cdot \frac{\eta_0}{\eta_1} \cdot (\frac{\alpha_0}{\alpha_1} \cdot \frac{S_1}{S_0} \cdot \frac{\sigma_1}{\sigma_0})^2 \)

Estimate ratio between volume and number of trades (\( \beta \)) using posted liquidity:

- \( \frac{\beta_{V,M,1}}{\beta_{V,M,0}} = (\frac{\alpha_1}{\alpha_0})^\gamma \)

- \( \frac{V_1}{V_0} = (\frac{S_1}{S_0} \cdot \frac{\sigma_1}{\sigma_0})^2 \cdot \frac{k_1}{k_0} \cdot \frac{\eta_0}{\eta_1} \cdot (\frac{\alpha_0}{\alpha_1})^{2-\gamma} \)
Volatile volume

\[
\frac{V_1}{V_0} = \left( \frac{11189}{11059} \cdot \frac{0.00375}{0.00438} \right)^2 \cdot \frac{2.6}{3.6} \cdot \frac{0.27}{0.37} \cdot \left( \frac{1}{0.5} \right)^{2-\gamma}
\]

\[
\frac{V_1}{V_0} = 0.75 \cdot 0.527 \cdot (2)^{2-\gamma}
\]

\[
\gamma = 1
\]

\[
\frac{V_1}{V_0} = 0.80 \text{ (realized 0.85)}
\]
Tale of the tape

- Average cost of each trade
- Group by amount traded and average
- Average results by amount over time
What this talk was about anyway?

Market design

- Exchanges need to keep all customers equally unhappy
- Tick value and $\eta$ helps to determine spread, liquidity, cost/market impact
- Presence of informed traders increases $\eta$, spreads
- Dashboard of factors to measure and monitor

What $\eta$ measures

- Not only mean reversion
- Predictive power of imbalance
- Relative proportion and sign of depletions by cancel and trade and refills
- $1 - 2 \cdot \eta$ as relative value of top of book (first place in queue)
What is next?

- Link to Queue Reactive model
- Expand model to other futures
- Even price level changes are a natural experiment
Books, papers, website