Fire sales, price-mediated contagion and systemic risk.

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Overview

1. Introduction: Price-mediated contagion and endogenous risk
2. Modelling fire sales
3. Is it relevant?
4. Monitoring
5. Comparison of fire sales and leverage targeting models
6. Conclusion
Crisis of 2007-2008: Direct contagion (e.g. counterparty credit risk or funding relations) cannot explain the magnitude and breadth of contagion, across sectors, countries and asset classes that was observed.
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**Goal:** Develop models for macro stress testing that can quantify such second round effects in a realistic and robust way. (“Stresstesting 3.0”)
Systemic stress testing

System:
- $N$ banks, $K$ illiquid asset classes, $M$ marketable asset classes
- $\rightarrow N \times K$ illiquid assets portfolio matrix (network): exposure to common shock
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Data: $N = 90$, $M = 148$, $K = 75$. 
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Mechanism:

1. **Shock** to illiquid assets
2. **Deleveraging** of marketable assets by some institutions
3. **Feedback effects** via price-mediated contagion
   - $\rightarrow$ potentially triggers more deleveraging (cascade).

Mathematically this is a discrete time non-linear dynamical system.
Questions

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- What can regulators do to monitor and mitigate this channel of contagion? (Acharya et al (2014), ECB (2013))
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Modelling fire sales
### Model balancesheet

#### Illiquid assets
- Residential mortgage exposures
- Commercial real estate exposure
- Retail exposures: Revolving credits, SME, Other
- Indirect sovereign exposures in the trading book
- Defaulted exposures
- Residual exposures

#### Marketable assets
- Corporate bonds
- Sovereign debt
- Direct sovereign exposures in derivatives
- Institutional client exposures: interbank, CCPs, ...

**Table:** Stylized representation of asset classes in bank balance sheets.  
(Data: European Banking Authority Stress Test)
A stress scenario is defined by a vector \( \mathbf{\epsilon} \in [0, 1]^K \) whose components \( \epsilon_\mu \) are the percentage shocks to asset class \( \mu \).

Gradual increase of the shock from 0% to 20%.
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Gradual increase of the shock from 0% to 20%.

Four scenarios:

1. Spanish residential and commercial real estate losses
2. Northern Europe residential losses
3. Southern Europe commercial real estate losses
4. Eastern Europe commercial real estate losses
Fire sales model

- Total value of illiquid holdings: $\Theta^i_t := \sum_{\mu=1}^{K} \Theta^i_{\mu t}$.  
- Securities: $\Pi^i_t := \sum_{\mu=1}^{M} \Pi^i_{\mu t}$.  
- Common Equity Tier 1 capital: $C^i_t$  
- Initial loss: $L^i_0 := \sum_{\mu=1}^{K} \Theta^i_{0 \mu} \epsilon^\mu$  

When a bank exceeds the leverage constraint, $\lambda^i_t > \lambda^{\text{max}}$, it engages in fire sales of magnitude $\Gamma^i_t \in [0,1]$: 

$$(1 - \Gamma^i_1^t)\Pi^i_0 + \Theta^i_0 - L^i_0 = \lambda^i_{\text{new}}$$ 

which yields in the fire sales model: 

$$\Gamma^i_1^t = C^i_0 \left( \lambda^i_0 - \lambda^i_{\text{b}} \right) \Pi^i_0 \lambda^i > \lambda^{\text{max}}.$$
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\]

which yields in the fire sales model:

\[
\Gamma^i_1 = \frac{C^i_0(\lambda^i_0 - \lambda^i_b)}{\Pi^i_0} \mathbb{1}_{\lambda^i > \lambda_{\text{max}}},
\]
The price of an asset undergoing a forced liquidation at $t$:

$$S_{t+1}^\mu = S_t^\mu \exp \left( -\delta^{-1}_\mu \sum_{j=1}^{M} \Pi_t^j \Gamma_{t+1}^j \right),$$

where the market depth

$$\delta_\mu \sim \frac{ADV_\mu}{\sigma_\mu},$$

- **ADV**: average daily volume
- **$\sigma_\mu$**: daily volatility
Modelling fire sales

Estimated market depth

![Histogram showing market depth distribution](image)
Is it relevant?
Fire sales losses and market depth
Is it relevant?

**Indirect exposures and stress test outcomes**

**Figure:** Source: EBA (public) & authors calculations.
Is it relevant?

Indirect exposures and stress test outcomes
Is it relevant?

Monitoring
Linearising the price impact function yields

\[ L_t^i \approx (1 - (1 - \alpha)\Gamma_{t+1}^i) \sum_{\mu=1}^{M} \sum_{j=1}^{N} \delta_{\mu}^{-1} \prod_{t}^{i\mu} \prod_{t}^{j\mu} \Gamma_{t+1}^{j} \]

\[ = (1 - (1 - \alpha)\Gamma_{t+1}^i) \sum_{j=1}^{N} \omega_{ij} \Gamma_{t+1}^{j} , \]

where \( \omega_{ij} := \sum_{\mu=1}^{M} \prod_{0}^{i\mu} \prod_{0}^{j\mu} \delta_{\mu}^{-1} \) is the liquidity weighted overlap of portfolios \( i \) and \( j \). This gives rise to a weighted and undirected “liquidity weighted overlap network” given by the symmetric (positive semidefinite) matrix:

\[ \Omega := \Pi D^{-1} \Pi^{\top} . \]
European banking system: liquidity weighted overlap
Figure: European banking system: Liquidity weighted overlaps. Source: EBA (public)
Figure: European banking system: Nominal overlaps. Source: EBA (public)
Figure: Source: EBA (public) & authors calculations
Figure: Threshold model: Fire sales losses as function of the initial shock and the market depth. Source: Statistics Norway.
A Systemic Vulnerability Indicator

Figure: Minimum shock required to trigger large fire sales cascades, as a function of time and market depth. Source: Statistics Norway.
A Systemic Vulnerability Indicator

Figure: Minimum shock required to trigger large fire sales cascade, average over market depths. Source: Statistics Norway.
Comparison to “leverage targeting” models
Figure: Leverage targeting response function (dashed) and two variants of the fire sales (full and circles) response functions.
Comparison of fire sales and leverage targeting models

Fire sales losses and market depth

[3D graph showing the relationship between fire sales loss (% of system equity), shock size (%), and multiplier market depth.]
Comparison of fire sales and leverage targeting models

Fire sales losses and market depth

![Fire sales losses and market depth graph](image_url)
Figure: Fire sales loss for different scenarios and different model combinations.
### Sensitivity to initial stress scenario

<table>
<thead>
<tr>
<th>Scenario combination</th>
<th>Sample correlation coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 &amp; 2</td>
<td>0.0840</td>
</tr>
<tr>
<td>1 &amp; 3</td>
<td>0.2130</td>
</tr>
<tr>
<td>1 &amp; 4</td>
<td>-0.1449</td>
</tr>
<tr>
<td>2 &amp; 3</td>
<td>-0.0509</td>
</tr>
<tr>
<td>2 &amp; 4</td>
<td>0.0394</td>
</tr>
<tr>
<td>3 &amp; 4</td>
<td>-0.0149</td>
</tr>
</tbody>
</table>

**Table:** Sample correlations between the initial loss vectors from the stress scenarios. The four stress scenarios are very different in terms of which banks are hit by the corresponding shock.
Figure: The pairwise sample correlation between the fire sales loss vectors of different scenarios as a function of the initial shock. Threshold model full lines - leverage targeting dashed lines.
Comparison of fire sales and leverage targeting models

Sensitivity to initial stress scenario

**Figure:** The evolution of the pairwise sample correlation during the fire sales cascade for a given scenario. Threshold full - leverage targeting dashed.
Conclusion
Account for fire sales losses “without fire sales model?”
Conclusions

- We presented a fire sales model in a network of institutions with common asset holdings and one-sided portfolio constraints;
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Exposure to price-mediated contagion leads to the concept of indirect exposure to an asset class → the risk of a portfolio depends on other large, leveraged and overlapping portfolios;
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Liquidity-weighted overlaps lead to a bank-level indicator that may be used for monitoring and for quantifying the contribution of a financial institution to price-mediated contagion;
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Liquidity-weighted overlaps lead to a bank-level indicator that may be used for monitoring and for quantifying the contribution of a financial institution to price-mediated contagion;

The phenomenon of fire sales calls for the collection of portfolio holdings data on a broad scale (banks and shadow banks)
Conclusions

Even with optimistic estimates of market depth, moderately large macro-shocks may trigger fire sales which then lead to substantial losses across bank portfolios, modifying the outcome of bank stress tests;
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- Contagion through fire sales cannot be accounted for simply by applying a larger macro-shock to bank portfolios;
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- Even with optimistic estimates of market depth, moderately large macro-shocks may trigger fire sales which then lead to substantial losses across bank portfolios, modifying the outcome of bank stress tests;
- Contagion through fire sales cannot be accounted for simply by applying a larger macro-shock to bank portfolios;
- Results in our model differ significantly from results obtained in “leverage targeting” models.
Thank you!
Liquidity and leverage. 

Stress tests to promote financial stability: Assessing progress and looking to the future. 

An agent-based model for financial vulnerability. 

Fire sales, price-mediated contagion and systemic risk. 
*Working Paper*.
